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## **NARRATIVE ELEMENTS OF CHILDREN'S ARGUMENTATIONS IN PRIMARY MATHEMATICS CLASSROOMS**

### **Abstract:**

Results from two related research projects about processes of argumentation in primary mathematics classrooms will be presented. The central research interest is to examine the relationship between the participation of students in argumentative processes and their individual content-related development. Hereby, the focus is on mathematics teaching and learning situations in regular classroom settings. Illustrated by the interpretation of two joint solving processes of third-graders this paper shows the narrative character of these processes. The theoretical relevance and some practical implications of this approach will be outlined, finally.

### **Introduction**

Some results from two related research projects about processes of argumentation in primary mathematics classrooms will be presented. The projects are "Arguing in primary mathematics classrooms" sponsored by the state of Baden-Württemberg, Germany in 1994 and 1995, and "The reconstruction of 'formats of collective argumentation' in primary mathematics education" sponsored by the German Research Foundation (DFG) since October 1996.

The central research interest is to examine the relationship between the participation of students in interactional classroom processes and their individual content-related development. Hereby, the focus is on mathematics teaching and learning situations in regular elementary classes. The theoretical orientation of these projects is based on ethnomethodology (GARFINKEL 1967), symbolic interactionism (BLUMER 1969) and cultural psychology (BRUNER 1996). From these perspectives classroom situations are seen as *processes of interaction*: Students and teachers contribute to their accomplishment according to their insight into the sense and purpose of these events. They act as it seems *sensible* and *tenable* to them. For this, they *interpret* these classroom situations: they reflect, set up and review hypotheses, and make rational decisions; common features are found which temporarily enable them to cooperate. In such a classroom situation they develop their content-related understanding in order to be mutually regarded as responsible and capable and in order to participate in the joint creation of this interaction. Thus, in such a situation their mind is challenged, which they employ and develop by the way.

In both projects the research approach concentrates on (collective) argumentation (see KRUMMHEUER 1995; KRUMMHEUER & YACKEL 1990). This paper stresses the narrative character of this process. In the first chapter the view on learning as a social phenomenon will be outlined. Here, the concepts of "culture" and "classroom culture" play a crucial role. The second chapter deals with the narrative feature of those classroom cultures. In the third part two examples of collective task solving processes among three third-graders will be presented in which the narrative character of their joint argumentation process will be explained. The last chapter summarizes the results, integrates them in the larger research-context and shows practical implication.

## 1. The social constitution of learning

In the discussion about conditions of learning it is more frequently stressed that the entire phenomenon of human learning is not spanned if one confines one's studies to the interior, mental processes of the learner. Learning is also a social process which takes place in the interaction between human beings:

... where human beings are concerned, learning ... is an interactive process in which people learn from each other (BRUNER 1996, p. 22; see also BAUERSFELD 1995, BRUNER 1990, ERICKSON 1982, and KRUMMHEUER 1992).

Thus, learning is socially constituted, or, in other words: interactive processes are vital parts of the "nature" of learning.

This insight led BRUNER 1986 to formulate the feature of a psychology which constitutionally considers social elements for psychic processes, as for example in the course of learning. He defines this as *cultural psychology* (p. 35) and explains that each individual development must be expressible in the particular symbolic system of a given culture. For this, the members of a culture not only have the general means of their language but, additionally, they can also employ specific culturally accomplished ways to interpret the psychological disposition of individuals. BRUNER 1990 defines this as *folk psychology* (e.g. p. 33ff). Regarding the teaching- learning-process in a classroom situation he speaks of *folk pedagogy* (BRUNER 1996, p. 46). These two concepts include the mostly implicit basic assumptions of a culture about the psychological functioning of its members.

All cultures have as one of their powerful constitutive instruments a folk psychology, a set of more or less connected normative descriptions about how human beings 'tick', what our own and other minds are like, what one can expect situated action to be like, what are possible modes of life, how one commits oneself to them, and so on. We learn our culture's folk psychology early, learn it as we learn to use the very language we acquire and to conduct the interpersonal transactions required in communal life (BRUNER 1996, p. 35).

In this quotation it is also stated that such folk psychological insights are acquired at an early age and that they are already linked with the learning of one's own native language. The folk psychology is acquired through narrative interaction, in other words, their insights are learned from stories. BRUNER designates this way of "learning from stories" as an independent mood. On the one hand he contrasts this mood with a logical and scientific way of thinking, on the other hand, he regards it as a learning mechanism through which a child develops fundamental views and perspectives of the world it lives in (s. 1986, p. 11ff and 1996, p. 39f).

In this sense, learning is not only the appropriation of culture, it is implemented in its co-creation. Especially with regard to primary education it is often stated that basic cultural techniques such as reading, writing and arithmetic are taught and acquired here. From the developed perspective of cultural psychology, this seems to be an insufficient point of view: Children do not only learn the content of culture. Rather, through their contributions in reading, writing, and calculating they also create "a" or "the" culture. Concatenating these two aspects one arrives at what could be described as *classroom culture*. Participation in this double sense

integrates the social constitution of learning. Thus, **classroom culture is a culture of subject matter and a culture of learning.**

## 2. The narrativity of classroom culture

Obviously, in primary education children like to listen to and to tell stories. The argument presented here goes beyond this empirical evidence and claims that the children learn by these stories. Hereby they learn the content of different school subjects. The classroom culture of our primary schools is characterized by narrativity: frequently, the different contents are presented in a narrative style, and the social constitution of classroom-learning can be described in related models of participation in situations of story-telling. This is also relevant for mathematics classes, and the analysis of processes of interaction concerning this subject matter can demonstrate the importance of this thesis in general.

In the following, firstly, some characteristics of such narrations in the observed mathematics classes will be presented. The aim is to clarify which aspects of narrativity are most relevant in mathematics classroom-interaction. It will be shown that it is of an argumentative nature and that argumentation dissolves in a narrative presentation (see also the concept of "reflexive argumentation" in KRUMMHEUER 1995). This means: **The narrative classroom culture of primary education is based on rationality, and the social constitution on classroom-learning is the participation in the interactional accomplishment of argumentative, narratively structured sequences of actions.**

This thesis does not imply that in classroom-situations "stories" will be told endlessly and that beyond educational goals in native language classes, classroom-education intends to teach children the telling of stories. Rather it is that the negotiated theme in a classroom-interaction emerges more frequently in such a way that one can reconstruct aspects of a narrative process. Thus the concept of "narration" is used here in order to describe a specific phenomenon of everyday classroom-conversation. It is not meant in the sense of literary science.

According to BRUNER 1990 one can identify four characteristics for narrative accomplishments:

1. "sequentiality"
2. "factual indifference between the real and the imaginary"
3. "unique way of managing departures from the canonical", and
4. "dramatic quality" (p. 50).

Here, the first and third points are of special interest. The claimed narrativity of classroom-culture is seen in the patterned sequentiality of classroom-interaction. The specificity of an event, such as the elaborated solving process for a new mathematical task, is presented in relation to the canonical management of such events or problems.

Classroom-processes display some specificities in which they differ from usual narrations:

- Frequently, students and teacher complement each other in the role of the story-teller. Thus, there are no definite roles of the "listener" and the "teller". Usually, several persons are engaged in generating a story.
- Not only stories about the past are told, but also stories about something new emerge. Usually one associates with the concept of "narration" the image that something that is already over comes to the fore. One can describe this as the *presentation* of facts (KALLMEYER & SCHÜTZE 1977, p. 159 f). In our observed group-work about mathematical problems we see also a narratively characterized interaction, during which the children accomplish their single steps of calculation and at the same time

express what they do: in such cases they "tell" or "narrate" how they come to their solution, or better: how one can come to a solution. One could rather call this the *constitution* of facts. In this constitution one can identify typical elements of a narration, which is the ordered presentation of a concrete case, in which a problematical situation is managed (see e. g. BRUNER 1986, p. 16 ff; 1990, p. 47 ff; KRUMMHEUER 1997).

.In the following, more evidence will be given to this theoretical approach by reconstructing in detail two classroom-episodes.

### 3. Two examples

Two examples might help to illustrate and differentiate the thesis: Two or three children work together in order to solve a given problem. A successful cooperation demands two different cognitive achievements from these children. They have to clarify

- what shall be done at what time and
  - who shall do it at what time
- Both cases have to do with the decision about the right moment whereas
- the first point deals with the insight within a sequence of solving steps and their chronological order and
  - the second point has to do with the sequence of interactional moves and the chronological change of speakers.

The structure of actions and interactions with regard to the first issue can be called an "academic task structure" (ERICKSON 1982). It is based on the understanding of the situation of the problem as shared among the students. It is not identical with logical considerations about a sequence of solving steps according to the subject matter of mathematics (see VOLLMER & KRUMMHEUER 1997). A second issue can be described in reference to ERICKSON as a "social participation structure". Both structures are mutually dependent (see ERICKSON 1982, p. 156 and VOLLMER & KRUMMHEUER 1997).

In the following, I will illustrate some aspects of the academic task structure (ATS) by presenting two examples of the mentioned research projects. The first example will help to clarify four general aspects of the ATS. The second example will focus on one of these aspects which will be called the innerlogic or the plausibility of a solution.

#### 3.1. The shirt-problem

The boys Daniel, Slawa and Stanislaw from a third grade are confronted with the presentation of numbers at the back of T-shirts which represent the first parts of a number sequence. Their task is to determine the fifth element of this sequence which is:

{3 - 8 - 15 - 24 - ?}

For this sequence, Slawa can quickly give solution:

47 Slawa (*pointing at the picture*) Here comes five, here comes seven´..

49 Slawa here comes (.) nine´

52 Slawa He gets an eleven-

53 Daniel Why eleven´

54 Stanislaw Why´

55 Slawa Well eleven. Look´, (*precariously whispering*) how much plus three,

- 56 look', at this number. five-
- 57< Daniel Well', from three to eight are five.
- 58< Slawa (*directed to Daniel and still pointing at the picture*) here comes already
- 59 seven', seven-
- 60 Daniel seven-
- 61 Slawa nine' (.) eleven.
- 62 Stanislaw (*inarticulate*) well yeah.
- 63 Slawa Eleven plus twenty-four. add it here. then one gets (*figures about 2 sec*) thirty-five.

From a mathematical stance, one can view in Slawa's solution the thematization of the general concept of the sequence of differences and the first four numbers of a specific sequence of differences {5 - 7 - 9 - 11}. The boy cannot name them. He does not define them explicitly and in a certain way he is not talking about them, but through them. His two classmates cannot follow him. Slawa is obliged to explain; generally, he reacts in the way just described: He names the four elements of the sequence of difference. One short episode might demonstrate this:

- 77< Slawa This are five. here (*points at paper*) then seven', here comes nine
- 78< Daniel five (*mumbles inarticulately*) from eight to fifteen are seven'
- 79 Slawa add always two to it.
- 82< Slawa Thus here comes eleven', Daniel (*points at number sequence*) here
- 83< Daniel seven' yes from, yes
- 84 comes eleven to that number
- 85 from fifteen to twenty-four are nine.
- 86 Slawa Thus here you get thirty-five . (*inarticulate*) thirty-five.
- 87 Stanislaw Whoop-
- 88 Daniel Yes, nine'

One recognizes how Daniel in <78, 84 and 88> agrees to the numbers 5, 7 and 9 as the difference between the given elements of the initial number sequence. He and Stanislaw as well do not conceptualize the numeration of these numbers as the elements of a number sequence which emerges by finding the differences. Even Slawa's meta-comment about the rule for this sequence of differences in <79> does not help. Slawa's finding of the solution, his presentation and his justifications are narratively oriented. In order to understand his solution one must, firstly, recognize the phenomenon of a sequence of difference and, secondly, the defining characterization  $x_{n+1} = x_n + 2$  while repeating the numbers 5, 7, 9, 11. If somebody cannot infer this argument from the numeration of the numbers, he does not understand the sense of the story at all.

Summarizing, the four following conclusions can be drawn from the interpretation of this episode:

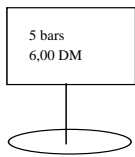
1. The mathematical concepts which are necessary for understanding the ATS are not introduced explicitly. In a narrative way, they are rather pointed at opaquely. Not all students will be able to recognize the ATS by this way of presentation. The plausibility of this solving process might be inscrutable for them.
2. For the accomplishment of the different steps of the ATS the boys need certain mathematical competencies such as addition and subtraction of positive integers.

3. No meta-commentary about the functionality of the ATS or certain steps of the ATS will be given. This will be seen to characterize narratively organized interaction.
4. The presentation of the solution steps proceeds mainly by verbalization. The boys do not use alternative presentations such as visualization or embodiments. This also is a characteristic for narratively organized processes of interaction in mathematical group work.

These four issues describe aspects of the ATS and its narrative generation in group work. In general these are "(a) the logic of subject matter sequencing; (b) the information content of the various sequential steps; (c) the 'meta-content' cues toward steps and strategies for completing the task; and (d) the physical materials through which tasks and tasks components are manifested and with which tasks are accomplished" (ERICKSON, 1982, p. 154).

### 3.2. The chocolate-task

The main interest of this paper being the argumentative elements in group work, the focus of the next example is on the first point: the hidden innerlogic. The thirdgraders Linda and Esther work on the following problem:

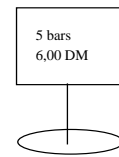


*3 bars of milk-chocolate cost  
3,30 DM.*

*(a) How much do 7 bars cost?*

*(b) 5 bars of nut-chocolate cost 6,00 DM.*

*Are 5 bars of milk-chocolate cheaper?*



"DM" is the abbreviation for "German Mark".

- |     |        |   |
|-----|--------|---|
| 1   | Esther | <i>(writes their names on working-sheet) (reads task) Three bars of milk-</i>     |
| 2   |        | <i>chocolate cost three Mark thirty.</i>  |
| 3   |        | <i>How much do seven bars cost.</i>   |
| 4   | Linda  | <i>(looking at teacher) Can we write down our calculations right here´</i>        |
| 5   | Esther | <i>Hey, look here, five bars cost six Mark´ (points thereby at the picture at</i> |
| 6   |        | <i>the right margin, which belongs to part b of the problem; after this she</i>   |
| 7   |        | <i>points again to part a)</i>  |
| 8   | Esther | <i>nine thirty´, are then for si...nine thirty</i>                                |
| 9   | Linda  | <i>three ... (not reconstructable) (both murmur incomprehensibly)</i>             |
| 10< | Esther | <i>... so, they are six´ bars of chocolate´</i>                                   |
| 11< | Linda  | <i>nine ... (turns away and murmurs something to herself) (4 sec)</i>             |
| 12  |        | <i>(both think) (8 sec)</i>   |
| 13< | Esther | <i>Just three bars cost (points at left margin of sheet)</i>                      |
| 14< | Linda  | <i>twelve sixty.</i>  |

Esther presents in line 8 a first approach for a solution: 9,30 DM is supposed to be the price for six bars of chocolate. It is assumed here that she takes six Mark as the price for five bars, as it is indicated on the right picture, and she takes 3,30 DM from the left one, apparently ignoring the information that this is the price for 3 bars. Linda's contributions will not be analyzed, here.

Thereafter, both children seem to think for a while silently. Linda seems to begin to reject her first attempt at a solution. This will become clearer, later. Contrarily, Esther's proposal of 12,60 DM will be understood as a somehow consistent continuation of the former approach: The mentioned amount of 9,30 DM was interpreted here as the sum of 6 DM for five bars and 3,30 DM for one bar. Thus adding another 3,30 DM to 9,30 DM reveals the result of 12,60 DM and stands for the price for seven bars of chocolate.

If one takes into account the difficulties one has as an observer attempting to understand this part of the transcript, the apparent ease of the exchanges between the two girls is striking. They do not need any further clarification and comment. It seems as if their actions speak for themselves. Obviously, the addition of 6,00 DM + 3,30 DM transports the justification for this approach simultaneously. Obviously, for the two girls the rationality of their procedure is "self-explanatory" - an explicit explanation is not needed.

At a first glance, the impression persists that the work of the two children is relatively uncoordinated and isolated. The analysis shows, however, that there is an exchange between them. At the end Linda carries on an approach which was introduced by Esther. Both children are aware of the action of each other. In a pragmatist sense one can describe their interaction as cooperative: they present calculations and trust that the mental execution meets with the approval of their partner. As we shall see, they confront the given calculation with a new one in the same way, when wanting to criticize or correct it.

It might be mentioned that shortly after the girls recognize that one has to separate the two parts of the problem and that one has to calculate solutions separately. They develop an ATS in which first of all they determine the price for one bar of chocolate and then conclude the price for seven bars. I mention this because one might feel committed to criticize the formulation of the problem, but one should take into account that the two girls find the right solution relatively quickly. In a certain sense one assumes that in these social encounters mistakes might very often be produced and that the children learn by this nevertheless.

#### **4. Social learning conditions in classroom interaction and an implication for enhancing the classroom culture**

The basic insight of this research is that in classroom teaching and in group work as well a proved folk psychology of learning in this classroom culture becomes apparent which for sure is not given by nature or God but which has two very important features: It functions in everyday classroom situations and it has a rationality.

The rationality of actions expresses itself in the pursuing or novel creation of a sequence of working steps. With regard to ERICKSON 1986 it is called the "academic task structure" (ATS). This is a sequence of actions as it is accomplished *by the participants* in their process of interactional negotiation. In primary mathematics classes, this interactive realization occurs often in a narrative style: the conducted calculations are told according to the sequence of the ATS in as much as the necessary competencies can be integrated. Typically, the inner logic of the total approach within such narratively realized academic task sequences is not explicitly thematized, but it is expected as a specific achievement of the participants. They have to *infer*

this inner logic from the specific presentation of the narrations (for more details see KRUMMHEUER 1995, 1997). This does not usually happen successfully and often not in its entirety. Learning which is related to novel concepts and insights does not happen automatically. But, on the other hand, this kind of narrative classroom culture is characterized by a great stability in everyday primary school teaching and learning situations and there are many students who daily proceed successfully in their content-related learning development by participating in this classroom culture.

With regard to those students who do not proceed successfully, one issue will be emphasized, here: It is the fact that writing and application of other illustrating tools are missing. The observed interaction processes in the two projects are solely based on oral exchange. BRUNER 1996 speaks with regard to this point of the necessity of an externalization tenet.

Externalization, in a word, rescues cognitive activity from implicitness, making it more public, negotiable, and 'solidary'. At the same time, it makes it more accessible to subsequent reflection and meta-cognition (BRUNER 1996, p. 24 f).

The starting point of my argument is that in the project episodes preferentially verbal productions can be observed. Generally, the quick evaporation and the situative uniqueness of verbal accomplishments impedes the reflection on such interactive procedures - at least for some, the so-called "weak" students. Complementing such reflections with a written presentation of not only the result, but especially of the process of working seems helpful. BRUNER 1996 refers to the concept of the *œuvre* of the French psychologist Ignace Meyerson. *Œuvre* does not mean a somehow standardized scientific presentation. It rather means that the children by themselves find a productive form of written presentation of their thoughts. *Œuvres*, produced in such a way, facilitate easier listening and possible repetitions, if necessary.

"Creative and productive writing" in such a sense is not only a category of native language classes, but in general a platform for reflection on classroom related processes of symbolization. It is not the question if the children should write down something that is correct in the sense of the subject matter, but rather that the children are supposed to find means of presenting their thoughts which lasts over a longer span of time.

Such classroom culture provides *all* participants with well-founded possibilities to negotiate meaning productively and to produce shared meaning. The specific problem might be to identify forms of externalization which enable all students and (not only) the teacher or researcher to pursue a specific solving process. With regard to arithmetic one can refer here to standardized iconic ways of presentation. However, they need to be assessed and enhanced for this special use of providing reflection for the students.

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