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PROBLEM SOLVING BEHAVIOUR OF PRIMARY SCHOOL STUDENTS IN REAL-LIFE SITUATIONS PRESENTED WITH POSTER AND TEXT

– RESULTS OF AN EMPIRICAL INVESTIGATION –

Abstract:

The paper describes an investigation about problem solving strategies in real-life contexts. 8-to 9-year old children were offered multiplicative problems in three different contexts - „buying goods for a classroom party“, „buying bottles of juice for a punch“, and „buying tiles for a doll’s house“ - which were presented in different forms: as poster task (poster with picture and text), as project, as simulating role-play, and as word problem. The results of this qualitative analysis show a great variety of pattern and arithmetic strategies among the students, differences in familiarity with the contexts and dependency on the form of presentation. This paper concentrates on arithmetic strategies in the first situational context presented as poster task and uses other results by way of comparison.

1. Theoretical background

In recent years, real-life situations and childrens’ problem solving behaviour have been studied by many researchers. They have observed that children solved real-life problems correctly without making use of classroom mathematics (Carraher, Carraher & Schliemann 1985, 1987; Lave 1988; Lave & Wenger 1991; Saxe 1991; Scribner 1984). At the same time, children are incapable of applying classroom mathematics in real-life tasks (Greeno & Kintsch 1985). Although a lot of research has been done to understand the development of multiplicative concepts (eg., Greer 1988, 1992; Nesher 1988; Schmidt & Weiser 1995; Vergnaud 1983, 1994) and the problem-solving strategies of primary school children dealing with multiplicative problems (eg. Anghileri 1989; Bönig 1995; Brown 1992; Burton 1992; Kouba 1989, Selter 1994; Steffe 1988), there is less investigation concerning the combination between these strategies and real-life arrangements. In order to modify classroom teaching, it is necessary to understand how children think while working on these problems. With this objective, in 1994, we started an investigation which was supported by the German Research Community (DFG).

2. Description of the investigation

2.1 Aim of the investigation

The aim of the investigation was to discover the behavioural patterns and problem solving strategies of primary school students when solving non-standard multiplication tasks which were similar to each other in their arithmetic structure but differed in the situational contexts and in the form of presentation.

2.2 Inquiries of research

- Which *arithmetic strategies* do children use when solving the task? Which behavioural patterns do they resort to?
- Which *heuristical strategies* do children use while considering different ways of reaching the solution?
- How do students *co-operate* with each other?

In addition to this, we investigated compariative questions with reference to presentation and context:

- Is it possible to recognise behavioural patterns and arithmetical strategies typical of each *presentation*?
- Is it possible to recognise behavioural patterns and arithmetic strategies with reference to each *context*?

2.3 Research design

Table 1: Overview of research design

<table>
<tr><th>Context
Form of presentation</th><th>Buying goods for a classroom-party</th><th>Buying bottles of juice for a punch</th><th>Buying tiles for a doll house</th></tr>
<tr><td>Project
(with real objects)</td><td colspan="3" rowspan="4">Video recordings
of children working in pairs and groups
followed by interviews</td></tr>
<tr><td>Simulating role-play</td></tr>
<tr><td>Poster task (with pictures and text)</td></tr>
<tr><td>Word problem</td></tr>
</table>

As the shadows in table 1 already indicate, the results presented in this paper will mainly concentrate on the arithmetic strategies in the situational context „classroom party“ and the presentation form „poster-task“. Additionally, results of the other contexts as poster-tasks and also results of the other presentation forms to the context „classroom party“ are included to answer the comparative questions.

2.3.1 Contexts

We developed non-standard tasks in three different familiar contexts. All three tasks involved buying goods which were available only in packets or bottles. The task was to find out how many packets or bottles were needed to buy and to write the number in a shopping list. Therefore, the children had to determine the number of items given in one packet first. The three chosen tasks have similar arithmetic structures but different mathematical contents: numbers in the case of the classroom-party, volume in the case of the fruit punch and area in the case of the doll's house. We considered these contexts to be familiar to the children but also to be highly individual and therefore different experiences.

2.3.2 Form of presentation

The forms in which we presented the tasks vary in degree of closeness to real-life:

The presentation as a *project* comes closest to a real-life situation. In this kind of setting it is possible to actually carry out the tasks with or without material. In the context „classroom party" the children were required to write a shopping list and then to buy the things needed for a carnival party in their class.

In the *simulating role-play* dealing in the context „classroom party", a shopping list written by a fictive class of 18 children was given to the children. They could act as if in a real supermarket, because all the things were presented in their genuine packings. The task was to determine the number of packets needed for those 18 children.

In the pictorial presentation, the *posters* show shelves with things arranged on them in the contexts „classroom party" and „fruit punch" (for an example see figure 1) and the doll's-house in the third situation. The children had to choose the right things and to find a possible number of packets or bottles. There are still several possible ways of solving the problems but the activities are limited to pictures and symbols.

The fourth form of presentation is traditional *word-problems*. While solving these problems, the activities are mainly limited to symbols.

2.3.3 Arithmetic structure of the tasks

All the tasks have multiplicative structures: $x \cdot b = a$ or the more difficult variation $x \cdot b > a$. For the children, the second variation seemed either insoluble or a division-problem with remainder. In the context "classroom-party", the students had to choose goods for *18* children. Therefore the arithmetic structure was $x \cdot b \geq 18$ with the constraint that the packet contained *(b)* items where $b \ \varepsilon \ \{2, 3, 4, 5, 6, 7,9\}$ items. The structure in the context "buying juice-bottles" was $x \cdot b \geq a$ with $b \in \{2, 5, 7\}$ and a given in the recipe. The context "doll's-house" had the same underlying structure: b was the number of tiles per packet, given as *3, 6 or 8* but a had to be determined by measuring the area of the rooms.

2.4 Methodology

For the analysis of behavioural patterns and problem-solving strategies of primary school children, we used case studies in the form of video recorded observations followed by interviews. The interpretation was based on qualitative methods (Maier; Voigt 1991).

The method most accepted in the recent years for the analysis of video recordings is the interpretation of transcripts (Maier; Voigt 1994). In addition to verbal communication, we transcribed some of the activities where necessary (Wollring 1994). According to Beck and Maier (1994), there are four ways of text interpretation: “category-based” interpretations, “category-building” interpretation, “explorative-paraphrasing” and “systematic-extensional” interpretation. We used the category-based method for the interpretation of arithmetic strategies. The following categories were developed by Ruwisch (1998) based on Anghileri (1989), Kouba (1989), Burton (1992) and others:

1. *Counting*
2. Use of *number patterns* (multiplication tables)
3. Use of (repeated) *addition*
4. Use of *multiplication and division facts* (equations)
5. Use of *mixed strategies* and others

Since this paper concentrates on results to the arithmetic strategies, the other forms of analysis are mentioned only briefly: For analysing the heuristic strategies (Anderson 1983; Franke 1986; Pólya 1949; Schoenfeld 1985), an explorative-paraphrasing interpretation was necessary. For the analysis of the co-operation, the individual case studies were interpreted in the systematic-exploratory way.

2.5 Structure of the investigation

The tasks presented with *posters* were worked on in 1996 in three different classes at the end of the second school year (age 8). These children had already been introduced to multiplication and division with small numbers. The children were presented a text which described the situation (see frame 1), were shown a poster (see figure 1 for an example) and were given a working sheet, they had to fill in (see figure 2 for an example).

Frame 1: Instructional text

The Classroom Party

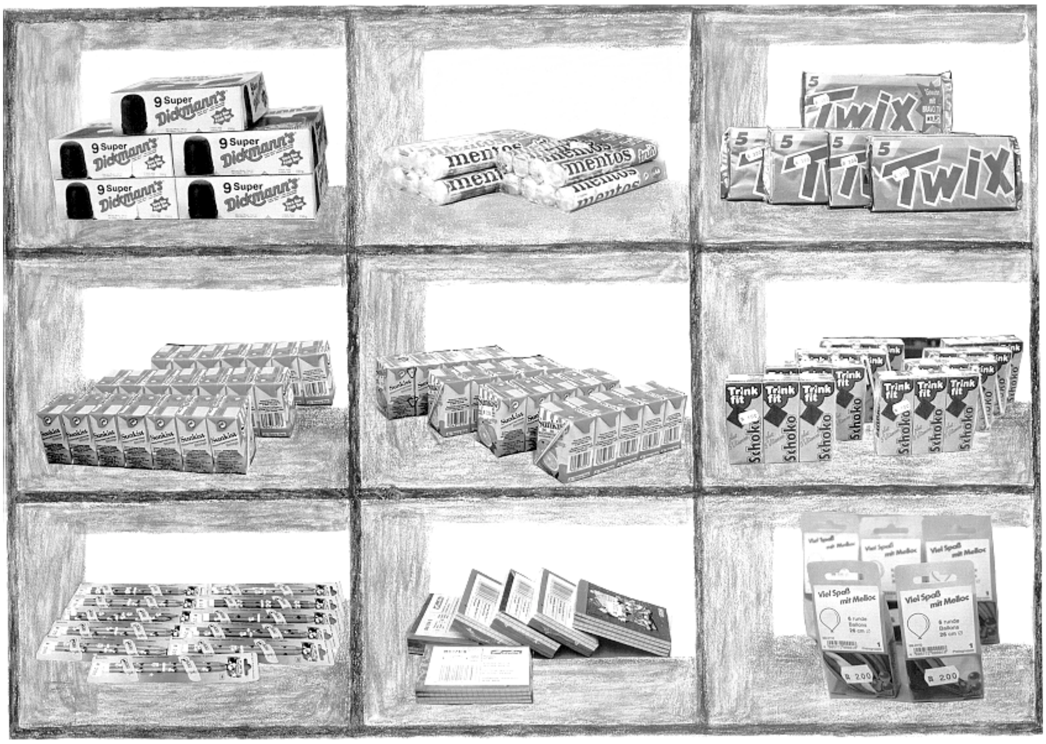
The children of class 2c are planning a classroom party. They have thought about the things they need for the party. Laura wishes to go shopping for all the 18 children of the class. Every child should get something of every article.

Fill in the shopping list for Laura!

Figure 2: Original and translated shopping list

Einkaufsliste	**shopping-list**
... Packungen Schokoküsse	... packs chocolate-coated éclairs
... Packungen Twix	... packs chocolate bars
... Packungen Limo groß/ ... Packungen Limo klein	... packs lemonade big/ ... packs lemonade small
... Packungen Kakao	... packs chocolate drink
... Packungen Bleistifte	... packs pencils
... Packungen Blöcke	... packs writing pads
... Packungen Luftballons	... packs balloons

Figure 1: Poster in the situational context „classroom party“



The children solved the problems in pairs. We recorded their behaviour on video. After they had finished we asked them how and why they had solved the problem in that particular way. One week later, the same children solved a word-problem with another context (see table 2).

Table 2: Number of subjects in the poster-tasks and in the word-problems

Presentation Number of pairs	Poster-task	Word-problem
7 Pairs	Classroom-party	Fruit punch
7 Pairs	Doll house	------------------
9 Pairs	Fruit punch	Classroom-party

In the study of the *simulating role-play* 122 children (66 second graders, age 7-8, and 56 third graders, age 8-9) were included. At the time of the investigation, the second graders had not yet been introduced to multiplication or division, whereas the third graders had already learnt multiplication and division facts of problems with small numbers. The children were given a short oral introduction into one of the situations described above. They also worked in pairs and were videotaped. This working phase was followed by a re-interview. Within two weeks the children were confronted with two of the three situations.The results of this study have been described in Ruwisch 1998.

The task presented as a *project* was carried out only in one class (age 8/9). A group of four children bought goods for a “carnival-party” of their class.

3. Results of the investigation

3.1 Arithmetic strategies in the situational context „classroom party“ presented as poster-task

The problem concerning „buing goods for a classroom party“ can be divided into seven smaller problems: each of the seven things on the shopping list. These seven problems were of a similar nature and could therefore be solved in the same manner. The children had always to make sure that they bought each of the items for each of the 18 children.

We observed that most of the children worked in the following four steps:

1) They looked if each of the seven things from the shopping list can be found in the poster.
2) They found out the number of items in each packet.
3) They counted the number of packets for their class (18 children).
4) They completed the shopping list.

Most of the children recognised the problem to be a multiplication task and were able to interprete the situation in mathematical terms. 2 of the 7 pairs of students working on this task did not correctly consider the aim of the task. So they did not show any comparable arithmetic strategies in their work. Since the poster showed lemonade in packets of two sizes, with 7 and with 5 items, all students of the remaining 5 pairs solved the lemonade problem twice. But Mentos (sweets), which were also illustrated on the poster, were overlooked by the children, because they had not been mentioned in the shopping list. Thus in the case of altogether 40 calculations, it was possible to recognise the following arithmetic strategies (see table 3).

Table 3: Arithmetic strategies in the context "Classroom-party"

	Counting	Number patterns	Additive facts	Multiplicative facts	Mixed strategies	Not identifiable
Frequency absolute	2	17	—	14	5	2
Frequency relative (in %)	5	42.5	—	35	12.5	5

We often observed that children mentioned a muliplication equation and so concluded the number of packets (35 %). But most of the tasks were solved by using number patterns and at the same time counting the number of packets with their fingers (42.5 %). If the students did not know the number patterns completely they continued by counting (mixed strategies). They never showed that they were adding. It was observable that the children tried to repeat the same strategy for each problem as far as it was possible. But both strategies mentioned could not be used in the same manner for problems with a remainder (eg. in the case of Twix). A typical way of handling these problems is shown by Marcel:

> Marcel said:
>
> "Five items in one packet, mh? Then we must take four, because five cannot be broken up. Yes, we need four!"

Some of the children seperated out the problems with remainder and solved them later. Others asked how the problem could be solved. In the end all these children were able to overcome this difficulty without help by using their experience in daily life:

> "What remains, we can give to the teacher."

More than 2/3 of the students were able to consider all conditions mentioned above. If one of the conditions was not considered, the task could no longer be worked on properly:

> Steffen and Hatchi eg. counted all the packets of every product on the poster and wrote the number on the shopping list. Then Hatchi added all the numbers and got 100 as the answer. Since 100 is a lovely number, he was convinced that the answer was correct. These two boys had not considered the number of children (18) they had to buy for and the number of items in each packet.

This example shows that while solving a problem, the children had their own ideas even if they did not correspond with the aim of the task.

3.2 Results of comparison between the presentation forms

The children who solved tasks in different forms of presentation used the same strategies. Not only the arithmetic strategies but also the heuristical strategies and the co-operation between the children were the same.

The main difference between the presentations was to be found in the identification with the context. During the project, although the children knew that I was paying for the goods, they still wanted to consider the prices. They spoke about cheap and expensive goods and wanted to go to different shops. The other presentations were not so real for them and so they were not interested in the prices. Also they said nothing about whether they found the goods tasty or if they preferred to buy some other goods.

An interesting observation was that, in case of the poster-task, the children did not write down the calculation but in the case of the word-problem, most children used a pattern they had learnt in school.

3.3 Results of comparison between the contexts

The children did not find all situations equally familiar. Some of them asked questions about the text or wanted some words to be explained. After having understood the context, they could solve problems even if they were not familiar.

Table 4: Arithmetic strategies in the context „Buying bottles of juice for a punch”

	Counting	Number patterns	Additive facts	Multiplicative facts	Mixed strategies	Not identifiable
Frequency absolute	1	11	1	8	8	1
Frequency relative (in %)	3	36	3	27	27	3

Table 5: Arithmetic strategies in the context „Buying tiles for a doll's- house”

	Counting	Number patterns	Additive facts	Multiplicative facts	Mixed strategies	Not identifiable
Frequency absolute	15	—	3	3	—	—
Frequency relative (in %)	71	—	14.5	14.5	—	—

The tables 4 and 5 give an overview of the applied arithmetic strategies in the different contexts.

As in the case of shopping for the classroom-party, the main strategies used in the situation

“Buying bottles of juice for fruit punch” were number patterns and multiplicative facts, and in the situation “Buying tiles for the doll-house” the strategies were counting and using multiplicative facts. In the last situation, the strategy used was addition in 3 cases. In this situation, the children drew tiles of the rooms. They used the picture to count. One pair used addition to solve the task for all three rooms.

4. Conclusive remarks

The most interesting finding of this investigation was that number patterns and multiplicative facts were the dominant mathematical strategies. This observation does not correspond to observations in investigations of other scientists. In an investigation by Bönig in Germany (1995) addition was observed as the dominant strategy. In their research Anghileri (1989) and Kouba (1989) recognised counting as the dominant strategy. Anghileri presumes that counting is the preliminary stage of using number patterns.

The strategies used by the children probably depend on those introduced in the classroom. In Germany, teachers of mathematics often emphasize the learning by rote of multiplication tables too soon. It was interesting to note that the children did not use addition to solve the problems although they should have been introduced to multiplication in the form of shortened additions. We did not observe the children using the distributive law like for example $7 \cdot 5 = 5 \cdot 5 + 2 \cdot 5$. It appears that even today, the learning of multiplication tables is the dominant way and is not necessarily supported by an understanding of the underlying strategies. Irrespective of the situation and the presentation form, the children recognised that the problem involved a multiplication task. However, if they did not know the result by multiplying, they preferred counting to adding.

Further results of the investigation have been described in Franke (1998).

References

ANDERSON, J.R.: The Architecture of Cognition. Cambridge: Harvard UP, 1983.

ANGHILERI, J.: An Investigation of Young Children’s Understanding of Multiplication. In: Educational Studies in Mathematics, 20, 4, 1989, pp. 367-385.

BECK, Chr. & Maier, H.: Zu Methoden der Textinterpretation in der empirischen mathematikdidaktischen Forschung. In: Maier, H. & Voigt, J. (eds.), 1994, pp. 43-76.

BÖNIG, D.: Multiplikation und Division. Empirische Untersuchungen zum Operationsverständnis bei Grundschülern. Dissertation Münster 1993. Münster; New York: Waxmann, 1995.

BROWN, S.: Second-grade Children’s Understanding of the Division Process. In: School Science and Mathematics, 92, 2, 1992, pp. 92-95.

BURTON, G.M.: Young Children's Choices of Manipulatives and Strategies for Solving Whole Number Division Problems. In: Focus on Learning Problems in Mathematics, 14, 2, 1992, pp. 2-17.

CARRAHER, T.N.; CARRAHER, D.W. & SCHLIEMANN, A.D.: Mathematics in the Streets and in School. In: British Journal of Developmental Psychology, 3, 1, 1985, 21-29.

CARRAHER, T.N.; CARRAHER, D.W. & SCHLIEMANN, A.D.: Written and Oral Mathematics. In: Journal for Research in Mathematics Education, 18, 2, 1987, 83-97.

FRANKE, M.: Zum Arbeiten mit arithmetischen Schüleraufgaben im Mathematikunterricht der Unterstufe. Erfurt 1986 (unpublished manuscript).

FRANKE, M.: Kinder bearbeiten Sachsituationen in Bild-Text-Darstellung. In: Journal für Mathematik-Didaktik, 19, 1998, 30 pp. Journal für Mathem.Didaktik 1998, 89-122

GREER, B.: Nonconservation of Multiplication and Division: Analysis of a Symptom. In: Journal of Mathematical Behavior, 7, 3, 1988, pp. 281-298.

GREER, B.: Multiplication and Division as Models of Situations. In: Grouws, D. (ed.): Handbook of Research on Mathematics Teaching and Learning. New York u. a.: Macmillan, 1992, pp. 276-295.

KINTSCH, W. & GREENO, J.G.: Understanding and Solving Word Arithmetic Problems. In: Psychological Review, 92, 1, 1985, pp. 109-129.

KOUBA, V.: Children's Solution Strategies for Equivalent Set Multiplication and Division Problems. In: Journal for Research in Mathematics Education, 20, 2, 1989, pp. 147-158.

LAVE, J.: Cognition in Practice. Mind, Mathematics and Culture in Everyday Life. Cambridge u. a.: UP, 1988.

LAVE, J. & WENGER, E.: Situated Learning. Legitimate Peripheral Participation. Cambridge: UP, 1991.

MAIER, H.; VOIGT, J. (eds.): Interpretative Unterrichtsforschung. Köln: Aulis, 1991.

MAIER, H.; VOIGT, J. (eds.): Verstehen und Verständigung. Arbeiten zur interpretativen Unterrichtsforschung. Köln: Aulis, 1994.

NESHER, P.: Multiplicative School Word Problems: Theoretical Approaches and Empirical Findings. In: Behr, M. & Hiebert, J. (eds.): Number Concepts and Operations in the Middle Grades. Reston: NCTM, 1988, pp. 19-40.

PÓLYA, G.: Schule des Denkens. Vom Lösen mathematischer Probleme. Berlin: Francke, 1949.

RUWISCH, S.: Lösungsstrategien und Handlungsmuster von Grundschulkindern beim Bearbeiten multiplikativer Sachsituationen. Dissertation. Gießen 1998.

RUWISCH, S.: Children's Multiplicative Problem-Solving Strategies in Real-World Situations. PME 22. Stellenbosch 1998, 73-80.

SAXE, G.B.: Culture and Cognitive Development: Studies in Mathematical Understanding. Hillsdale: Erlbaum, 1991.

SCHMIDT, S. & WEISER, W.: Semantic Structures of One-step Word Problems Involving Multiplication or Division. In: Educational Studies in Mathematics, 28, 1, 1995, pp. 55-72.

SCHOENFELD, A.H.: Mathematical Problem Solving. Orlando u.a.: Academic Press, 1985.

SCRIBNER, S.: Cognitive Studies of Work: Introduction. In: The Quarterly Newsletters of the Laboratory of Comparative Human Cognition, 6, 1/2, 1984, pp. 1-4.

SELTER, Chr.: Eigenproduktionen im Arithmetikunterricht der Primarstufe. Dissertation Dortmund 1994. Wiesbaden: DUV, 1994.

SIEGLER, R.S.: Strategy Choice Procedures and the Development of Multiplication Skill. In: Journal of Experimental Psychology:General, 117, pp. 258-275.

STEFFE, L.P.: Children's Construction of Number Sequences and Multiplying Schemes. In: Behr, Merlyn & Hiebert, James (eds.): Number Concepts and Operations in the Middle Grades. Reston: NCTM, 1988, pp. 119-140.

VERGNAUD, G.: Multiplicative Structures. In: Lesh, R. & Landau, M. (eds.): Acquisition of Mathematics Concepts and Processes. New York: Academic Press, 1983, pp. 127-174.

VERGNAUD, G.: Multiplicative Conceptual Field: What and Why? In: Harel, G. & Confrey, J. (eds.): The Development of Multiplicative Reasoning in the Learning of Mathematics. Albany: SUNY, 1994, pp. 41-59.

WOLLRING, B.: Fallstudien zu frequentistischen Kompetenzen von Grundschulkindern in stochastischen Situationen. In: Maier, H. & Voigt, J. (eds.) 1994, pp. 144-181.

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