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RESULTS OF A LONGITUDINAL STUDY ON MATHEMATICAL ACHIEVEMENTS OF GERMAN AND ENGLISH STUDENTS

Abstract:

Within a joint project of the Universities of Kassel and Exeter, from 1993 to 1996 a longitudinal study on the state and development of achievements in mathematics of German and English students from the lower secondary level was carried out. The same cohorts of English and German students were followed over three years, from the beginning of year 8 until the end of Year 10 (respectively Year 9). They were tested three times in the topic areas Number, Algebra, and Functions/Graphs/Geometry. Selected results of the study are presented and interpreted in this paper.

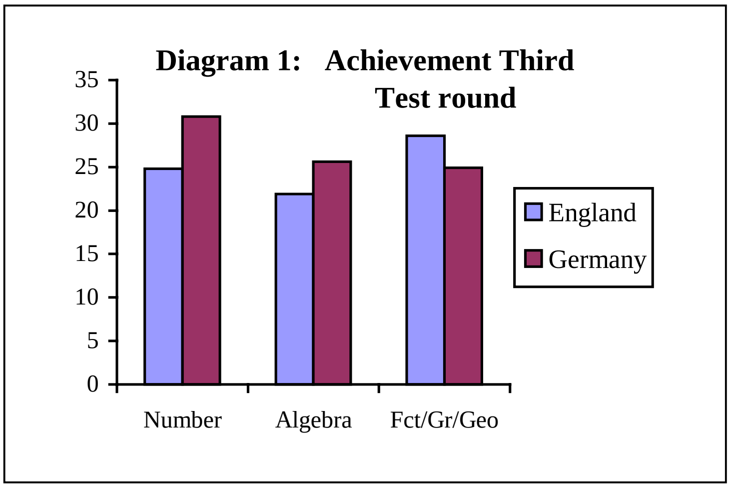
1. Conception of the study

Two of the reasons for carrying out international comparison studies are, to give an insight into the strengths and weaknesses of educational systems and teaching conceptions, and to stimulate reforms in one's own educational system.

In our study, mathematical achievements from two student groups - chosen so as to be as representative as possible - in Germany and England were measured at three different times, namely at the beginning of Year 8 (in England Year 9), at the beginning of Year 9 (in England Year 10), and towards the end of Year 10 (in England Year 11). Students from German lower type secondary school, so-called *Hauptschule*, were tested already at the end of Year 9, since many of these students leave school at the end of that year. In Germany, the sample was stratified according to the different school types and included five federal states, while the total sampling included 34 classes with about 800 students. In Britain a sample was set up with various criteria, such as type of school and results of national contests, with 850 students. Schools from the private school sector as well as from the state school sector with different socio-economic background were included.
Testing was carried out in three topic areas, Number, Algebra and Functions/Graphs/ Geometry, as well as with selected groups of students for Mathematics in Context. This was done with time-limited tests (40 minutes each, in Number and Algebra without pocket calculator), which were based on the mathematical knowledge required at the end of the lower secondary level in both educational systems. Tests were developed on the basis of a detailed curriculum analysis in both countries. Examples of the items are displayed in the appendix. At the beginning all students had to undergo a ‘Potential test’ in order to find out their ‘mathematical ability’ and to enhance comparisons (concerning the conception of this test see Blum et al., 1993, for the results see Blum et al., 1994).
In addition to evaluating the level of achievement at these three times, the aim of this study was the measurement and interpretation of progress made in-between. The interpretation of the results is based on extensive, qualitative classroom observations, which were conducted in a sub-project. Thus we observed about 250 lessons in English mathematics classes and around 100 lessons in German mathematics classes (for a separate description of the study’s results see Kaiser, 1999). For details concerning methodology and results of the first test round, we refer to Blum et al., 1993, and Kaiser/Blum, 1994.

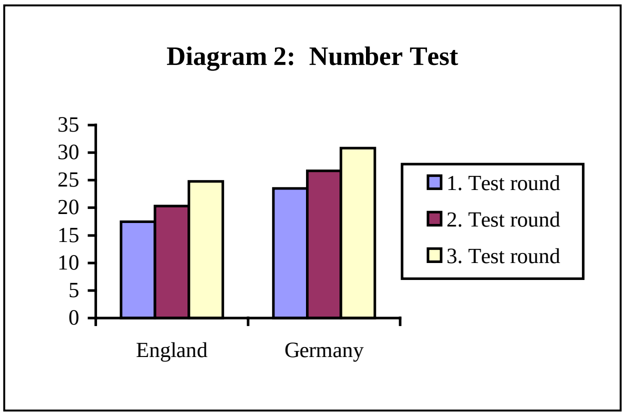
2. Global results of the study

By the end of the lower secondary level German and English students showed significantly different topic area achievements: German students had better results in Number, the English students in Functions/Graphs/Geometry, while there was no significant difference in Algebra. The average scores for both groups are displayed in diagram 1 (with a maximum score of 50 marks).



Differences already appeared during the first test round, where achievements in the different topic areas developed as described below.

In the topic area **Number** - in which most significant differences between both groups occurred - the German students achieved recognisably better results at the beginning of the study (see diagram 2).

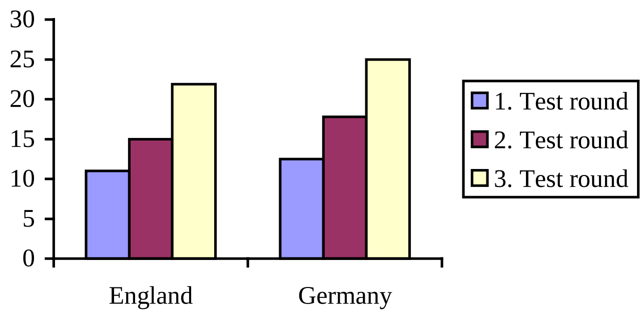


The differences remained quite similar, because both groups made learning progress rating 7.3 marks. Considerable differences in achievement over all three test rounds became obvious, especially in calculations with fractions and decimal numbers. When considering context-bound items, the differences are significantly smaller.

The greatest progress has been made, by both sets of the students, in the same topic areas, in calculations with decimal numbers and scientific notation with real numbers (especially from the second to the third test round). However, the opposite effect was also apparent (although only on a small scale): The German students' achievement declined slightly in calculations with natural numbers, mainly from the second to the third test round, whereas English students displayed this mainly from the first to the second test round in applied calculations with natural numbers.

In **Algebra**, German students obtained better results at the beginning, and during the following school years differences increased even more, as German students improved their achievements by 12.5 marks, contrasting with the English by 10.9 marks (see diagram 3).

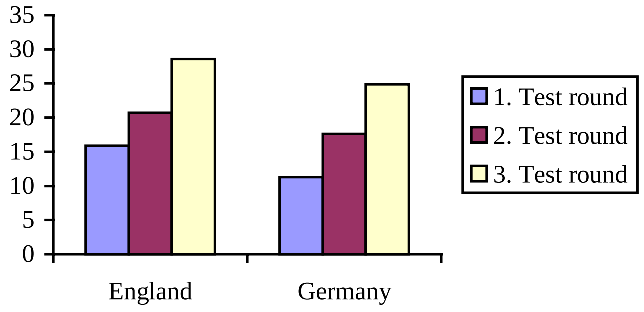
Diagram 3: Algebra Test



The German students had the greatest lead at the end of the study with manipulation of linear, quadratic and higher terms, while English students were better at the solution of systems of linear equations. In other topic areas, such as sequences and quadratic equations, both groups made quite similar progress. Basically most significant learning progress had been made by both groups in the same topic areas, namely in manipulation of linear terms from the first to the second test round, and in manipulation of non-linear/quadratic terms, as well as with powers, from the second to the third test round.

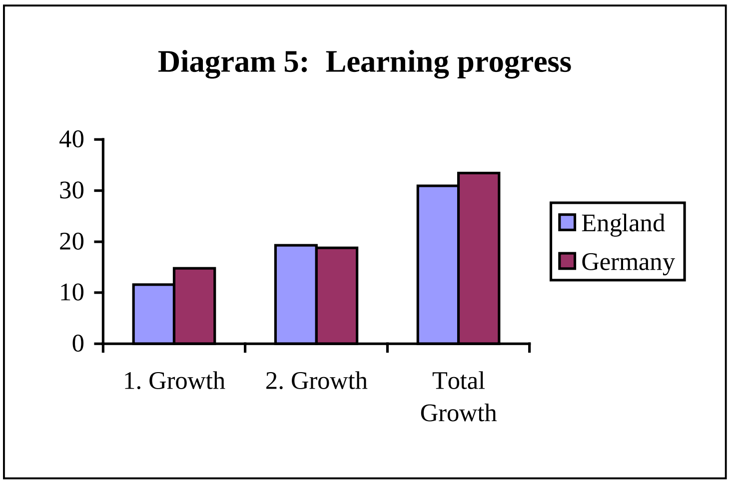
In the topic area covered by the **Functions/Graphs/Geometry** test English students achieved far better results. However, differences decreased slightly during the years, as German students, with 13.7 marks, made slightly greater learning progress than the English-ones with their 12.7 marks (see diagram 4).

Diagram 4: Fct./Graph./Geom. Test



The greatest differences in favour of English students became obvious in symmetry of plane figures, recognising elementary graphs of functions, and in dealing with real graphs, whereas in calculations on solids and co-ordinate systems only slight differences were found. The greatest learning progress was made by both groups in the last test round, namely in volume and surface area of cylinders, graphs of quadratic functions, and in recognising graphs of elementary functions. Moreover, German students made great progress in calculations on the circle, but went back a few steps in co-ordinate systems.

On the whole it can be seen, that in Germany the learning progress with a total of 33.4 marks is higher than in England with 30.4; much of this progress, for German students, was made between the first and the second test round (see diagram 5).



These study results can be *interpreted* as follows:
Differences of achievement in the tests reflect specific emphases that both countries set in curriculum and teaching. For instance, **number** work is a central topic in German mathematics teaching, in all school types, especially until Year 8, where the main stress is put on fractions and calculations with decimal numbers, whereas much less importance is placed on this in English mathematics teaching. Likewise, **algebra** is of much higher importance in German than in English mathematics teaching, especially at the higher type secondary school of the German tripartite system, the so-called *Gymnasium*. However, some topic areas are introduced earlier in English than in German mathematics teaching, which might have caused the small differences between the German and the English students. The reasons for the overall better results in the test on **functions/graphs/geometry** are twofold: On the one hand **geometry** is explicitly a main topic area in England, especially with respect to constructive aspects. On the other hand, although **functions** are more important in Germany, the main emphasis of our test had been put on **graphs**, which is more stressed in England.

The different learning progress results from the different curricular structure of the two countries: In English teaching, by reason of a kind of ‘spiral-shaped’ curriculum, many topics are treated earlier, a fact which caused a strong ‘catch-up effect’ among German students. Due to the small role of repetitions in the German curriculum, forgetting effects among German students were to be seen, particularly from the second to the third test.

There were also significant differences in all three tests between the various **types of schools within the German tripartite system**. However, differences in the area of number are less significant than in algebra and in functions/graphs/geometry, where students of the lower type secondary school, so-called *Hauptschule*, generally achieved low results. This too reflects different curricular emphases in the German tripartite school system. A further evidence of this is that, in the *Hauptschule*, from the second to the third test round, even achievement setbacks in number and geometry were registered, which could be explained partly by lacking motivation and by achievement stress due to pre-vocational selective exams for apprenticeship.

Considerable **country-specific differences** can be recognised in the distribution of the results achieved. For instance, we can find among English students at the beginning of Year 8 a pronounced tip, representing best achievement, a flat area representing average achievement, and a large area of lower achievement. Achievements diverge further until the beginning of Year 9, and differences are even more obvious until the end of Year 10. Generally, achievement is improving for most students, though there remains a part of lower achievement which cannot be neglected; the top band of achievement is growing larger also. The diagrams 6.1 to 6.3 illustrate this development with the example of the Number test, but this pattern can be found in the same way for all three topic areas.

Diagram 6.1: ENGLAND; Number Test, First Test round

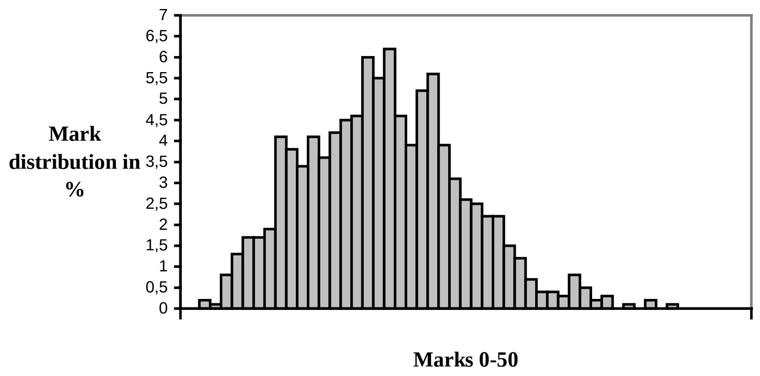


Diagram 6.2: ENGLAND, Number Test, Second Test round

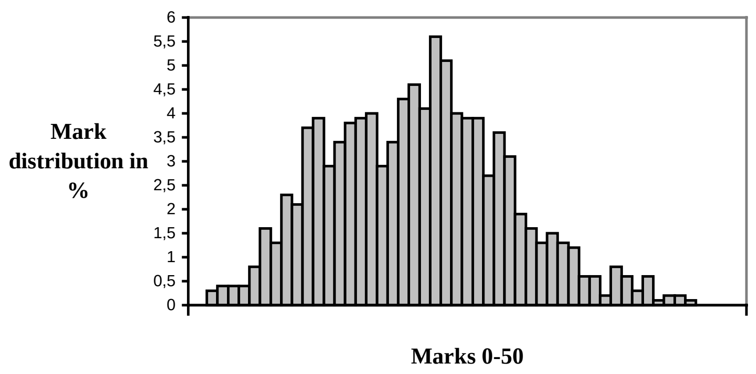


Diagram 6.3: ENGLAND, Number Test, Third Test round

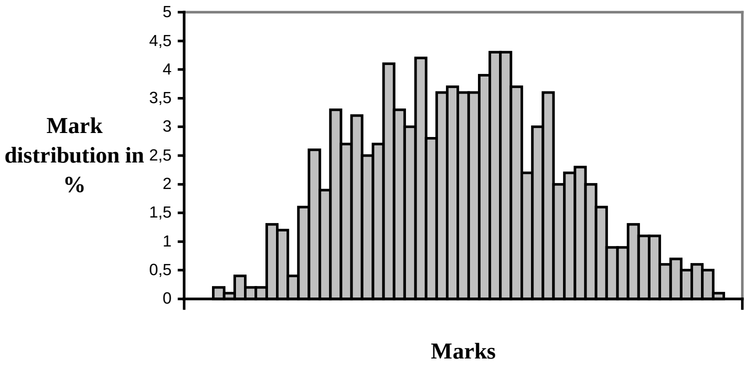


Diagram 7.1: GERMANY, Number Test, First Test round

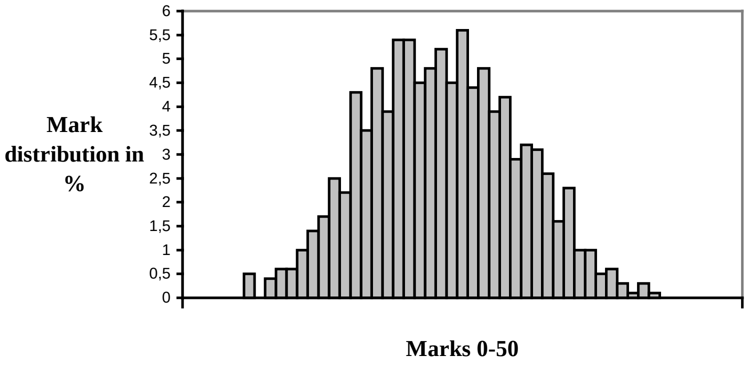


Diagram 7.2: GERMANY, Number Test, Second Test round

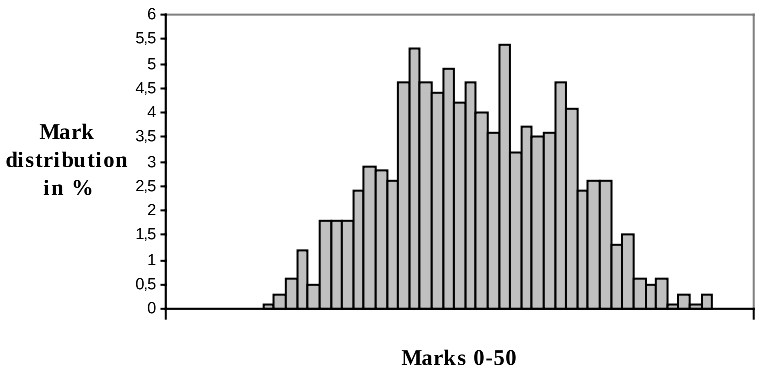
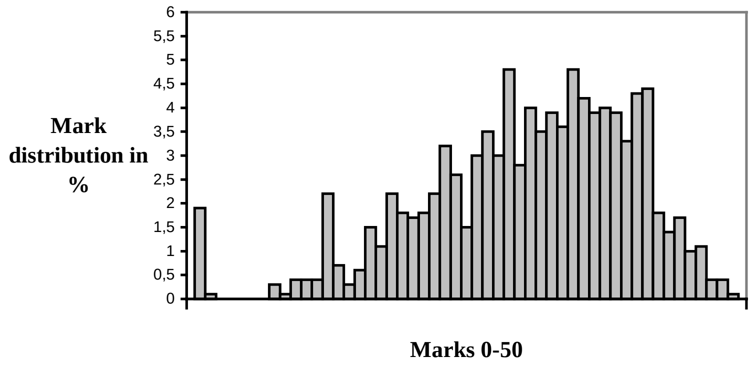


Diagram 7.3: GERMANY, Number Test, Third Test round



Among German students, a different pattern can be observed. At the beginning of Year 8 there is a broader area of average achievement, but neither a clear superior achievement area, nor a broader area of low achievement is found. At the beginning of Year 9 distribution of achievement remains quite similar. Towards the end of Year 10 achievement development becomes significantly divergent, with a greater part of students in the superior third and a smaller number of students in the lower third of achievement. The diagrams 7.1 to 7.3 illustrate the development in Germany.

The differences can be interpreted according to the background of different educational traditions and philosophies of the English and German school system. Roughly speaking, the English tradition of education is mainly characterised through its elite orientation and its emphasis on the individual. In contrast to this, German educational traditions are based on a characteristic trend of egalitarianism, where teaching is connected to year groups, where all students (within the tripartite school system) proceed together at the same pace in the learning process.

3. Results in detail

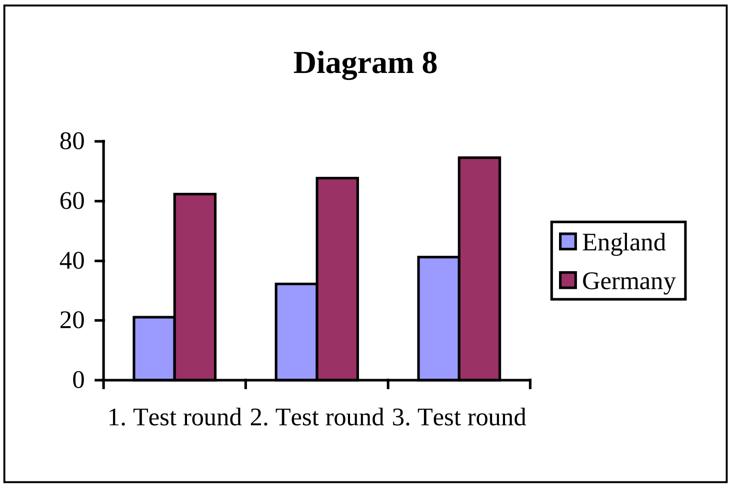
The global descriptions of chapter 2 will now be exemplified using results from selected items. From each of the three topic areas Number, Algebra, Functions/Graphs/Geometry two sample items will be discussed, which represent a certain class of items or a specific ability component and illustrate a general trend.

First, concerning the *Number test*: as previously described, the most significant differences in favour of German students were found in the ability component of calculations with decimal numbers and with fractions.

This will be demonstrated with the following example which was posed in all three test rounds without modification:

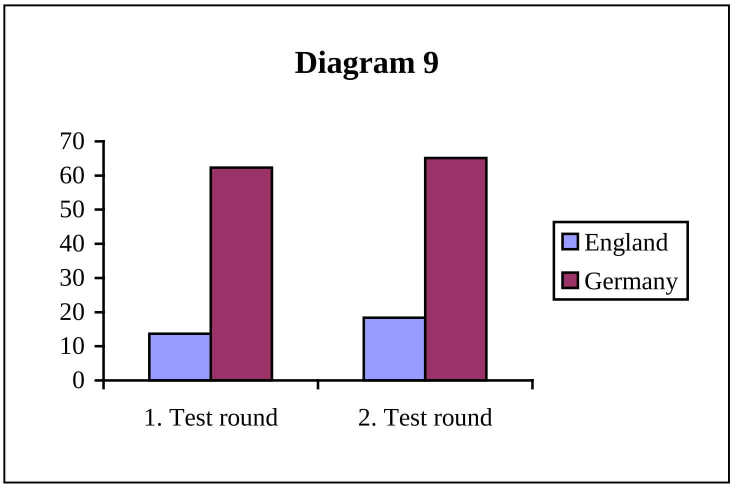
21. $\frac{1}{2} - \frac{1}{3} =$

The following results were achieved (diagram 8):



Similar results were also obtained with the following item (diagram 9):

33. $\frac{1}{3} : \frac{1}{9} =$

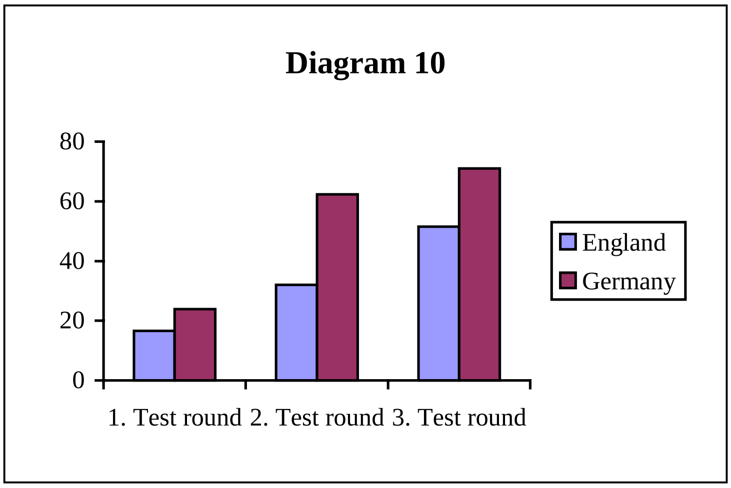


The results reflect - as already stated - different curricular emphases and the earlier usage of pocket calculators in English mathematics teaching. On the whole the results of the German group, concerning simple calculation exercises, can be judged as more or less satisfactory, though one must consider that approximately one third of the students cannot solve easy calculation problems with fractions, as exemplified in item 21.

In the *Algebra test*, the greatest differences to the advantage of German students are found in the ability component of term manipulation, that is, in manipulating linear, quadratic or non linear terms.
This can be shown with the following example:

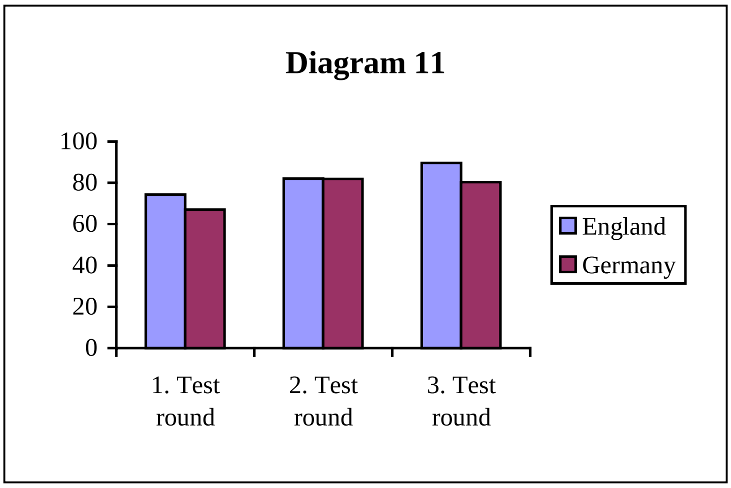
11. Simplify (a) 2 (a+2a)

This example demonstrates a clear increase of achievement among German students from the first towards the second test round, whereas English students catch up from the second to the third test round (see diagram 10).



This result too seems to be rightly explainable through the different importance given to algorithmic skills in both countries. Once more the German results in calculations are quite respectable. However, the situation changes in the area of 'applied' problems, which also can be solved in a purely content-related way. This will become clear by the following example, for which German students achieved lower results in year 8 and 10 (see diagram 11):

3. Rachel thinks of a number. She doubles it, takes away one and gets three.
   What was her number?

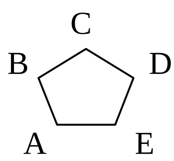


In the *Functions/Graphs/Geometry test* the greatest differences in favour of English students came out in the topic areas symmetry, angles, and straight lines. The following example demonstrates this:

7. ABCDE is a regular pentagon.

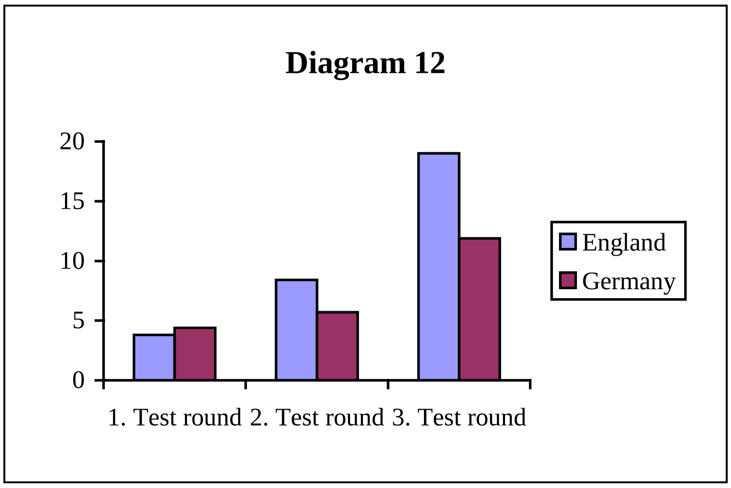
   What is the size of the interior angle

   angle $\angle$ ABC ?



Not to scale

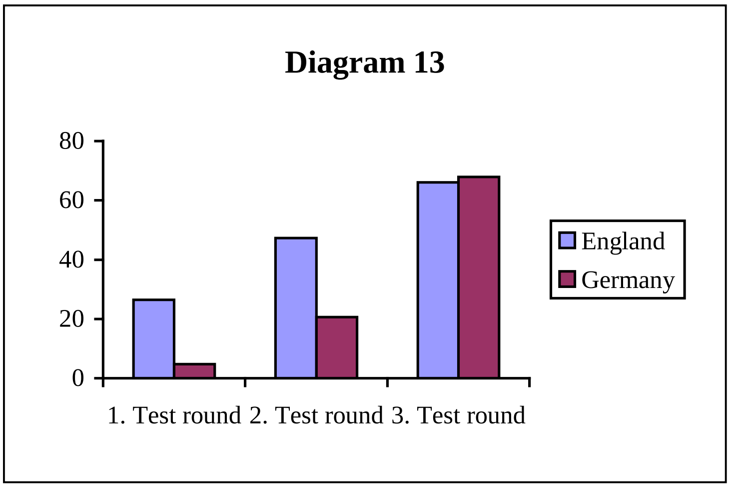
In diagram 12 the differences in achievement between both student groups to the advantage of the English students are clearly obvious, and with time expand further.
The overall very low solution level of the item can presumably be explained by the fact that its thematic background - particularly in Germany - is not embodied in the curriculum. On the whole, the results in this test can again be well explained with the different curricular importance given to geometrical topics in both countries. Geometrical topics have lost much of their importance in English mathematics courses too, but not as much as in German teaching.



The most significant progress of achievement among German students becomes obvious for the ability component of plane figures (in this case particularly the circle) and solids, mainly from the second to the third test round. This will be illustrated with the following example:

9. (a) A circus ring is designed as a circle with diameter 15 metres.
   What is the length of the circumference of the ring?
   (Give your answer in metres to 2 decimal places.)

In diagram 13 different developments in the solving frequency of both students groups are easily visible.



The great increase in achievement among German students, particularly from the second to the third test round, as well as the great lead of the English students in both previous test rounds, can be well explained by the different curricular position given to the topic ‘circle’ in mathematics teaching in both countries. Thus, in English mathematics teaching the topic circle is taught much earlier than in Germany, where it is planned to be taught in Year 10. However, the failure rate of a third for this easy word problem (the formula was given, pocket calculator allowed) seems to be unsatisfactory.

4. Summary of the German results, and prospects

To finish with, we would like to point out briefly some results of the *Mathematics in Context Test*, which we used only selectively and which entails broader mathematical comprehensive problems in context, for example with ‘money matters’, ‘mosaic stones’ or, ‘posting parcels’; the last example is displayed in the appendix. The German students generally had great difficulties with these non-routine problems; especially when asked to provide explanations, they showed little motivation and endurance.

Conclusion: Our study shows that German students achieved good results with problems for which they could use learned and intensively exercised algorithms from arithmetic and algebra. Their achievements were much poorer in geometrical problems or those demanding non-routine tasks, which required some autonomy. This corresponds to results, obtained from our curriculum analysis and classroom observations, that the implemented and even more the realised curriculum in Germany puts much emphasis on reproductive calculation skills, whereas abilities such as mathematising, interpreting, or reasoning, are quite neglected. Such abilities cannot be developed only by transfer, they must be fostered deliberately.

Globally we may state (though it may sound trivial), that mathematics teaching in Germany achieves effects corresponding to its priorities. On the one hand this is encouraging: teaching **has** effects, and these can be influenced. On the other hand, obviously not all effects are desired to the same extent, and on the whole the teaching results are insufficient.

These findings are in good accordance with results from the Third International Mathematics and Science Study (TIMSS) for the lower secondary level (see Baumert/Lehmann et al., 1997). The fact that in our study - in contrast to TIMSS - German pupils globally had significantly better achievements than British pupils is presumably due to the fact that our tests are more strongly oriented towards the curricula which is actually taught, and therefore entails a much greater quantity of routine items than TIMSS.

In further studies we intend to examine, among other things, the abilities of German and English students to use mathematics in real-world contexts. In addition, we will establish stronger connections between our study and TIMSS, by specific subject-oriented analyses.

REFERENCES

BAUMERT, J.; LEHMANN, R. et al. (1997). TIMSS - Mathematisch-naturwissen-schaftlicher Unterricht im internationalen Vergleich. Opladen, Leske + Budrich.

BLUM, W. et al. (1993). British/German Comparative Project: Some Preliminary Results. In Teaching Mathematics and its Applications, 12, 1, 13-21.

BLUM,W. et al. (1994). Entwicklung und Erprobung eines Tests zum Vergleich der mathematischen Leistungsfähigkeit deutscher und englischer Lernender in der Sekundarstufe I. In: Journal für Mathematik-Didaktik, 15**,** 1/2,149-168.

KAISER, G.; BLUM, W. (1994). Vergleich mathematischer Leistungen deutscher und englischer Lernender in Klasse 8. In: mathematica didactica, 17, 2, 17-52.

KAISER, G. (1999). Comparative Studies on Teaching Mathematics in England and Germany. In: KAISER, G. & LUNA, E. & HUNTLEY, I. (Eds.) (1999). International Comparisons in Mathematics Education. London, Falmer Press, 140-150.

Appendix

Selected items of the NUMBER TEST

13. $900 \div 30 = ?$

23. $\frac{2}{5}$ of a mass is 30 g. What is the mass?

25. Express $\frac{1}{8}$ as a percentage.

31. $490 \div 0.7 = ?$

32. Taking 8 km as 5 miles, estimate the distance of a 3000 metre race in miles.

42. Simplify as far as possible $\frac{\sqrt{147}}{\sqrt{3}}$

45. 8% of the total fuel capacity of a lorry is equal to 9.60 litres.
What is the total fuel capacity?

Selected items of the ALGEBRA TEST

11. Simplify (c) $(12a^3) \div (4a)$

15. A ball is dropped from a height of 12 metres. It bounces on the ground and reaches $\frac{3}{4}$ of the original height. It continues to bounce in this way, each time rising to $\frac{3}{4}$ of the previous height.

What height does the ball reach after (a) one bounce
(b) three bounces?

19. Solve
$$x + 2y = 3$$
$$2x - y = -4$$

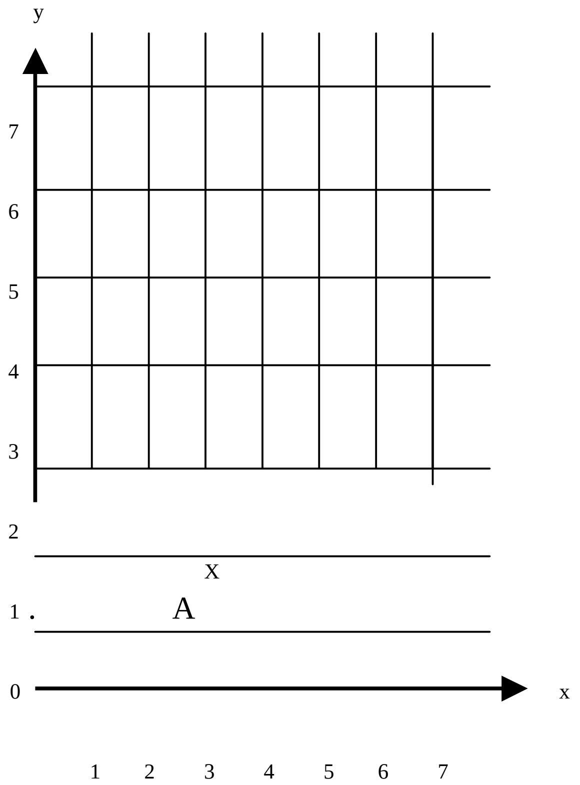
22. Multiply out:

(b) $(x + a)(x - a)$

23. (a) $x^6 = \left(x^2\right)^n, \quad n = ?$

Selected items of the FUNCTIONS/GRAPHS/GEOMETRY TEST

1.



(a) Write down the coordinates of A.

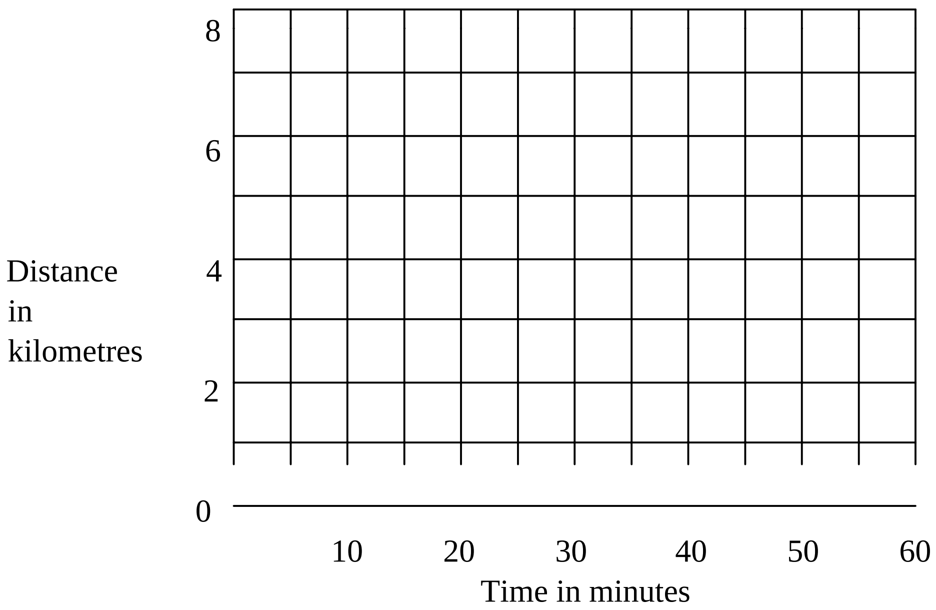
(b) B is the point (6, 5).
Mark the point B on the grid.

5. **M A T H S**

(a) Which of the letters above have **just one** line of symmetry?

(b) Which of the letters have **two** lines of symmetry?

11. (a) Karen is taking part in a sponsored walk. She jogs at 12 kilometres per hour but has to rest for 10 minutes after every 10 minutes of jogging. Draw a time/distance graph below to show her progress.



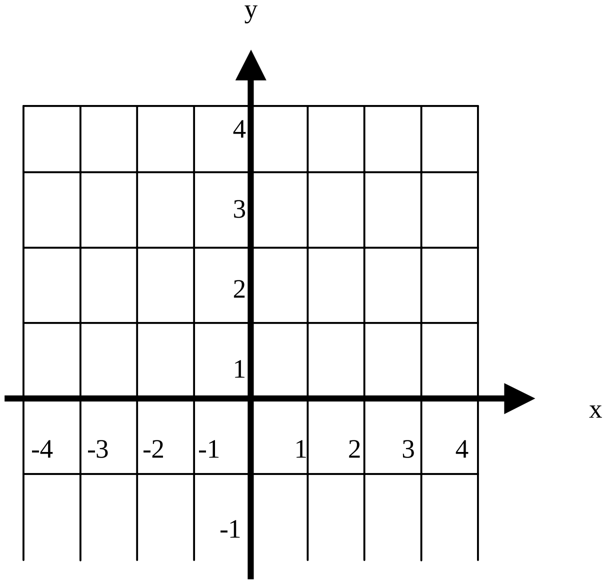
(c) How far has she gone after one hour?

13. A solid cylinder has a radius of 14 cm and a length of 8 cm.

(a) Find its volume, giving your answer to the nearest cm^3.

(b) Find the total surface area, giving your answer to the nearest cm^2.

14.



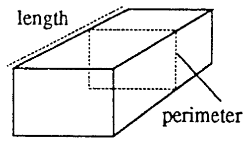
(a) On the grid above, draw the lines with equations:
(i) $x = -2$ (ii) $y = 1$ (iii) $y = x$.
Label each line clearly.

(b) Write down the equation of another line that is parallel to $y = x$.

Topic 1 Posting Parcels

Parcels sent by *Parcel Post* in the UK must satisfy the following conditions.

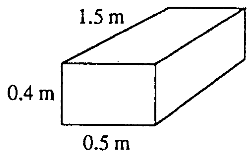
> Maximum length: 1.5 m
> Maximum total length and perimeter combined: 3 m



For cuboids take the length as the largest dimension.

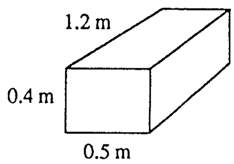
1. Do the following parcels meet these conditions? ***Show all working.***

(a)



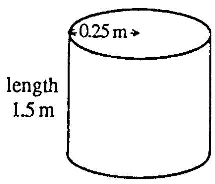
Yes / No

(b)



Yes / No

2. Does this cylindrical parcel meet the conditions above? ***Show all working.***



Yes / No

3. What are the dimensions of the cuboid with **maximum volume** which meets the conditions given above?

dimensions: ☐ by ☐ by ☐