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HOW DO PRIMARY SCHOOL PUPILS "SEE" AND STRUCTURE TWO-DIMENSIONAL REPRESENTATIONS OF CUBE CONFIGURATIONS? CASE STUDIES WITH ANALYSES OF EYE MOVEMENTS DURING SPATIAL STRUCTURING PROCESSES.

Abstract:

A task on recognizing spatial relationships was used to study spatial imagery ability in primary school pupils. Visual information intake and processing, while determining the number of cubes in two-dimensional (2D) oblique views of various cube configurations, was studied by registering eye movements. This new method in the field of mathematics education was supplemented by follow-up interviews. In this way, case studies could demonstrate individual strategies for spatially structuring 2D depictions of cube configurations. These strategies were based not only on different levels in the development of spatial imagery abilities but also in the competence of comprehending numbers.

1. Spatial Imagery and Spatial Structuring Strategies

In line with Wollring's (2001) conception of spatial imagery ability, the comprehension of spatial relationships can be understood as the ability "to visualize configurations of spatial objects and the viewer" (Wollring, 2001, p. 138). An individual either reverses the object configuration mentally as a whole, thus endowing it with a structure, or adopts another position relative to the objects (see also Wollring, 1996, p. 476).

The present study focuses on the decoding processes when working on pictures of spatial objects. Developing an understanding for representations of space is a problem for pupils, because they are confronted with a wealth of pictures every day. Unlike plane geometry, graphic representations of three-dimensional (3D) objects and the real spatial relations they represent are not isomorphic. In this sense, a drawing of an object in parallel projection, like those in the present study, represents a mathematical tool that children first have to develop the ability to use.

The reverse process of *coding* 3D information in drawings has been already documented in children. Ingram and Butterworth's (1989) study has shown that one of the ways in which 3- to 8-year-olds present the relative positions of objects in a spatial arrangement is to use a temporal ranking in their drawings (see, also, Stückrath, 1963). Wollring (2001) has also found similar phenomena during the coding of spatial configurations when analysing children's drawings.

For the present study of the decoding process, the available empirical findings can be reversed to produce the following hypothesis: *The temporal sequence of fixations during the perception of a 2D oblique view of a cube configuration will provide information on the type and sequence of mental constructions of positions of elements in the overall configuration and thus on the development of structuring competencies.*

1.1 Spatial Imaging Abilities in the Analysis of Cube Configurations

Ben-Chaim, Lappan and Houang (1985) as well as Battista and Clements (1996) have studied how children and adults determine the number of cubes in real and graphic cube configurations. Ben-Chaim et al. (1985) abstracted a total of four idiosyncratic strategies from interviews and multiple-choice tests dealing with rectangular solid cube configurations (drawings presented isometrically) that were given to fifth- and sixth- grade pupils. These strategies consist exclusively in counting the cube surfaces that are visible in the display and counting the cubes that can be seen in the drawing either with or without a subsequent doubling of this total. This study focused on the reasons behind solutions being inadequate reconstructions.

Battista and Clements (1996) developed a model containing a hierarchy of successive concepts for structure comprehension from a quantitative and qualitative analysis of interviews (with 48 third- and 78 fifth-grade pupils) about oblique views of cube configurations. In pupils of this age, the model proposes that the mental comprehension of structure develops from seeing cube configurations in uncoordinated ways to increasingly seeing them as layers of cubes. The authors assumed that successful participants end up by mentally decomposing such cube configurations into layers, in order to determine the number of cubes. Correspondingly, their studies focused exclusively on rectangular solid cubes that suggest such layering strategies, while completely neglecting more complex, non-convex cube configurations. Therefore it cannot be ruled out that their findings (e.g. that 18% of the third-grade pupils mentally decomposed the solid rectangular cubes into layers compared to 58% of the fifth-grade pupils) are also influenced by their choice of a specific research design.

1.2 Structuring Processes in 2D Tasks

In their most recent study, Battista et al. (1998) have applied a modified research design to focus on the structuring processes underlying the pupils' abilities observed in 1996 (see Battista et al., 1998, p. 503). They proposed that all processing of quantitative tasks in spatial situations is based on the process of spatial structuring which they defined as a "mental operation of constructing an organisation of form for an object or set of objects" (Battista et al., 1998, p. 503). Because they considered this shaping of spatial structurings to precede the ability to count meaningfully (Battista et al., 1998, p. 504), they reduced the complexity of their tasks to plane problems, and analysed the structuring competencies of second-grade pupils (7- to 8-year-olds). They asked them to determine numbers of squares in 2D rectangular fields in which the pattern of squares was only suggested through different kinds of partial information. In two successive interviews (one in spring, the other in autumn), 12 second-grade pupils completed two different sets of tasks (Battista et al., 1998, p. 506; see Figure 1).

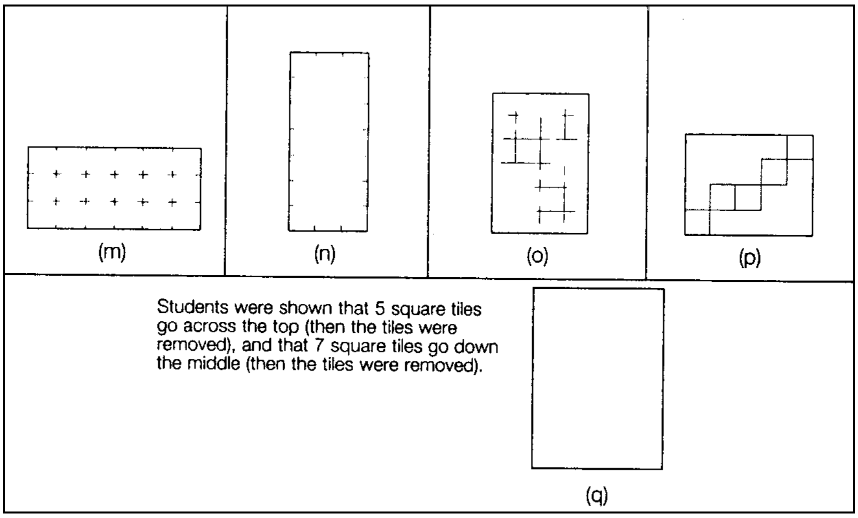


Figure 1: Selected interview tasks (modified figure based on Battista et al., 1998, p. 507).

The pupils were asked to report how many square tiles they needed to fill the rectangle completely (the original prediction). They were then given a rectangle with the same shape and size, and asked to mark where they would place the individual square tiles before once more reporting how many square tiles fitted into the rectangle (the drawing prediction). Finally, they had to pave the rectangle with square tiles and once more determine how many square tiles they needed (see Battista et al., 1998, p. 506).

On the basis of their observations while this task was being processed, the authors classified a total of five hierarchical levels of structuring rectangular fields of squares. These levels describe a progression from a complete lack of structuring in rows or columns, over a locally restricted structuring in rows or columns up to a highly evolved, internalised and iterative row-per-column structuring process (Battista et al., 1998).

Battista et al.’s (1998) study does not just confirm that different types of spatial structuring processes are formed gradually in children. It also confirms that specific interpretations of modes of presentation (in this case the row and column structure of fields of rectangles composed of square tiles) first have to be constructed individually. This conforms to a constructivist view of mental operations. It implies that an individual has to actively construct spatial structures for sets of objects by accumulating information that is not just linked directly to perception (see Battista et al., 1998, p. 531). Nonetheless, the question of how children determine the number of cubes in pictures of cube configurations when interpreted from a constructivist perspective still has to be considered largely unresolved.

2. The Method for Registering Eye Movements

The method of observing behaviour directly by registering eye movements is particularly firmly established in linguistics and cognitive science as well as in various applied fields (e.g. advertising efficiency research, see Leven, 1991). However, eye movement studies using tasks related to mathematics or mathematics education are still rather rare (for studies on reading mathematical tasks, see : De Corte & Verschaffel, 1986, 1987; Verschaffel, De Corte & Pauwels, 1992; symmetry phenomena: Locher & Nodine, 1987; 3D interpretations of Necker cubes: Ellis & Stark, 1978; mental rotations: Putz-Osterloh & Lühr, 1979; Just & Carpenter, 1985, 1987 b).

The history of the registration of eye movements has produced two parallel research orientations; the analysis of the characteristics of eye movements in order to develop theories on eye movement control, and the use of data on eye movements to develop models explaining perceptual processes and accompanying cognitive processes. Our approach belongs to the latter, applied research orientation.

Studies using the registration of eye movements are based on several methodological assumptions (see Just & Carpenter, 1980; Rayner & Sereno, 1994) that also highlight the critical aspects of the method. For example, eye movements are assumed to be goal-directed, and are applied, either consciously or subconsciously, to select or structure information from the perceptual field. The focus of attention corresponds to the direction of gaze (the eye-mind assumption). For static visual material, information intake occurs only while the eye fixes upon an object and not during saccades (Houtmans & Sanders, 1983). Saccades serve exclusively to reposition the eye. The duration of fixation corresponds to the duration of central processing (the immediacy assumption). Furthermore, the sequence of eye movements reflects the serial processing of visual information. According to Just and Carpenter (1980, p. 331), it is generally assumed that internal cognitive processes proceed in synchrony with fixations. Fixations that last longer than average are then viewed as indicators of strong cognitive activity for constructing an appropriate mental representation (see Pomplun et al., 1995). However, these assumptions are still subject to a degree of controversy in the scientific literature (see Rayner & Sereno, 1994). It is impossible, for example, to rule out with certainty that some of the processes have not taken place already during an earlier fixation; in other words, the occurrence of a so-called parafoveal preview effect. Different models of the control of eye movements have also been discussed.

As a result, registering eye movements cannot be conceived as a perfect record of the mental activities accompanying a comprehension process. Nonetheless, it is a serviceable method, in the sense of an applied research orientation, for obtaining information on precisely this process, because it accompanies perceptions directly and offers an opportunity to describe the course of information intake and processing through objectively measurable process variables.

3. Use of Eye Movement Registration

The present study is designed to overcome the subjective aspect when interpreting sequences of eye movements and pupils' articulations by first using a set of tasks to derive hypotheses on individual solution strategies. These are then tested in a second set of tasks after a break of roughly one week. In order, in turn, to control the inter-subjectivity of the interpretations of patterns of eye movements for each set of cube pictures, eye movement data were supplemented by retrospective interviews in the next part of the study that followed directly. This provided further indications on the formation of specific solution strategies and how they are determined by both subjective variables and task variables. The children were asked about one of the tasks they had just processed, and their behaviour was observed (e.g. "Which of the cube pictures can you remember? How did you work out the number of cubes in this construction?"). In addition, a further part of the experiment gave the children an opportunity to demonstrate how well they could reconstruct the task by assembling it with real cubes.

The data in the present study were collected with Eye Tracker recordings (OMNITRACK system). These registered the temporal sequence of the co-ordinates of a fixation on the monitor and their duration as well as the accompanying diameter of the pupil. The data were further analysed and interpreted with a specially developed computer program named "Vision".

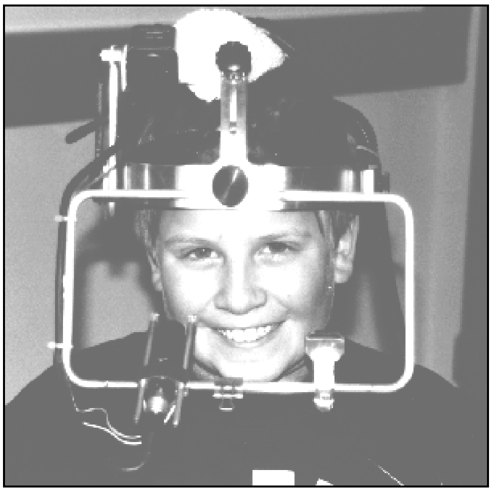


Figure 2:
A child in the study wearing the OMNITRACK eye-tracker helmet.

As well as computing statistics, one particular advantage of this software is that it illustrates the gaze sequences by plotting the individual fixations as superimposed circles on the visual display. These are then joined together by arrows indicating the sequence in which they occurred. The diameter of each circle varies as a function of the duration of fixation, which is also entered in ms time units in the circle. This delivers time-based documents that, just like the transcripts in interaction analysis research, can be subjected to a sequential interpretation (sequences of images of individual focussings of attention for each cube configuration).

4. Eye Movements as Indicators of Spatial Structuring Processes

The task of determining the number of cubes from oblique views of cube configurations was embedded in a learning environment of real cube-shaped wooden building blocks. This understanding was conveyed to the children in advance with a short introductory interview.

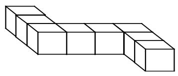
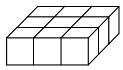
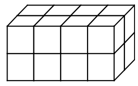
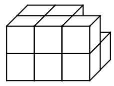
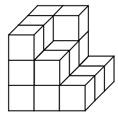
picture	 			
Structural type	**Typ LP**: linear or plane configuration	**Typ S**: 3 D (spatial) configuration with hidden cubes		
		Convex	Non-convex plus partly hidden cubes	Non-convex

Figure 3: Selected examples of the pictures with different structural types of cube configuration.

It can be assumed that each individual concrete spatial structuring is influenced by (at least) two variables; the level of task complexity (dimensionality, number of cubes, convexity or non-convexity of the cube configuration), and the individual level of development in spatial imagery abilities along with competence in comprehending numbers (quantifying sub-items and unitising). More highly developed competence should reveal itself in the children's ability to not only identify the individual cubes, but also to speed up the process of determining the number by identifying relations between individual cubes and other substructures (pairs and triples of cubes, bars, layers) and employing simultaneous assessment.

This resulted in the following set of questions: Which structural elements do children identify within a picture of a cube configuration, and how do they process them? What is the relationship between individually selected structural elements and the structural characteristics of the cube configuration? Which counting processes accompany the structurings? Can intra-individual spatial structuring strategies be identified that are typical for individual children and (relatively) independent from the item's degree of complexity?

In the following, selected case reports will be used to show that the method of eye movement analysis can provide answers to the questions above. It should be noted that while descriptions of the individual structuring processes refer to the real cube configurations, what is actually being discussed are the mental constructs used in the mental representation of each picture compiled by the children.

4.1 Examples of Inter-Individual Differences in the Structured Counting of Linear Cube Configurations

In the following, the paths taken by the eye movements of two children when processing the same task are presented in the way generated by the Vision program (see pp. 5).

<u>Example 1</u>: Fabian (11.5 years old, fourth grade)

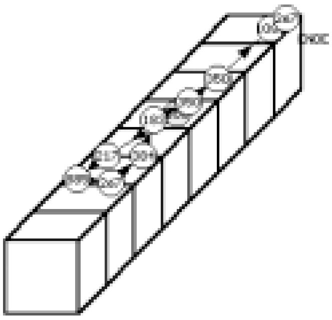


Figure 4:
After a short orientation from back to front, he makes his first fixation of above-average duration of 885 ms on the upper edge of the second cube from the front. He then continues to scan with fixations on all the remaining individual cubes. After 4.67 s, he reports the correct number of cubes (mean fixation duration of 433 ms, total fixation duration of 4.34 s).

Fabian's pattern of fixations across the picture can be interpreted as an example of a strategy of forming a cube grouping (in this case, a unit of two) plus individual cubes, combined with counting in sections and further counting in ones.

<u>Example 2</u>: Jan-Hendrik (8.4 years old, second grade)

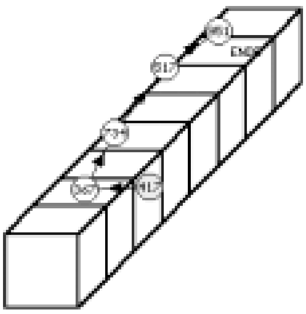


Figure 5:
He fixates step by step from front to back on bordering sub-units of twos. After 4.2 s, he reports the correct number (mean fixation duration of 597 ms, total fixation duration of 2.99 s).

Jan-Henrik decodes such linear arrangements of cubes through a complete formation of sub-units, which, in this example, are identical to each other. In line with his structuring, he also counts in twos. This makes this scanning pattern an example of the strategy of forming cube groupings and counting in sections.

These examples illustrate different forms of structuring as a function of individually different levels of competence in comprehending numbers. However, by only analysing the fixation sequences in these examples, one cannot assume conclusively that the structurings proposed here are actually based on spatial imagery abilities. Hence, the pattern of eye movements alone does not suffice to decide whether a structuring actually occurred on the basis of cube or surface features.

In both linear and 2D cube configurations (Type LP; see Figure 3), the correct number could be determined simply by counting the parallelograms in the picture without any need for a mental generation of a 3D configuration. Putz-Osterloh and Lühr (1979) have already drawn attention to the fact that some of the processing strategies applied to mental rotations of a cube do not imply spatial abilities but are of a heuristic nature. Such strategies only prove to be ineffective for reconstruction when participants have to process specific task items in which spatial imagery is an indispensable precondition for success. Therefore, a description of the individual ability to perform spatial structurings can be developed only after combining eye movement patterns and the pupil's articulations in follow-up interviews on pictures of 3D cube configurations (Type S; see Figure 3).

4.2 Case Studies on Individual Processing Strategies

The complex combinations in which spatial structuring processes can occur in completely individual ways will be illustrated on the eye movement patterns of two children, Carmen and Martin. Both children have difficulties in learning mathematics that are particularly apparent in the field of arithmetic.

Example 3: Carmen (7.1 years old, first grade)

Carmen finds it difficult to construct relationships between the numbers from 1 to 20. She nearly always computes through counting procedures. One example of this is the way she generally solves addition tasks by counting further with the help of her fingers. Carmen also does not have any strong preference for either her left or her right hand.

In the present study, Carmen exclusively applied decoding procedures and counting principles that refer to the individual cubes when decoding the cube configurations presented in oblique perspective. Carmen counts linear arrangements of cubes completely, and often counts the individual cubes for a second time before reaching her correct answer. With 2D convex cube configurations, she successfully counts the individual cubes by structuring in bars. She decomposes non-convex, 2D cube configurations into non-congruent and non-parallel sub-units that she then counts as individual cubes. Her success depends on how far she manages to count systematically in line with her structuring.

However, with 3D cube configurations, Carmen reveals a spatial decoding that is not longer effective for a reconstruction. With drawings of non-convex 3D cube constructions, Carmen concentrates on counting only the visible cubes without any continuous reference to specific sub-units of the cube constructions. It is clear that she is capable of interpreting the graphic representation of the individual cubes three-dimensionally. However, her ability is not sufficient to transform the relationship of a single cube to neighbouring cubes completely in order to produce the mental space of the total group of several cubes needed for an effective reconstruction. Hence, her decoding of the depth dimension is limited to a depth of one cube.

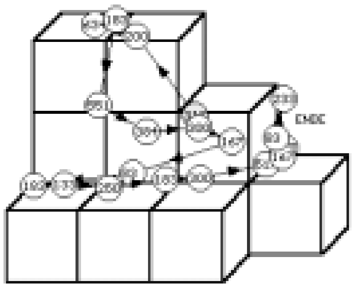


Figure 6:
Carmen fixates single cubes from top to bottom in visible, horizontal bars of two and three, and then, finally, the individual cube to the far right of the figure. After 6.53s, she determines the number of cubes as 9 instead of the correct number 12 (mean fixation duration of 270 ms, total fixation duration of 5.14 s).

With regularly structured 3D configurations, Carmen tends to orient towards the cubes that are visible from various perspectives.

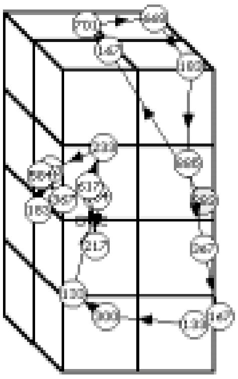


Figure 7:
Carmen's scanning pattern traverses the visible cubes on the top and front surfaces of the rectangular solid. From the front surface, she then fixates the left side of one of the cubes several times. However, this scanning pattern is not extended to the side face of the entire rectangular solid and therefore does not seem to play any role in the structuring process. After 7.3 s, she determines the number as 10 (mean fixation duration of 350 ms, total fixation duration of 6.66 s).

Carmen's decoding process across visible cubes is also incomplete in the following cube construction. Counting is restricted to sub-units that can be identified from the top view. For example, Carmen's attention focuses on the regular "step formation". As a result, she counts only the cubes on the steps whose two side surfaces take the form of parallelograms.

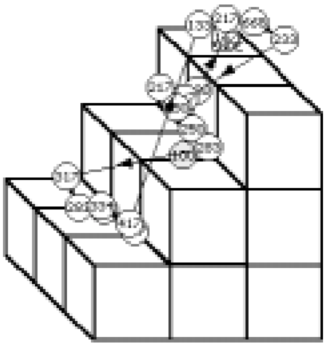


Figure 8:
Carmen fixates on the single cubes of selected y bars of three. She then selects those y bars that each have two bordering side faces that are visible as parallelograms. This procedure leads her to determine the number of cubes as 9 instead of the correct 18 after 6.32 s (mean fixation duration of 272 ms, total fixation duration of 4.90 s).

Carmen's spatial structuring with visible cubes proves to be locally confined to a depth of one cube with a lack of global co-ordination ability. She is unable to organise the structurings she performs in different directions into a spatial relationship that would enable her to avoid, for example, counting neighbouring sub-units twice.

Even after she has put together wooden blocks to form the same configuration as the one in the picture during the follow-up interview, she still determines the number of cubes in this concrete construction, that she had built herself, just as unsystematically and with only partial reference to specific structural units. In this staircase configuration (see Figure 8), she tries to count the cubes by touching each one with her finger in horizontal layers from bottom to top, but fails to recognise when and where she has counted one complete layer. Carmen can only ascertain the correct number of cubes by gradually dismantling the configuration cube by cube while counting simultaneously. She expressed her surprise at the number of cubes obtained in this way compared with her previous report by taking the picture in her hand and turning the page over while stating, “Well, you can’t see it from the back”.

<u>Example 4</u>: Martin (10.9 years old, third grade)

Martin is 10.9 years old and able to decompose numbers and also apply operative strategies meaningfully. However, his arithmetic operations in the number space from 1 to 20 are still not completely automatic. It is noticeable that he almost always uses counting strategies to work out these tasks, but he frequently makes (+1) or (-1) errors. He has no problems in discriminating between left and right when referring to his own body, but is uncertain when faced with an object in front of him.

The findings support the hypothesis that Martin too does not possess sufficient strategies for decoding depth in these perspective drawings. This is confirmed by the observation that he can carry out meaningful structurings with pictures of linear or plane formations of cube configurations and get the number right most of the time. His mode of counting frequently falls back on counting in ones. Once he has identified a bar in cube pictures with a linear or plane arrangement, he first counts the cubes one at a time. However, if a visible, parallel and congruent bar is present, he can then assess it simultaneously.

On pictures of 3D cube combinations, in contrast, Martin tends to focus on the sub-units of cubes that are visible in the picture and reveals problems with reconstructing the concealed cubes—in relation to the real wooden cube environment. Unlike Carmen, he forms certain structuring units (e.g. bars of two and three), and he can combine them to form larger, higher-order units such as layers—as long as all the cubes involved are visible in the picture. However, he is then no longer able to grasp further, congruent structuring units that are not completely visible. For example, with the picture of a rectangular solid, he cannot identify a layer that is congruent with the front layer that he has already addressed in the configuration.

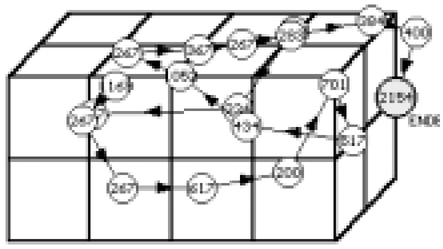
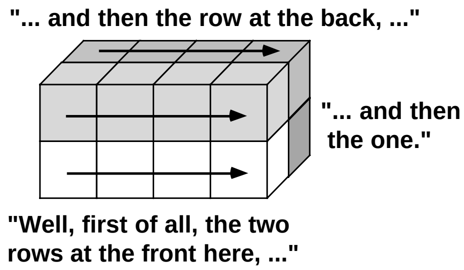


Figure 9:
Martin fixates initially on the upper x bar of four in the front xz layer followed by the congruent lower x bar. Then he registers the rear x bar of four located in the upper xy layer, but changes to the side view of the rectangular solid and fixates the two cubes of the rear x bar of two. After 12.45 s, he reports the number 14 instead of the correct 16 (mean fixation duration of 555 ms, total fixation duration of 10.55 s).

In general, this scanning pattern can be interpreted as a strategy of structuring over visible bars with a quasi-simultaneous assessment of the number of cubes per bar supplemented by visible single cubes. In the follow-up interview, Martin basically confirmed this procedure.

time	Protocol of follow-up interview (translated) Martin: aged 10.9 years	Interpretation of problem-solving behaviour in individual steps
0:48:33	*And how would you solve that for this construction?*	
0:48:47	Well, first of all, the two rows at the front here, and then the row at the back, and then the one.	
0:48:54	*Ah, show me how you work that out.*	
0:48:59	Four plus four is eight, plus four is twelve plus one is thirteen.	Martin strokes his finger across each bar of four and taps the single cube in Position (4,2,1). This is how he assesses the two front bars of four first (“four plus four”). Then he adds the visible upper rear x bar of four (“plus four is twelve”) and the one visible cube on the rear bottom row (“plus one is thirteen”). He does not collect and count any of the three invisible cubes in the hidden y bar of four.
0:49:07	*Hhmm, OK!*	

As far as his perception of numeracy is concerned, Martin’s articulations in the follow-up interview indicate that the structural elements he has formed here are assessed simultaneously (“the two rows”, “four plus four”). However, his eye movements reveal that, in this situation, he did not grasp these bars “in one glance” but tended to mentally reconstruct and count the cubes individually (see Figure 9). He tries to solve the task by structuring and counting those sub-units that are visible from various perspectives. However, even in the follow-up interview, he fails to achieve a decoding of depth that leads to an effective reconstruction in the chosen learning environment. Nonetheless, compared with Carmen, he is capable of spatially co-ordinating the structuring units formed from various perspectives.

Both case reports reveal that children develop individually differing spatial structurings, and that these also remain relatively constant personally when dealing with comparable types of cube configuration.

5. Consequences and Outlook

The different structuring strategies presented in the form of case reports all satisfy the definition of spatial imagery ability cited in the introduction. In this task, individual processing strategies prove to be relatively stable across different items. Many children such as Jan-Hendrik or Fabian structure the cube configurations according to a “building block” strategy; in other words, they break down or build up the configuration mentally in individually different ways in order to assign it a structure. At the same time, they reveal their conception of the spatial arrangement of cubes within a cube configuration through a temporal progression of building up or breaking down sequences. Carmen, however, does not possess this type of dynamic spatial ability. She orients predominantly towards the static aspects of the configuration that can be seen in the picture, but her co-ordination is insufficient, and she makes only slight attempts to decode depth. Martin is already able to form more comprehensive structuring units for visible elements, but is limited in his ability to co-ordinate them on the dimension of one layer of cubes.

These examples reveal that various components are involved in forming the ability to recognise spatial relations; structuring competence, depth decoding, and a competent comprehension of numbers in the sense of quantifying sub-items. Battista and Clements (1996, p. 291) have already ascertained that the individual spatial structuring of a rectangular solid determines the mechanism used to ascertain the number of cubes it contains. The spatial structuring can impact positively on the counting process, but it can also impede a correct determination of number or even prevent it completely. Hence, the mental construction of an appropriate representation of a 3D cube configuration requires a complex interaction between numerical and spatial structuring processes. Mathematics education could benefit from an explanation of these interacting processes through further studies of these tasks.

Because it can record spontaneous and intuitive learning processes, the method of registering eye movements proves to be more suitable for documenting the real counting processes than the oral comments on a problem-solving procedure elicited at the meta-level of reflection. Nonetheless, it needs to be pointed out that influences from the two different research environments also cannot be ruled out as a potential explanation for the observed differences in the individual counting process for one and the same item. The relevance of the eye movement registration method in research is revealed particularly by the way it can depict different strategies when solving tasks in spatial geometry. Transferring the method to other fields might permit an explanatory modelling of other sets of abilities associated with spatial abilities. This also makes the method a serviceable instrument for observing learning processes in these fields. It helps to assess individual progress in learning. Furthermore, it may also contribute to analysing the effectiveness of different training options in terms of the modification of cognitive processes. It can be concluded that the registration of eye movements is also a valuable methodology for studying issues in mathematics education.

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