Single-Peakedness and Coalition-Proofness*

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Abstract

We prove that multidimensional generalized median voter schemes are coalition-proof.

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1 Introduction

It is well-known that the majority rule is not transitive. In order to guarantee transitivity we have to restrict the preferences of the voters. The first well-known restriction is single-peakedness, which was introduced by Arrow [1951] and Black [1948]. The median voter scheme over the domain of single-peaked preferences was shown to be compatible with Condorcetś rule. Moulin [1980] has introduced generalized median voter schemes over one-dimensional sets of alternatives. His paper includes, among other results, both the characterization of all strategy-proof voting schemes, and the characterization of anonymous, strategy-proof, and Paretian generalized median voter schemes. He also characterized the family of schemes which only satisfy anonymity and strategy-proofness. Border and Jordan [1983] extended generalized median voter schemes to multi-dimensional sets of alternatives. As far as we know, the latest generalization of Moulin [1980] is due to Barberá, Gul and Stacchetti [1993]. They consider generalized median voter schemes over multi-dimensional sets of alternatives. As expected, they restrict their analysis to multi-dimensional single-peaked preferences. One of their important results is that multi-dimensional generalized median voter schemes are characterized by strategy-proofness. We prove in this work that multi-dimensional generalized median voter schemes are also coalition-proof. For the notion of coalition-proofness see Bernheim, Peleg, and Whinston [1987]. Coalition-proofness may be regarded as an interesting stability property to be satisfied by voting schemes, because in many cases the voters may have the opportunity to communicate prior to vote. Therefore, generalized median voter schemes being coalition-proof means that they generate "agreements" which are immune to self-enforcing improving deviations. Peleg [1998] shows that pivotal mechanisms are not coalition-proof. We now shall explain and motivate our result.

Let N be a set of $n = 2k+1$, $k \geq 1$, voters, let B be a (finite) set of alternatives, and let P_0 be a fixed linear ordering of B. Assume that the preferences of the members of N on B are restricted to be single-peaked with respect to P_0. Then, the median voter scheme is strategy-proof and Paretian. Moreover, the median voterś peak is an outcome of a strong Nash equilibrium (with respect to the true preferences). Thus, under the foregoing assumptions, the median voter scheme is group strategy-proof. This result remains true, if we replace the median voter scheme by a generalized median voter scheme (see Moulin [1980]). However, Barberá, Sonnenschein, and Zhou [1991] show that multi-dimensional generalized median voter schemes are not coalitionally strategy-proof. In this paper we address the following problem: What is the strongest kind of group stability which is satisfied by all generalized median voter schemes? We solve the foregoing problem in Sections 4 and 5: Theorem 4.1 proves that every multi-dimensional generalized median voter scheme is coalition-proof. Furthermore, in Section 5 we give an example of a generalized median voter scheme which is not strongly coalition-proof.

We now briefly review the contents of this paper. Section 2 contains preliminary definitions and Section 3 introduces generalized median voter schemes. The proof of the coalition-proofness of multi-dimensional generalized median voter schemes is presented in Section 4. An example of a generalized median voter scheme which is not strongly coalition-proof, is given in Section 5. Finally, some remarks are contained in Section 6.

2 Definitions and Notations

A game in strategic form is a system $G = (N, (A_i)_{i\in N}, (u_i)_{i\in N})$ where N is a finite set of players; A_i, $i \in N$, is the (non-empty) set of strategies of i; and $u_i : \times_{j\in N} A_j \to R$ is the payoff function of player $i \in N$. (Here R is the set of real numbers.) Let $S \subset N$, $S \neq \emptyset$. We denote $A_S = \times_{i\in S} A_i$ and $A = A_N$. If $x \in A$ then x_S denotes the restriction of x to S.

Let $G = (N, (A_i)_{i\in N}, (u_i)_{i\in N})$ be a strategic game, let $S \subset N$, $S \neq \emptyset$, and let $x \in A$. The **reduced game** of G with respect to (w.r.t) S and x is the game $G^{S,x} = (S, (A_i)_{i\in S}, (u_i^x)_{i\in S})$, where $u_i^x(y_S) = u_i(y_S, x_{N\setminus S})$ for all $y_S \in A_S$ and $i \in S$.

Let $G = (N, (A_i)_{i\in N}, (u_i)_{i\in N})$ be a strategic game. $x \in A$ is a Nash equilibrium (NE) of G if, for every $i \in N$, $u_i(x) \geq u_i(y_i, x_{N\setminus\{i\}})$ for all $y_i \in A_i$. We now define coalition-proofness by induction on the number of players.

Definition 2.1 *(1) In a single player game G, $x \in A$ is a* **coalition-proof Nash equilibrium** *(CPNE) if and only if it is an NE.*

(2) Let $n > 1$ and assume that CPNE has been defined for games with fewer than n players. Then

(a) For any game G with n players, $x \in A$ is **self-enforcing** *if, for all $S \subset N$, $S \neq \emptyset, N$, x_S is a CPNE in the reduced game $G^{S,x}$.*

(b) For any game G with n players, $x \in A$ is a CPNE if it is self-enforcing and if there does not exist another self-enforcing strategy vector $y \in A$ such that $u_i(y) > u_i(x)$ for all $i \in N$.

Clearly, a CPNE of a game G is an NE of G. The following definition is closely related to Kaplanś definition of semi-strong equilibrium (see Kaplan [1992]).

Definition 2.2 *Let $G = (N, (A_i)_{i\in N}, (u_i)_{i\in N})$ be a strategic game and let $x \in A$. x is a* **strong** *CPNE if*

(1) x is an NE of G;

(2) for every $S \subset N$, $S \neq \emptyset$, and every NE y_S of $G^{S,x}$, there exists $i \in S$ such that $u_i(x) \geq u_i(y_S, x_{N\setminus S})$.

Clearly, a strong CPNE of G is a CPNE of G.

NEś , CPNEś and SCPNEś are ordinal concepts, that is, they are generalized in a straightforward manner to ordinal games $(N, (A_i)_{i\in N}, (p_i)_{i\in N})$, where N and A_i are defined as above and p_i is a preference (i.e. a complete and transitive binary relation) on A. If C is a set and $f : A \to C$ is an "outcome function", then every profile $(P_i)_{i\in N}$ of preferences on C induces a profile $(p_i)_{i\in N}$ of preferences on A by ap_ib iff $f(a)P_if(b)$ for all $a, b \in A$ and $i \in N$. We write $(N, (A_i)_{i\in N}, f, (P_i)_{i\in N})$ for $(N, (A_i)_{i\in N}, (p_i)_{i\in N})$.

3 Generalized Median Voter Schemes

In this section we recall some definitions of Barberá, Gul and Stacchetti [1993] which are essential for our work.

Definition 3.1 *For integers $a \leq b$, $[a, b]$ will denote the set $\{a, a+1, \ldots, b\}$. An ℓ-dimensional* **box** *B is a cartesian product of ℓ integer intervals: $B = \times_{j=1}^{\ell} B^j$ where $B^j = [a^j, b^j]$ and $a^j \leq b^j$.*

Let B be an ℓ-dimensional box. We consider B as a metric subspace of the space R^ℓ with the L_1-norm. (The L_1-norm of $\alpha \in R^\ell$ is $\|\alpha\| = \sum_{j=1}^{\ell} |\alpha^j|$.) A linear order on B is a complete (and, thus, reflexive), transitive, and antisymmetric binary relation on B. If P is a linear order on B, then $\tau(P)$ will denote the (unique) maximum of P on B.

Definition 3.2 *A linear order P on a box B is* **multi-dimensional single-peaked with bliss point** *$\alpha \in B$ if and only if (i) $\tau(P) = \alpha$, and (ii) $\beta P \gamma$ for all $\beta, \gamma \in B$ satisfying $\|\alpha - \gamma\| = \|\alpha - \beta\| + \|\beta - \gamma\|$.*

If B is an ℓ-dimensional box, then we denote by $\pi = \pi(B)$ the set of all single-peaked preferences with bliss point in B. Let B be an ℓ-dimensional box and let $N = \{1, \ldots, n\}$ be a (finite) set of players.

Definition 3.3 *A* **social choice function** *is a map $\varphi : \pi^N \to B$. A social choice function φ is a* **voting scheme** *if there exists a function $f : B^N \to B$ such that*

$$\varphi(P_1, \ldots, P_n) = f(\tau(P_1), \ldots, \tau(P_n)) \quad \textit{for all} \quad (P_1, \ldots, P_n) \in \pi^N$$

(f will also be called a voting scheme).

We shall be interested in the following class of voting schemes. First we need an auxiliary definition.

Definition 3.4 *Let $B = [a, b]$ be a one-dimensional box and $N = \{1, \ldots, n\}$. A* **left-coalition system** *on B is a correspondence $W : B \to 2^N$ satisfying the following conditions:*

(1) If $\xi \in B$, $C \in W(\xi)$, and $D \supset C$, then $D \in W(\xi)$;

(2) If $\xi, \eta \in B$ and $\xi < \eta$, then $W(\xi) \subset W(\eta)$ and

(3) $W(b) = 2^N$.

Left-coalition systems induce voting schemes in a natural way. For each $\tilde{\alpha} = (\alpha_1, \ldots, \alpha_n) \in B^N$ and $\xi \in B$, let $C(\tilde{\alpha}, \xi) = \{i \in N | \alpha_i \leq \xi\}$ be the coalition to the left of ξ.

Definition 3.5 *Let $B = [a, b]$ be an integer interval and let $W(\cdot)$ be a left-coalition system on B. The voting scheme $f : B^N \to B$, defined as follows:*

$$f(\tilde{\alpha}) = \min\{\xi | C(\tilde{\alpha}, \xi) \in W(\xi)\} \quad \textit{for all } \tilde{\alpha} \in B^N$$

is called the **generalized median voter scheme** *(GMVS) induced by $W(\cdot)$. When $B = \times_{j=1}^{\ell} B^j$ is an ℓ-dimensional box, the voting scheme $f : B^N \to B$ is a GMVS if $f = (f^1, \ldots, f^\ell)$ and each f^j is the GMVS induced by some left-coalition system $W^j(\cdot)$ on B^j.*

4 GMVS's are Coalition-Proof

Let B be an ℓ-dimensional box, let $N = \{1, \ldots, n\}$, and let $f : B^N \to B$ be a GMVS. For $\tilde{P} = (P_1, \ldots, P_n) \in \pi^N$ we consider the strategic game

$$G(f; P_1, \ldots, P_n) = (N; B, \ldots, B; f; P_1, \ldots, P_n).$$

Here B is the set of strategies of player $i \in N$; f is the outcome function; and $P_1, \ldots, P_n$ are the preferences of the players on the outcome space. f is **coalition-proof** if for every $\tilde{P} = (P_1, \ldots, P_n) \in \pi^N$, the n-tuple $\tilde{\alpha} = \tilde{\alpha}(\tilde{P}) = (\tau(P_1), \ldots, \tau(P_n))$ is a CPNE of $G(f; P_1, \ldots, P_n)$.

Theorem 4.1 *Every GMVS is coalition-proof.*

Proof. We shall prove our claim by induction on the number of players n.

Step 1. $n = 1$.

Let $B = \times_{j=1}^{\ell} B^j$ be an ℓ-dimensional box and let $f : B \to B$ be a GMVS. If $P \in \pi(B)$ then $\tau(P)$ is a dominant strategy in $G(f; P) = (N; B; f; P)$, because f is strategy-proof. Hence $\tau(P)$ is an NE of $G(f; P)$.

Assume now that every GMVS with k players, $1 \leq k < n$, is coalition-proof. Let $N = \{1, \ldots, n\}$, let $B = \times_{j=1}^{\ell} B^j$ be an ℓ-dimensional box, let $W^j : B^j \to 2^N$ be a left-coalition system on B^j, $j = 1, \ldots, \ell$, and let $f : B^N \to B$ be the GMVS which is induced by $W^j(\cdot)$, $j = 1, \ldots, \ell$. Furthermore, let $P_1, \ldots, P_n \in \pi(B)$, and $\alpha_i = \tau(P_i)$, $i = 1, \ldots, n$. We shall prove that $\tilde{\alpha} = (\alpha_1, \ldots, \alpha_n)$ is a CPNE of $G(f; P_1, \ldots, P_n)$.

Step 2: $\tilde{\alpha}$ is self-enforcing.

For each $S \subset N$, $S \neq \emptyset, N$, and each $j = 1, \ldots, \ell$, define the (reduced) left-coalition system $W^j_{S,\tilde{\alpha}}$ on B^j by

$$T \in W^j_{S,\tilde{\alpha}}(\xi) \Leftrightarrow T \cup \{i \in N \setminus S \mid \alpha^j_i \leq \xi\} \in W^j(\xi)$$

for all $T \subset S$ and all $\xi \in B^j$. As the reader may easily verify $W^j_{S,\tilde{\alpha}}$ is a left-coalition system on B^j (w.r.t. the set of players S). Denote by $f^{S,\tilde{\alpha}}$ the GMVS which is induced by $W^j_{S,\tilde{\alpha}}$, $j = 1, \ldots, \ell$. Then $G(f^{S,\tilde{\alpha}}; (P_i)_{i \in S}) = (S; B^S; f^{S,\tilde{\alpha}}; (P_i)_{i \in S})$ is the reduced game of $G(f; P_1, \ldots, P_n)$ w.r.t. S and $\tilde{\alpha}$. By the induction hypothesis $\tilde{\alpha}_S = (\alpha_i)_{i \in S}$ is a CPNE of $G(f^{S,\tilde{\alpha}}; (P_i)_{i \in S})$. Because this is true for each proper subset of N, $\tilde{\alpha}$ is self-enforcing.

Step 3: $\tilde{\alpha}$ is a CPNE.

Assume, on the contrary, that $\tilde{\alpha}$ is not a CPNE. Then, there exists $\tilde{\beta} \in B^N$ such that (i) $\tilde{\beta}$ is self-enforcing (in the game $G(f; P_1, \ldots, P_n)$), and $f(\tilde{\beta}) \neq f(\tilde{\alpha})$; and (ii) $f(\tilde{\beta}) P_i f(\tilde{\alpha})$ for $i = 1, \ldots, n$. We denote $s = f(\tilde{\alpha})$ and $t = f(\tilde{\beta})$. Let $s = (\xi^1, \ldots, \xi^\ell)$ and $t = (\eta^1, \ldots, \eta^\ell)$. We distinguish the following possibilities.

(4.1) There exists $m \in \{1, \ldots, \ell\}$ such that $\xi^m < \eta^m$. Let $Q = \{i \in N \mid \alpha^m_i \leq \xi^m$ and $\beta^m_i > \xi^m\}$. Q is non-empty because $\xi^m < \eta^m$. Without loss of generality $Q = \{1, \ldots, r\}$ and $\alpha^m_1 \leq \ldots \leq \alpha^m_r$. Now replace sequentially, in $\beta^m = (\beta^m_1, \ldots, \beta^m_n)$, β^m_i by α^m_i, $i = 1, \ldots, r$. There exists k, $1 \leq k \leq r$ such that $f^m(\alpha^m_1, \ldots, \alpha^m_{k-1}, \beta^m_k, \ldots, \beta^m_n) = \eta^m$ and $f^m(\alpha^m_1, \ldots, \alpha^m_k, \beta^m_{k+1}, \ldots, \beta^m_n) = \zeta < \eta^m$. By the choice of k, $\alpha^m_k \leq \zeta$. Thus, all the members of $Q^* = \{1, \ldots, k\}$ strictly prefer $\alpha^m \mid Q^*$ to $\beta^m \mid Q^*$ at $\tilde{\beta}$ ($\alpha^m \mid Q^* = (\alpha^m_i \mid i \in Q^*)$ etc.). That is, Q^* can improve upon $\tilde{\beta}$ by playing $(\alpha^m \mid Q^*, \beta^{-m} \mid Q^*)$, where $\beta^{-m} = (\beta^j \mid j \in \{1, \ldots, \ell\} \setminus \{m\})$.

(4.2) There exists $m \in \{1, \ldots, \ell\}$ such that $\eta^m < \xi^m$. Let $Q = \{i \in N \mid \alpha^m_i \geq \xi^m$ and $\beta^m_i < \xi^m\}$. Clearly, $Q \neq \emptyset$. Without loss of generality $Q = \{1, \ldots, r\}$ and $\alpha^m_1 \geq \ldots \geq \alpha^m_r$. Now replace sequentially, in $\beta^m = (\beta^m_1, \ldots, \beta^m_n)$, β^m_i by α^m_i, $i = 1, \ldots, r$. For some k, $1 \leq k \leq r$, $f^m(\alpha^m_1, \ldots, \alpha^m_k, \beta^m_{k+1}, \ldots, \beta^m_n) = \zeta > \eta^m$, and $\zeta \leq \alpha^m_k$. Thus, all the members of $Q^* = \{1, \ldots, k\}$ strictly prefer $\alpha^m \mid Q^*$ to $\beta^m \mid Q^*$ at $\tilde{\beta}$.

We call a coalition Q **regretful** if there exists $m \in \{1, \ldots, \ell\}$ such that Q can improve upon $\tilde{\beta}$ by playing $(\alpha^m \mid Q, \beta^{-m} \mid Q)$. $f(\tilde{\alpha}) \neq f(\tilde{\beta})$ implies that (4.1) or (4.2) is true. Hence, we have proved the existence of a non-empty regretful coalition. Let T be a (non-empty) regretful coalition of minimum size. The following claim is true.

Claim 4.2. For each $m = 1, \ldots, \ell$, $f((\alpha^m|T, \beta^{-m}|T), \tilde{\beta}^{N \setminus T}) P_i f(\tilde{\beta})$ for all $i \in T$.

Proof of Claim 4.2. Let $1 \leq m \leq \ell$. We denote

$$T_- = \{i \in T \mid \alpha^m_i < \eta^m\}, \quad T_0 = \{i \in T \mid \alpha^m_i = \eta^m\}, \text{ and } T_+ = \{i \in T \mid \alpha^m_i > \eta^m\}.$$

We have to consider seven cases.

(4.3) $T_- \neq \emptyset$, $T_0 \neq \emptyset$, and $T_+ \neq \emptyset$. Without loss of generality $T_0 = \{1, \ldots, r\}$, $T_- = \{r+1, \ldots, r+k\}$ and $\alpha^m_{r+1} \leq \ldots \leq \alpha^m_{r+k}$, $T_+ = \{r+k+1, \ldots, q\}$, where q

is the number of members of T, and $\alpha^m_{r+k+1} \geq \ldots \geq \alpha^m_q$. First, for $i \in T_0$ replace β^m_i in $\beta^m = (\beta^m_1, \ldots, \beta^m_n)$ by α^m_i. Clearly $f^m(\alpha^m|T_0, \beta^m|N \setminus T_0) = \eta^m$. Now replace sequentially in $(\alpha^m|T_0, \beta^m|N \setminus T_0)$ β^m_i by α^m_i for $i = r+1, \ldots, r+k$. By the minimality of T and (i), i=1,2, of Definition 3.4 $f^m(\alpha^m|T_0 \cup T_-, \beta^m|N \setminus (T_0 \cup T_-)) = \eta^m$. (The rôle of (i), i=1,2, of Definition 3.4 is to guarantee that the order of replacement, first T_0 and then T_-, does not matter.) Similarly, we may show, by replacing sequentially $\beta^m \mid T_+$ by $\alpha^m \mid T_+$, that $f^m(\alpha^m|T, \beta^m|(N \setminus T)) = \eta^m$.

A careful examination of the proof of (4.3) reveals that if at least two out of the three sets T_-, T_0 and T_+ are non-empty, then $f^m(\alpha^m|T, \beta^m|N \setminus T) = \eta^m$. Thus it remains to consider the following three cases.

(4.4) $T_0 \neq \emptyset$, $T_- = T_+ = \emptyset$. Clearly, in this case $f^m(\alpha^m|T, \beta^m|N \setminus T) = \eta^m$.

(4.5) $T_- \neq \emptyset$, $T_0 = T_+ = \emptyset$. Again, an examination of the proof of (4.3) reveals that $\zeta = f^m(\alpha^m|T, \beta^m|N \setminus T)$ satisfies $\zeta \leq \eta^m$ and $\zeta \geq \alpha^m_i$, $i \in T$. Hence, the claim is proved in this case.

(4.6) $T_+ \neq \emptyset$, $T_0 = T_- = \emptyset$. An examination of the proof of (4.3) reveals that $\zeta = f^m(\alpha^m|T, \beta^m|N \setminus T)$ satisfies $\zeta \geq \eta^m$ and $\zeta \leq \alpha^m_i$, $i \in T$.

Let T be a (non-empty) minimal (in size) regretful coalition. We conclude from Claim 4.2 that $f(\tilde{\alpha}|T, \tilde{\beta}|N \setminus T) \neq f(\tilde{\beta})$ and $f(\tilde{\alpha}|T, \tilde{\beta}|N \setminus T) P_i f(\tilde{\beta})$ for all $i \in T$. Therefore $T \neq N$, because, by hypothesis, $f(\tilde{\beta}) P_i f(\tilde{\alpha})$ for $i = 1, \ldots, n$. Now consider the reduced game $(B^T; f^{T,\tilde{\beta}}; (P_i)_{i \in T})$. By the induction hypothesis $\tilde{\alpha}|T$ is a CPNE of this game. Hence T has an internally consistent improvement upon $\tilde{\beta}$. As $T \neq N$ this is impossible because $\tilde{\beta}$ is self-enforcing. Thus, the desired contradiction has been obtained. **Q.E.D.**

5 An Example

We shall show by means of an example that GMVS's may not be strongly coalition-proof. Let $\ell = 3$, $B^j = \{0, 1\}$ for $j = 1, 2, 3$, and $N = \{1, 2, 3\}$. We define a GMVS f by means of the following left-coalition systems: $W^j : B^j \to 2^N$ is defined by $W^j(0) = \{S \subset N \mid S \text{ has at least two members}\}$ and $W^j(1) = 2^N$, for $j = 1, 2, 3$. Let $B = \times_{j=1}^3 B^j$ and let e^j be the j-th unit vector in R^3, $j = 1, 2, 3$. We define three additive ($u : B \to R$ is additive if $u(x+y) = u(x) + u(y)$ for all $x, y \in B$) utility functions on B as follows: $u_1(0) = 0$, $u_1(e^1) = 4$, $u_1(e^2) = -1$, and $u_1(e^3) = -2$; $u_2(0) = 0$, $u_2(e^1) = -1$, $u_2(e^2) = 4$, and $u_2(e^3) = -2$; $u_3(0) = 0$, $u_3(e^1) = -1$, $u_3(e^2) = -2$, and $u_3(e^3) = 4$. Let P_i be the preference relation represented by u_i, $i = 1, 2, 3$. Then P_i is single-peaked with bliss point e^i, $i = 1, 2, 3$. Now $f(e^1, e^2, e^2) = (0, 0, 0)$ because of our definition of $W^j(0)$, $j = 1, 2, 3$. However, $(0, 0, 0)$ is not Pareto optimal. Indeed, let $\hat{u}_1$ be defined by $\hat{u}_1(0) = 0$, $\hat{u}_1(e^1) = 1$, $\hat{u}_1(e^2) = 2$, $\hat{u}_1(e^3) = 4$, and let $\hat{u}_2 = \hat{u}_3 = \hat{u}_1$ also be three additive utility functions on B. Denote by $\hat{P}_i$ the preference relation represented by $\hat{u}_i$, $i = 1, 2, 3$. Clearly, $\tau(\hat{P}_i) = (1, 1, 1) = e$, $i = 1, 2, 3$, and $f(e, e, e) = e$. Also, $f(e, e, e) P_i f(e^1, e^2, e^3)$, $i = 1, 2, 3$. Moreover, because of our definition of $W^j(0)$, $j = 1, 2, 3$, (e, e, e) is an NE of the game $(B^N; f; P_1, P_2, P_3)$. Hence, the truthtelling strategy (e^1, e^2, e^3) is not a strong CPNE.

6 Concluding Remarks

In this paper we proved that the strongest kind of group stability satisfied by all multi-dimensional GMVS's is coalition-proofness. This result is very far from being a consequence of strategy-proofness. Indeed, Peleg [1998] shows that pivotal mechanisms are not coalition-proof. Also, our result does not follow from Dasgupta, Hammond, and Maskin [1979], because multi-dimensional GMVS's may not be group strategy-proof. We recall that Dasgupta, Hammond, and Maskin [1979] contains a detailed investigation of the relationship between strategy-proofness and group strategy-proofness. When the dimension is greater than one, our restricted domain of preferences is too small to yield the Dasgupta-Hammond-Maskin type of results.

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