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ON PUPILS' TEXTUAL EIGENPRODUCTIONS

Abstract:

More and more researchers in mathematics education recommend mathematical writing by pupils as an important activity, in addition to their verbal contribution to classroom communication. A written text in which pupils express their own mathematical ideas in their own language, i. e. by use of words and formulations which are within their individual active language competence and performance, will be called here a 'textual eigenproduction' (TEP)[1]. After a brief characterisation of TEPs and discussing their didactical functions, the article deals with the questions of guiding pupils to write eigenproductions and of interpreting relevant texts by the teacher.

1. Character of textual eigenproduction

What are textual eigenproductions (TEPS), and what forms can they take?

In everyday mathematics, classroom pupils have rather a lot of writing to do. However, the main part of their writing is restricted to noting steps of the solution process and results of arithmetical or algebraic tasks, such as calculating the value of number terms, transforming terms containing variables, and solving equations. These formalised protocols almost always follow rather exclusively fixed algorithmic procedures and standardised patterns of symbolic representation; for this reason we do not call them TEPs. Protocols of problem solving procedures, however, can easily be extended to TEPs when the pupils – using all forms of language actively available to them, including their everyday language – give a detailed description of how they consider the problem content, and what their aim(s), ways, and method of solution are. In addition, they point out reasons and justifications for their methods of solutions and their results. The TEPs arising from such activity can be called 'commented problem-solving protocols' (see POWELL & RAMNAUTH 1992).

Of course, TEPs must not remain restricted to this one type. DAVIDSON & PEARCE (1983) distinguish five categories of mathematical writing: reproductions of a presented text (direct use of language); translations from symbolic into verbal language and formulations of the results of word problems or verbalisations of the steps of an algorithm (linguistic translation); the reconstruction of a presented text, summaries of a talk, diary texts or explanations of a mathematical concept (summarising); applications of a mathematical idea in a new problem context or invented word problems on a particular issue (applied use of language); and use of language to explain or transmit knowledge not having been a topic encountered previously in the classroom (creative use of language).

I myself prefer to classify TEPs in the following way:

- commented problem solving protocols (as described above);
- reports about mathematical investigations (aims, steps and measures taken, results produced);
- detailed descriptions and explanations of mathematical concepts or algorithms;
- texts defining mathematical concepts, formulating hypotheses, arguments or proofs in relation to a mathematical theorem;

[1] The English word "eigenproductions" is derived from the German word "Eigenproduktionen", used for the first time by SELTER (1994)

– texts initiated by a specific situation requiring the communication of mathematical facts and relations in written form, e. g. descriptions of a complex geometrical drawing in such a way that a classmate is able to reproduce the figure on the basis of this description alone.

Writing, in the sense of TEP, may happen from time to time in the mathematics classroom; it also can become a regular activity for the pupils. WAYWOOD (1992) reports about journal writing, while MILLER (1992) lets pupils write an improvised text about algebraic problems in every mathematics classroom. GALLIN & RUF (1993) call the texts their pupils produce regularly 'journey diaries'; these writings deal with 'core (mathematical) ideas' on which they are expected to reflect and to make conjectures. PHILLIPS & CRESPO (1996) organised a sustained exchange of 'penpal letters' between pupils and teacher trainees. KASPER & LIPOWSKY (1997) asked the pupils to write diaries telling about which mathematical problems in the classroom they liked mostly (and why) and which they did not like as much (and why). They were also asked to highlight what they felt they knew now and did not know before, which kind of learning difficulties they encountered in their learning, and what kind of actions they experienced which were a real help in overcoming these.

2. Didactical function of textual eigenproduction

There are many reasons why TEP should be introduced into pupils' mathematical classroom work, some of which are:

– TEP stimulates the individual pupil to analyse and to reflect on the mathematical concepts, relations, operations and procedures, investigations and problem-solving processes they are dealing with. Thus they can arrive at a greater consciousness, and a deeper mathematical understanding. Writing, more than speaking, leaves time for structuring observations and gathering and ordering thoughts. It enables pupils to explain, clarify, prove and extend their ideas. Writing lets them feel responsibility for the text they are going to produce, since it fixes the outcome of their thinking and acting and makes it accessible to subsequent control, at least by the teacher and the classmates;
– TEP is able to improve pupils' competence and performance in technical language since it leaves them time for reflecting on, and carefully choosing, their means of expression, and thus encourages them to make active use of technical terms and symbols (see MAIER 1989a, 1989b, 1993 and MAIER & SCHWEIGER 1999);
– TEP gives the individual pupil a chance to take control of their understanding of mathematical issues by means of reasoned and reflective feedback from the teacher and other pupils;
– TEP enables the teacher to assess previously and actually constructed knowledge and understanding of mathematical ideas in a more detailed and deeper way than would be possible with commonly written tests which are normally carried out in the manner of non-commented problem-solving protocols.

If pupils are required to produce texts which can give deep insight into their ways of mathematical acting, thinking, and understanding it has to be made sure that they address their TEPs to an audience – to someone who needs full information on the matter written about. Usually they tend to imagine the teacher as the main or even the only addressee of their writing, and he is assumed already to know all they have to communicate. Thus pupils feel that it is their ability to come up to quite specific expectations which is being examined. They therefore feel no need to give a detailed and explicit description and explanation. How can the process be managed so that the pupils address their TEPs to an 'ignorant' i.e. to someone who does not know the solution of the problem or the issue presented, and who, for that reason, must be informed not only in detail but also in understandable language? Possible ways in which this can be done include :

– in order to change their attitude, the writers can be encouraged to take on a role different from that of a pupil. A possible instruction may begin with the phrase “Imagine you were a father/mother, a teacher, ...” (see D’AMORE & SANDRI 1996 and D’AMORE & GIOVANNONI 1997);
– the TEP is written as a letter to a classmate who missed lessons for reason of illness and who needs to be informed about what has been learnt in his absence, or to the inhabitants of a country or a star who do not know mathematics in general or the particular issue in question. Examples of titles: “How we carry out the long division for 2753 / 5”, “How to draw through a certain point a straight-line which is perpendicular to another one”, “How a triangle can be reflected about an axis”, “How to construct a triangle the sides of which are of lengths 7 cm, 5.5 cm and 4 cm”;
– the TEP is designed as a diary, or as a poster for a mathematical exhibition. Possible titles: “Prime numbers present themselves”, “Filling in magic squares”, “About triangle numbers”;
– the TEP has the form of an article for an encyclopaedia. Examples of titles: “Fractions”, “Functions”, “Vectors”;
– sometimes it may also be helpful to de-familiarise the common problem-solving situation by means of open or incomplete tasks (see D’AMORE & SANDRI 1997, 1998).

3. Guiding pupils to textual eigenproduction

The use of TEP in the classroom, with its potential positive effect on the learning process and its evaluation, is certainly not just a matter of course. When the pupils are asked to produce texts of that kind they may feel hesitant and possibly get into difficulty in meeting this particular demand. This is mainly true if it is in a higher grade where they have to produce TEPs for the first time. There they already have firm ideas about what learning mathematics means and what kind of tasks are to be expected within the subject. Going away from normal problem solving by means of fixed algorithms can become a big hurdle for the pupils.

For that reason the writing of TEPs should be started as early as possible in the pupils’ school career, at least in grade two, and should remain a permanently trained activity. There are four kinds of help:

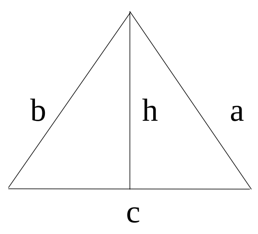
– in the learning of calculation and other kinds of mathematical knowledge, verbal expression as a kind of standardised, concise reading of technical symbols should be postponed to a later phase. Initially the pupils ought to depend extensively on talking even if they use everyday language;
– writing can be more easily started with short texts. The lengths and complexity of TEPs can be raised later, step by step;
– text production should be well motivated. Conceptual objects, for example, can be “personalised”, and the text can take the character of a game or a dispute. Possible titles include: “Pyramid and prism quarrel about their anatomy”, “Isosceles triangle and rectangular triangle in heavy discussion”, “Fractions and decimals point out their respective advantages”;
– it often seems helpful to prepare the production of text verbally.

4. About the interpretation of textual eigenproductions

In most cases TEPs hold much information about the individual pupil's mathematical knowledge and ideas. However this information is by no means evident and obvious. It has to be discovered in the text, and this is normally not possible without very careful interpretation. Thus the teacher needs not only good ideas for effective stimulation of the pupils, but they also have to provide adequate ways of working with the produced texts. Above all they have to be prepared and able to interpret and to analyse the TEPs carefully and competently. It is, indeed, not easy to identify the mathematical concepts, thoughts and ideas which underlie the particular text produced. It needs not only great attention but also a lot of experience and possibly some training as well.

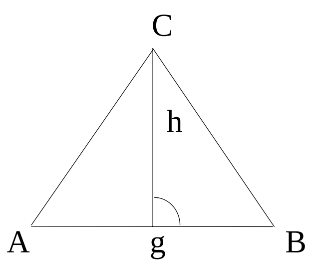
What are possible guidelines for this interpretation of TEPs? We have chosen some quite short TEPs as examples for pointing out and explaining the principles and strategies which should guide the interpretation. They are selected from a set of texts which were written by pupils of different classes from grades 8 and 9, stimulated by this instruction: *Imagine you were a father (a mother) .... Your young child, 7 years old, learnt from somewhere that every triangle has three heights and asks you: "Dad (Mum) what does that mean?" Nothing is more unfortunate than leaving young children's questions unanswered; therefore you decide to give the following reply:....*[2]. The five TEPs are as follow:

(I)



"The triangle has only one height and a baseline by means of which area and circumference can be calculated."[3]

(II)



"A triangle always has one height and this is always from the baseline to the point C. The height always starts perpendicularly from the baseline to the point C."[4]

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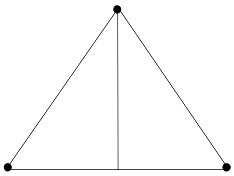
[2] The original German text has been: "Stelle Dir vor Du wärest ein Vater (eine Mutter). Dein 7jähriges Kind hat von irgendwoher erfahren, dass ein Dreieck drei Höhen hat. fragt Dich: ‚Papa (Mami), was heißt das?' Nichts ist unglücklicher als die Fragen kleiner Kinder unbeantwortet zu lassen. Daher entschließt Du Dich zu folgender Antwort: ...

[3] Original German text: „Ein Dreieck hat nur eine Höhe und eine Grundlinie, mit denen man die Fläche und den Umfang ausrechnen kann."

[4] Original German text: „Ein Dreieck hat immer nur eine Höhe und die ist immer von der Grundlinie zu dem Punkt C. Die Höhe geht immer senkrecht von der Grundlinie aus auf den Punkt C."

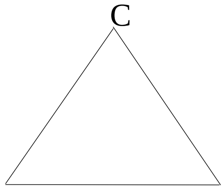
(III)



“Here I’ve drawn a triangle and when I now draw a line from the green corner downwards to the straight line this is called height. When I now draw a line from the orange corner upwards and from the green corner draw a line to the side the line upwards is also called height. When I do the same from the pink corner and from the green corner as well, the line upwards is again called height.”[5]

(IV) *“ A triangle is a plane figure, which has one height and, since the triangle can be turned around, it consequently has three heights.“*[6]

(V)



“The first height goes through the vertex C. It stands straight on the opposite segment. The second height goes through the vertex B. It stands straight on the opposite segment. The third height goes through the vertex A. It stands straight on the opposite segment.”[7]

According to which guidelines can they be interpreted?

a) Descriptive interpretation

This interpretation should primarily be describing rather than evaluating. The teacher ought to be interested in what the individual pupil really thought about the topic in question, i. e. in the mathematical concepts and mathematical knowledge indicated by the text. They should avoid drawing direct conclusions on the pupil’s general level of achievement.

Examples: The pupil who produced TEP (I) evidently regards lines in the triangle not as autonomous (geometrical) objects. The words “baseline” and “height” represent in his mind magnitudes which are needed to calculate the triangle (area and circumference), hence he interprets these concepts functionally. The triangle “has” the baseline and the height stated exactly, thus there is no reason to make further differences between them. In calculation both have the same function hence there is no need for differentiation. The pupil evidently uses the labels “baseline” and “height” synonymously, regards the baseline as the second height and talks about a “third height” although so far he had only mentioned one. The instruction clearly evoked in the pupil’s mind an algebraic rather than a geometrical imagination. Even the word “triangle” is used by him less as a name of a geometrical object but rather as a stimulus for selecting a certain algebraic procedure.

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[5] Original German text: „Ich habe ein Dreieck aufgezeichnet. Und wenn ich jetzt von dem Grünen Eck aus einen Strich nach unten ziehe zu der geraden Linie, heißt das Höhe. Wenn ich jetzt von der Orangen Ecke einen Strich nach oben ziehe und von der Grünen Ecke einen Strich auf die Seite ziehe, heißt die Linie nach oben auch Höhe. Wenn ich das gleiche jetzt auch von der Lila Ecke und von der grünen Ecke mache, heißt die Linie nach oben auch wieder Höhe.“

[6] Original German text: „Ein Dreieck ist eine Flächenfigur, dieses Dreieck hat 1 Höhe und weil man das Dreieck wenden kann hat es also 3 Höhen.“

[7] Original German text: „Die erste Höhe geht durch die Spitze C. Sie steht gerade auf der gegenüberliegenden Strecke. Die zweite Höhe geht durch die Spitze B. Sie steht gerade auf der gegenüberliegenden Strecke. Die dritte Höhe geht durch die Spitze A. Sie steht gerade auf der gegenüberliegenden Strecke.“

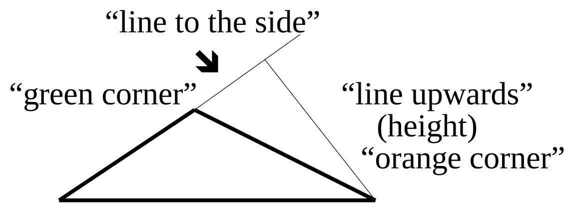
This is quite different in case of TEP (IV). In an expressively conceptual approach its author calls the triangle a “plane figure” i.e. she attributes it conceptually to the category of plane geometrical figures (polygons). The pupil also shows a quite clear intermodal conceptual imagination about height. In her opinion this is a line which only exists in a triangle, one side of which is horizontal, i.e. parallel to the top and bottom margin of the sheet. This is because the height has to be vertical, i.e. parallel to the right and left margins of the sheet. In spite of this quite narrow concept of height the text’s author succeeds in giving a reason for the triangle having three heights. She imagines turning the triangle in the plane so that, one after another, each of the three sides becomes horizontal. In this way it has one height for each of these sides, which means in total three heights. (It seems interesting that in this case a line of a triangle can lose its property to be a height when its position changes. On the other hand the line gets this property back at any time when the particular position is restored. This is sufficient to make the existence of three heights argumentatively certain.)

b) Open interpretation

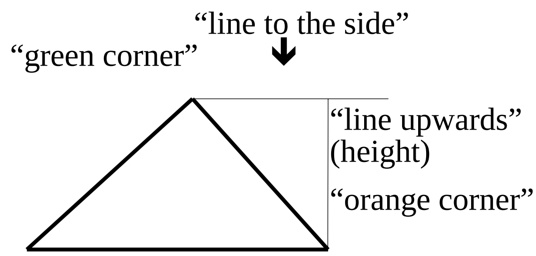
Texts are not unequivocal therefore it is not possible to decide exactly about the authors’ thinking and even about their particular level of achievement. Example:

In TEP (III) the height which goes from the “green” point to the “straight line” (means the horizontally drawn baseline) is relatively easily identified. But how shall we interpret the following sentence *“When I now draw a line from the orange corner upwards, and from the green corner draw a line to the side, the line upwards is also called height.”* There seem at least two possible interpretations, which can be made clear by the drawings as follow:

The pupil imagines a height line outside the triangle, perpendicular to an elongation of the opposite side.



The pupil has a height line in his mind which is parallel to the right and left margin of the sheet and runs towards a line parallel to the (horizontal) baseline through the “green” edge.



c) Global, not detailed interpretation

The text has to be analysed word by word, sentence by sentence in a detailed manner. This enables the teacher to differentiate in detail, and with sufficient exactness, their valuation and to relate it to the mathematical content, as well as describing the pupils’ mental processes which produced their text. In TEP (V), for example, it must be noted that the pupil does not say “through the opposite side” but describes that each height “stands straight on the opposite segment” (German: „steht gerade auf der gegenüberliegenden Strecke“). This can be seen as an exact equivalence from everyday language for the mathematical expression “is perpendicu-

lar to the opposite side”. In addition the text author talks about “the first”, “the second” and the “third height”. This is his idea of an indirect proof of the statement: “The triangle has three heights”.
In text (I) it should have been taken into account that the text describes “*triangle*” and “*height*”, talks about “*baseline*”, “*area*” and “*circumference*” and that it also mentions a “*third height*”, although until then it had only talked about a single one. In addition the author points out that he does not know anything about this third height. Also TEP (III) speaks in the I-form. As opposed to (I) it relates excessively to an added drawing in which the edges of the triangle are marked in colours. The pupil talks about the actions of drawing, where certain directions are attributed to lines: “*downwards*”, “*upwards*”, “*to the side*”, “*towards the straight line*”. It also forms if-then-sentences: *“If I draw...,then this upwards running line is called height”.*

Consequences

If the didactic function of TEPs described above ought to become reality, it seems vital that the teacher makes use of them in an appropriate manner. This means that he provides regular training for the pupils in text writing, and that he is prepared and able to interpret and analyse the texts in a descriptive rather than an evaluating way. This analysis must be done in detail and not in a general and selective manner, and he must be aware that every text is open to different interpretation. Evidently this ability is not a matter of course; it has to be learned and has to be trained.
In addition the teacher has to be convinced that his teaching and his organisation of learning processes can have, and normally do have, different effects on individual pupils. These differences are not only quantitative ones, i.e. different pupils pick up more or less from the instruction the teachers offers, but are, in the same way, of a deeply qualitative kind. This means that the pupils construct qualitatively different concepts, knowledge and mathematical thinking. The teacher should be eager to learn about these differences and draw conclusions from them for the planning of his classes.
Making pupils write TEPs, and appreciating this activity as a stimulus for learning and a means of assessment, has to be based on a particular philosophy of mathematics and mathematics learning. It demands a clear vision about what it means for the pupils to *do* mathematics.

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