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# INDIVIDUAL MENTAL REPRESENTATIONS OF GEOMETRIC PROPERTIES

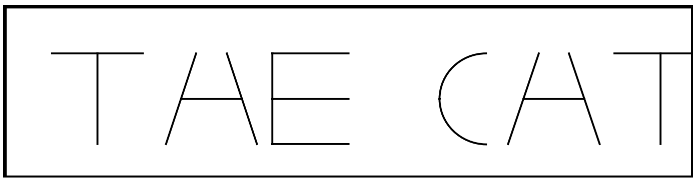
**Abstract:**

A real object in three-dimensional space, its two-dimensional projection, and its mental image from the perspective of a specific person are different entities. Psychological findings have revealed that people share some of the properties in their perception and their representation of an object. But there are also important inter-individual differences in perception and representation. Some of these differences are discussed on the basis of case studies with grade nine students. Intensive interviews were conducted. Afterward the students worked on problems which aimed at building cube-like objects and corresponding concept maps. These interviews allowed us or helped us to get insights into their procedural and declarative knowledge in the specific context. Possible links between procedural knowledge, declarative knowledge, and the student's performance on a test of spatial ability will be discussed.

## 1. Perception and Visualisation

In terms of mathematics, a spatial object and its two-dimensional representation, for example on a sketch pad or a computer monitor, are not isomorphic. All information on those points in three-dimensional space which are located on a straight line in the direction of the specific projection is comprised in a single point. This is certainly a severe loss of information about the properties of an object. Nonetheless, dealing successfully with situations which lack complete information is not an unusual problem for human beings. Propositional as well as spatial information is usually judged in its specific context and, if necessary, it will be completed by the individual in this context. The result of this process may be more or less meaningful, or it may still lack important bits of information. But there are many situations in which humans perform competently in adding information which fits well into a given context. This is confirmed by numerous research results. SELFRIDGE (1955; cf. ANDERSON 1989) provides such an example for context driven pattern recognition (cf. figure 1).

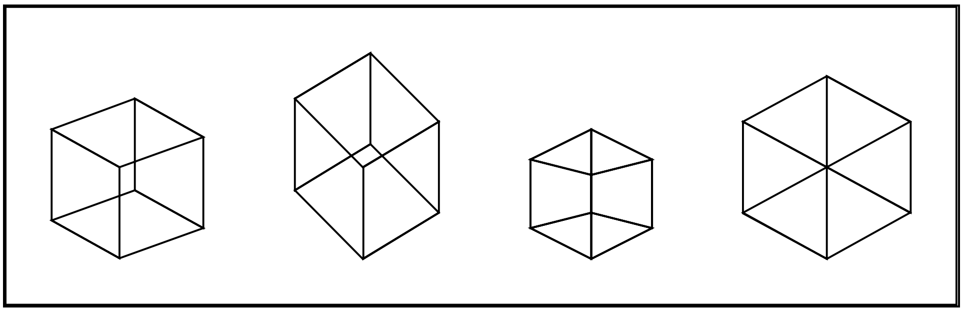
Figure 1: Context driven pattern recognition



The second sign or hypothetical letter is identical in both words or, technically spoken, in both patterns. But it would be interpreted by a reader as an "H" in the first pattern and as an "A" in the second pattern. Both are regarded as words and words are supposed to have their meanings. Accordingly, the signs are regarded as letters with a specific meaning in the specific context.

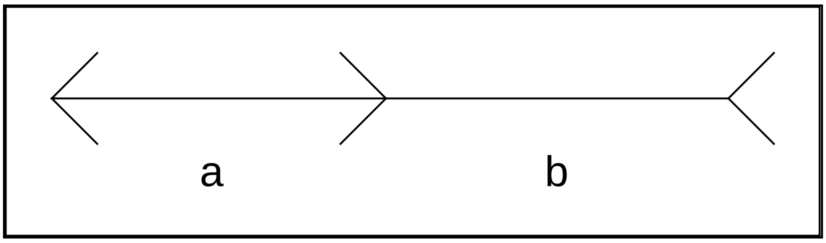
Not only propositional information but also spatial information may get its specific meaning with respect to the context in which it is used. The Kopfermann Cubes (see figure 2; cf. WILKENING 1988) may be regarded as two-dimensional drawings combined from a specific number of lines (for example the drawing on the very right side may be regarded as a pattern of six triangles) as well as representations of cubes or cuboids (the two drawings on the left side may be easily interpreted as spatial objects). These cubes were used in a research experiment by HOCHBERG & MCALISTER (1953). The authors were able to describe different reactions of their subjects to the different drawings. Most subjects interpreted the pattern to the very right as a two-dimensional pattern, and most of them regarded the pattern to the very left as the representation of a cube. If we stick to the actual information provided by each drawing there are only some lines on a sketch pad. The interpretation of most subjects overcomes this information and adds it up to a probably more meaningful pattern. This pattern may be two-dimensional or three-dimensional according to the point of view. Moreover, most people are able to change their point of view, and will see a two-dimensional or a three-dimensional object depending on their focus.

Figure 2: Spatial versus plane information: The Kopfermann Cubes



The Kopfermann cubes may serve as an example that the mathematical point of view stating that a non-isomorphic relation between a three-dimensional object and its two-dimensional representation must not be generalized in regards to human perception. The specific nature of human information processing with respect to spatial perception cannot be described merely in terms of mathematics. The examples discussed show that the interpretation of a drawing may be influenced by an individual's specific knowledge. This knowledge may lead to correct judgements even if the information actually provided is incomplete. On the other hand, even specific knowledge in a context is not a prerequisite for an adequate perception of this context. The Müller-Lyer illusion provides an example for the point that a stimulus may even conceal the objective fact a person wants to perceive (cf. figure 3).

Figure 3: Müller-Lyer illusion




This is a well-known illusion and most people are aware of the fact that a and b are identical in length. Nonetheless, what we actually see are two lines of different lengths. This erroneous perception is more powerful than the correct knowledge concerning their actual lengths. The Kopfermann cubes may be looked at in two different ways, namely as two-dimensional and as three-dimensional objects respectively. This is not possible for the two lines in Müller-Lyer's example. On the contrary, the knowledge about their true lengths will be automatically corrected by visual perception (WILKENING 1988).

An actual two-dimensional or three-dimensional object like a line or a cube may differ from the object which a person perceives, but both may also differ from a person's mental image of this object. Accordingly, we must distinguish between an object, its perception, and its mental representation. First of all, this mental image is an abstract image. It may reflect the object as it is seen by a person, but is not necessarily bound to its visual properties. Moreover, a mental image may reflect characteristics like the texture or shape of an object, but it is not necessarily caused by tactile perception (BROOKS 1968). In particular, a mental image is not a precise copy of the real object. Usually, there are fewer details in this image, and it may be altered more easily. Nonetheless, there are many operations performed on mental images which have analogies to operations in the real space. For example, let us have a close look at a cube. An image of this object on a sketch pad – or even on the may retina – provide information on its topview, frontview, and sideview, and thus it may provides two-dimensional information. In order to get a reliable representation of the cube which is an impression that includes information on all three dimensions we must take it into our hands and turn it around, or we must even walk around the cube. Afterwards, we will have information on all details of this specific object, but we need to distinguish this information from the mental image of a cube. This image usually lacks details and it is not very precise. But it is easy to refer to it in order to determine specific properties, for example the number of vertices or corners of a cube. Mental images allow a sudden change of focus in order to concentrate on specific properties of an object.

The various representations of a spatial object – regardless whether they are two-dimensional or three-dimensional in their nature or whether they are mental images – play a crucial role in research on spatial ability. They are part of most tests on spatial ability (e.g. AMTHAUER 1953, 1970; HORN 1962; GITTLER 1984a). Two-dimensional representations have been intensively analyzed as to their suitability for testing spatial abilities (GITTLER 1984b). Research has shown that subjects use different strategies for solving spatial problems, and not all of these strategies make use of spatial mental processes. The findings of PUTZ-OSTERLOH & LÜER (1975) suggest that some tasks of the LPS by HORN (1962), which is a non-verbal intelligence test, were solved by most subjects by using a strategy comparing plane properties. Other research results by PUTZ-OSTERLOH (1977) and PUTZ-OSTERLOH and LÜER (1979) identify items in three tests (LPS by HORN 1962; IST by AMTHAUER 1953; WAB by MEILI 1953) which are solved using spatial or plane strategies. In particular PUTZ-OSTERLOH and LÜER (1979) identified a variety of plane and spatial strategies which were actually used by their subjects in a specific problem solving situation. KÖLLER & ROST & KÖLLER (1994) used the IST-70 (AMTHAUER 1970) in order to identify strategies for solving spatial items. Their findings suggest two principal strategies, which they call holistic and analytic. A holistic strategy is performed if a subject uses mental rotation for his or her solution. An analytic strategy may be characterised through the use of non-transformational processes, like a comparison of the element's specific properties. Moreover, the findings suggest that a person usually prefers a specific strategy. This supports research results by BARRAT (1953), who analysed verbal descriptions of problem solving strategies for tests on spatial ability. He identified two strategies which differ with respect to the subject's point of view. Some of his subjects mentally rotated the stimulus whereas some others imagined themselves walking around the stimulus,

These experiments raise the question whether humans use specific strategies for solving specific problems or whether the environment influences the problem solving process. There are some findings which support the hypothesis that the problem solving environment is of minor importance. LEUTNER & KRETSCHMAR (1988) performed a geometry teaching experiment on parallel projection. Some of their subjects were working in a computer environment, some of them were working with real objects. These two groups were both separated into a subgroup in which the students worked hands-on, and into a subgroup in which the students watched a teacher demonstration. The experiment revealed no significant differences in a classroom test between the group using a computer and the group operating with concrete objects. Moreover, students who watched the teacher demonstration in the computer environment scored better than their class-mates who worked hands-on with the computer. A test on spatial ability turned out to be the best predictor for geometry achievement. Some of our own investigations support these findings. We performed experiments with students of different ages from primary to the lower secondary level. We used different mathematical problems like working with spatial objects, characteristic lines in triangles, or tasks on fractions. We also presented different environments like working with real objects versus paper and pencil, paper and pencil versus a computer environment, and concrete demonstration versus theoretical imbedding. In some investigations we used a test on spatial ability as pretest and posttest; in other investigations we made classroom tests on mathematical achievement. The investigations showed no evidence for any differences in performance between the groups working in the different settings (e.g. ALBRECHT 1993; REISS & ALBRECHT 1995). In particular, the findings suggest that hands-on experience is not a prerequisite for better student performance.

The research described above reveals desiderata for further investigations. Accordingly, we would like to investigate (i) what kind of strategies students apply when solving geometry problems, (ii) whether they are influenced by the environment the tasks are embedded in, (iii) what kind of declarative knowledge may be used by the students in their problem solving and (iv) whether there is a connection to spatial ability.

## 2. Methodological Design and Research Goals

### 2.1 Subjects

The three aspects of the representation of a geometric solid, namely as a three-dimensional object, as a two-dimensional picture, or as a mental image were investigated in twelve case studies with children aged 10 to 16. Our subjects were students in grades five and nine of a local secondary school. They were asked to participate in a 90 minute intensive interview which consisted of a 45-minute problem solving phase and a 45-minute concept mapping phase (NOWAK, GOWIN, & JOHANSEN 1983; REISS & HAUSSMANN 1990). These two parts were designed to provide information about the students' procedural knowledge as well as about their declarative knowledge on cubes and cube-like solids (cf. ANDERSON 1995, on details of the distinction between procedural and declarative knowledge).

### 2.2 Assessment of Procedural Knowledge

The problems presented in the first part may be described as cube construction tasks. A cube-like solid had to be constructed by using information on its front view, side view, and top view. Every solid consisted of eight parts, some of which were cubes and some of which were triangular prisms. These parts had up to four different colours. See figure 4 for an example of a problem in which the colours have been changed to black, white and two different shades of grey.

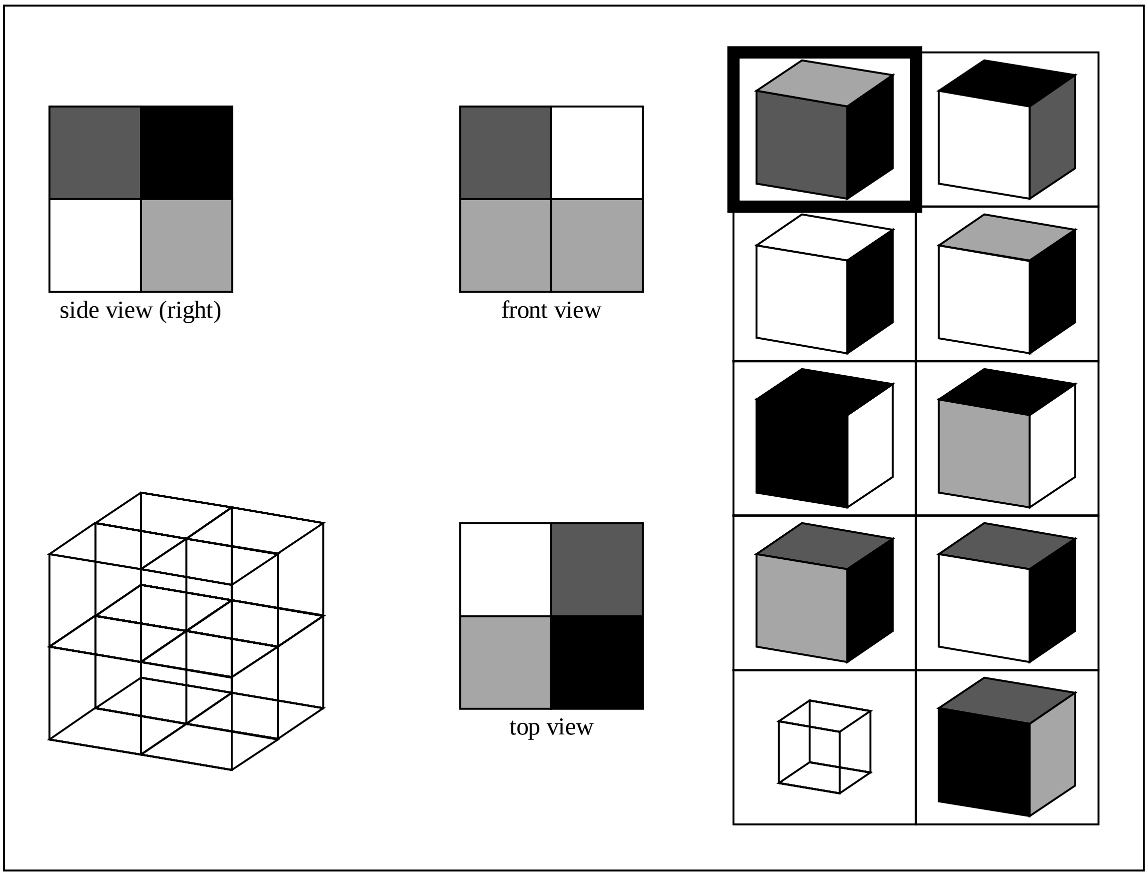



Figure 4: Cube construction task

The problems were presented to one group of students in a computer environment, and to the other group of students in an environment using wooden cubes and prisms. As an introductory task we asked the students to identify top view, front view, and side view of a 2 x 2 x 2 cube. While working on this problem they were encouraged to ask any questions concerning the problem and its specific form of presentation. The computer environment and its software were discussed during the work on this simple problem. This part of the interview was not interpreted. Afterwards the students were asked to solve two problems in which the solids had to be constructed. In the first one there were just squares in top view, front view, and side view. In the second one there were squares and triangles in the different views.

### 2.3 Assessment of Declarative Knowledge

In the second part of the interview the students were asked to express relations between some concepts which were used during the problem solving process. We presented the concepts of *cube, square, top view, side view, front view, vertex, corner, perspective, plane,* and *space* written on small cards. These cards were arranged by the students on a large sheet of paper with respect to their conceptual similarity: concepts that were similar had to be placed near to each other. Afterwards, pairs of them were linked – if possible – by a proposition which expressed the subjective relation between them. We asked the students to choose any two concepts and to find a proposition in which both of them would be used. When the students were finished the interviewer presented all pairs of concepts they had not yet mentioned and asked if there might be some relation between them.

Some weeks after the intensive interviews, all students took part in a test on spatial ability. We used the corresponding item of the 3DW (three-dimensional cubes) test by GITTLER (1984). This test was presented in a classroom setting.

The problem solving process as well as the concept mapping part of the interview was documented by means of video cameras.

## 2.4 Aims of the Study

This study aimed at different aspects of spatial knowledge. These aspects are closely connected to the presented problems. They may be summarised in the following research questions:

(1) Is it possible to identify the students' different strategies while they are working on the cube problems? If there are different strategies, can they be characterised as holistic and analytic?
(2) Are there different problem solving processes with respect to a computer environment or an environment where the students are confronted with real wooden cuboids?
(3) Is successful problem solving linked to specific declarative knowledge?
(4) Are there any hints that successful problem solving processes are linked to good results in a test on spatial ability?

We will compare the success in solving the cube tasks, the use of static and dynamic propositions (cf. REISS & WELLSTEIN 1996) in the concept map, and the test results from the test on spatial ability. Moreover, we will try to identify specific problem solving strategies.

## 3. Results

In the following we will present the results of six students, three females and three males of grade nine. In the problem solving part of the interview, the girls worked in a computer environment whereas the boys worked in an environment with concrete wooden cubes and triangular prisms. This was due to organizational reasons.

Table 1 presents the test results of the problem solving part. It includes the number of minutes a student needed for solving the problem or the number of minutes the student used until the problem was abandoned (tim). This table also presents the number of moves performed (mov), and the number of erroneous moves (err). We also indicate whether the student succeeded (suc: y) or not (suc: n). This table distinguishes between problem 1 and problem 2. Problem 1 is easier to solve than problem 2 because it makes use of triangular prisms, whereas the first problem is solved by using only small cubes.

| | Problem 1 | | | | Problem 2 | | | |
|---|---|---|---|---|---|---|---|---|
| | tim | mov | err | suc | tim | mov | err | suc |
| ALEX | 2:10 | 8 | 0 | y | 9:13 | 32 | 21 | y |
| LOUIS | 3:17 | 11 | 3 | y | 8:20 | 8 | 8 | n |
| JANE | 6:36 | 8 | 1 | y | 10:09 | 8 | 1 | y |
| ALICE | 6:31 | 9 | 1 | y | 12:12 | 9 | 2 | y |
| ELLEN | 6:33 | 7 | 0 | y | 32:42 | 21 | 7 | y |
| SAMUEL | 3:04 | 12 | 4 | y | 17:19 | 5 | 2 | n |

Table 1: Problem solving abilities with respect to the two problems presented

In their concept maps our students preferred propositions which described a characteristic property of a concept (cf. LICHTFELDT 1992). This description includes propositions like "... a cube has eight corners ..." in which the concept of *corner* is used to describe the concept of *cube*. We were able to identify two classes of propositions within this category – static and dynamic propositions. Static propositions express relations between concepts or properties of concepts by using verbs or verbphrases like "... has ..." or "... is a ...". Dynamic propositions describe properties or relations by expressing a mental process. Examples of dynamic propositions are "... you see a square if you look at a cube from the right ..." or "... you might observe a cube from top view ..." (REISS & WELLSTEIN 1996). Accordingly, static propositions may be more reproductive in their nature whereas dynamic propositions suggest that a subject used mental transformations in order to change a specific configuration (Wagener-Wender, as cited by WENDER & HABEL 1995).

Table 2 shows the number of static and dynamic propositions our subjects used in their concept maps. Moreover, their ranks in the test on spatial ability are included in this table. A "+" sign stands for performance above average, a "–" sign stands for performance below average compared to other students of their age.

In this small group students with the highest scores for spatial ability also scored high on the test regarding their use of dynamic propositions. Five of the six students used dynamic propositions frequently. This is not influenced by computer use or the use of wooden cubes and prisms in the first part of the interview. All three girls worked in the computer environment, all three boys worked hands-on with concrete wooden material. If we regard dynamic propositions as an indicator for mental transformation processes these processes are induced by the two-dimensional environment as well as by the three-dimensional environment.

| | static propositions | dynamic propositions | rank | average |
|---|---|---|---|---|
| ALEX | 11 | 25 | 90 | + |
| JANE | 11 | 25 | 86 | + |
| LOUIS | 8 | 22 | 86 | + |
| ALICE | 14 | 16 | 79 | + |
| ELLEN | 43 | 0 | 23 | – |
| SAMUEL | 19 | 18 | 6 | – |

Table 2: Use of static and dynamic propositions in the students' concept maps

In addition to this data, an analysis of the session protocols provides more specific information on the problem solving behaviour of each individual.

**Alex** (15;10) performs best in the test of spatial ability. His rank of 90 is above average. He also performs best in the first cube task and needs only a little more than two minutes to solve the problem correctly. He probably compares at least two relevant parts of the three views in order to find the right cube for a specific position. He needs about half a minute to find those cubes which have two or three relevant squares but he needs only ten to fifteen seconds to identify cubes with only one relevant square. Alex gets into difficulties while working on the second task. He concentrates on two sights only. Alex chooses cubes or prisms which add the correct information to any two of the frontviews, topviews or sideviews. He then controls the views and tries to correct faulty parts with respect to the third view, but he still concentrates on two views only. Eventually he develops a strategy, and focuses now on the sideview and the frontview. He gets a nearly correct solution as far as these two are concerned. After this he concentrates on the topview and changes the parts in order to get the correct view. Most probably, Alex uses an analytic strategy.

**Jane** (15;6) performs above average in the test of spatial ability. She works carefully on both problems. She avoids making any mistakes by taking her time solving the problems. This may indicate that she compares all three views in order to find the correct cube or prism. Nonetheless, there is some evidence that Jane also works intuitively. She tells the interviewer that she is not certain if she has completed the second task correctly. There is no way to distinguish whether Jane prefers a holistic or an analytic strategy. Because she used only a small number of moves, we can assume that she used advanced mental capabilities to work out the problems.

**Louis** (15;8) also performs above average in the test of spatial ability. Nonetheless, working on the cube tasks seems to be difficult for him. Obviously, he concentrates on only one view to find a suitable cube. Since the first task is quite simple, he succeeds with his strategy most probably by accident in a few moves. He has extreme difficulty while working on the second problem. He abandons his work after eight moves – none of which is correct.

**Alice** (15;4) performs above average in the test on spatial ability. She works on both problems with all her concentration. Alice needs some time to solve the tasks but she avoids faulty moves. She makes only two mistakes during problem solving, which are both corrected immediately. The first task seems to have been done by accident only. Most probably, Alice compares all three views before choosing a cube or a prism. It is not clear from her performance whether she uses a holistic or an analytic strategy.

**Ellen** (15;9) performs below average in the test on spatial ability. She takes her time while solving the first task, and needs between half a minute and two minutes for each move. There is no connection between the time needed and the number of squares to be considered in a specific move. The second task is a more challenging problem for Ellen. Her strategy takes into account only one or two parts of the cuboids. Like Alex she develops her strategy during the problem solving process. She takes into account two views and changes these parts afterwards with respect to the third view.

**Samuel** (16;8) performs below average in the test of spatial ability. Nonetheless, he has only minor problems while solving the first task. His mistakes occur most probably because he mixes up the frontview and the sideview in his first trial. He runs into problems while trying to solve the second task. He needs most of the time indicated in table 1 for discussing the solvability of the problem with the interviewer. Samuel states that there is no solution for this problem and refuses to work on it after fixing the first layer of cubes. The interviewer, who was not supposed to help the student was unable to motivate him to finish his work.

## 4. Discussion

The study focused on procedural and declarative knowledge of secondary school students in a geometry context. By means of case studies we tried to identify the specifics of problem solving processes in a computer environment and in an environment providing concrete material. The results indicate that there are a variety of individual differences in the students' procedural knowledge as well as in their declarative knowledge. According to the research questions, it is possible to give some preliminary answers.

The analysis of our students' problem solving processes indicates the use of different strategies. Alex and Ellen develop their strategy during problem solving. They pay attention to only two of the views in a first attempt. In a second attempt the preliminary result will be corrected with respect to the third view. We presume that this strategy is analytic in its nature. Both succeed in the problem solving process. Jane and Alice are also successful problem-solvers. They take into account that all three views thus build a mental image of the specific cube or prism which will fit into a specific slot. They might use mental rotation for this, and they might use a holistic strategy. Further experiments would be necessary to determine this. Louis and Samuel do not successfully complete the problems. Their strategy is probably characterized by the comparison of a single property of a cube or prism. This strategy may work on simple problems but is not appropriate for more complex problems. It is analytic but does not take all necessary information into account.

Comparing the problem solving environments shows there is no obvious difference between the subjects who used the computer and the subjects who worked with wooden cubes and prisms. A striking difference, namely that the girls needed more time than the boys for solving the first problem, is corrected since the girls performed better in the second task than the boys. There is no evidence from these case studies that an environment using concrete material indicates mental processes which differ from those in a computer environment. The representation of the problems in a two-dimensional or in a three-dimensional context probably has no influence on the mental images involved. In particular, we might suppose that a computer environment indicates similar mental images and thus similar mental transformations as the concrete environment.

A comparison of the data on procedural and declarative knowledge does not indicate any connection between specific knowledge of the concepts involved and success in problem solving. One might suspect a connection between the use of dynamic propositions and spatial ability. But six subjects are too small a sample to test such a hypothesis. Moreover, the data do not even suggest that good spatial ability is a sufficient prerequisite for successful problem solving. Further research is needed in order to get a reliable evaluation.

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