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TEACHERS' BELIEFS ON MATHEMATICS TEACHING ASSESSED WITH THE DIONNE METHOD: A CASE STUDY

Abstract

Our primary concern is the quantitive and qualitative investigation of teachers' views of mathematics and its teaching. As a result, Dionne's and Ernest's characterization of beliefs on mathematics serves as a theoretical background; the dominant perspectives of views on mathematics can be described as toolbox aspect, system aspect and process aspect. Originally, our test subjects numbered a total of thirteen experienced German teachers, here, however, we restrict ourselves to six persons. In particular the experiences with the Dionne method are discussed here, that is to say beliefs are interpreted as weight distributions for characteristic core representations, and are quantified by this method.

1. Introduction

The literature contains controversial and abundant contributions[1] concerning teachers' beliefs and concepts as well as the processes by which teachers change them in regard to their profession (see the survey articles of Thompson (1992) and Pehkonen & Törner (1996)). Marginal attention is dedicated, however, to the corresponding methodological questions concerning the research of these beliefs (Grigutsch 1994).

Originally, our test subjects amounted to a total of thirteen experienced German teachers, five of whom teach at the *Gymnasium*, two at the *Realschule*, one at the *Hauptschule*, and five at the *Gesamtschule*. Here we will limit ourselves to a study of six people.

1.1. Definition of Beliefs and their Role

The purpose of this subsection is to give a self-contained survey on the terminology used in this paper. For a survey of research conducted on teachers' beliefs, the reader is refered to the research synthesis of Thompson (1992) or Pehkonen (1994). However, there are hardly any articles focusing on the mathematical beliefs of German mathematics teachers.

An individual continuously observes and learns from the world around him or her. According to their experiences and prior perceptions, they make conclusions about different phenomena and their nature. The individual's personal knowledge, i.e. their beliefs, is a combination of these conclusions.

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[1] see http://www.uni-duisburg.de/FB11/PROJECTS/MAVI.html

The concept of mathematical beliefs has many definitions in the literature. Here we take (mathematical) beliefs to be one's subjective knowledge (which also includes affective loadings) of a certain object or concern for which indisputable grounds may not necessarily be found in objective considerations. One feature of such beliefs is that they can be held with varying degrees of conviction (Thompson, 1992). Beliefs cover personal convictions mixed with facts and external knowledge; thus the beliefs' subjective certainty ranges from truth-like facts to vague assumptions. This is not true in all cases: the individual is however aware of the truth-degree of beliefs. In this case we are speaking of unconscious beliefs.

The process of how and for what reasons a belief is adopted and defined by the individual is not well understood. The adoption of a belief may be based on some generally known facts as well as beliefs, and on logical conclusions drawn from them. But, in each case, the individual makes their own choice of the facts and beliefs to be used as reasons and their own evaluation of the acceptability of the belief in question. Recently this question was partly discussed in Schoenfeld's book (1998). Often for the individual there seems to be no objective distinction between facts and beliefs.

The individual compares their beliefs with new experiences and the beliefs of other individuals, and therefore these beliefs are subjected to continuous evaluation and hence undergo change. When an individual adopts a new belief, it will automatically form a part of the larger structure of their personal knowledge and of their belief system, since beliefs never fully develop independently (Green, 1971).

Green (1971) pointed out that beliefs always come in sets or groups, never in complete independence of one another. Thus we assume that an individual's beliefs form a structure. We will call this construct of beliefs a *belief system* or more generally their *views* of mathematics. This wide spectrum of beliefs around mathematics contains at least four main components which are also relevant for mathematics teaching: (1) beliefs about mathematics; (2) beliefs about oneself as a user of mathematics; (3) beliefs about teaching mathematics; and (4) beliefs about learning mathematics. These main groups of beliefs, in turn, can be split into smaller units. It is evident that these 'dimensions of beliefs' are interrelated. For more details on these ideas, see Pehkonen (1995).

Törner & Grigutsch (1994), and also elaborated in a more recent paper by Grigutsch, Raatz & Törner (1998) attempted to investigate belief system structures using factorial analysis. They used the term "mathematical world view" which can be found originally in Schoenfeld's discussions (1985). This is why we adopt in this paper the abbreviated term "views".

1.2. Beliefs versus Views on Mathematics and its Teaching

It is evident that views, in particular on mathematics, contribute an essential part to any belief system on mathematics. Thus there are numerous papers focusing on this aspect. For his research, Dionne (1984) used the following three perspectives of mathematics:

(T) Mathematics as a set of skills (*traditional* perspective). Doing mathematics is doing calculations, using rules, procedures and formulae;

(S) Mathematics as logic and rigour (*formalist* perspective). Doing mathematics is writing rigorous proofs, using precise and rigorous language and using unifying concepts;

(P) Mathematics as a constructive process (*constructivist* perspective). Doing mathematics is developing thought processes, building rules and formulae from experience and reality and finding relationships between different notions.

It seems obvious that these perspectives reflect guiding aspects of mathematics; however, different persons may evaluate these components differently, and so Dionne (1984) let his test subjects assign weights to these 'dimensions'. In his book, Ernest (1991) describes three similar views of mathematics, instrumentist, Platonist and problem solving. These correspond more or less to the three perspectives of Dionne (1984) (see above). Of course, there might be further relevant aspects characterizing mathematics (see the book of quotations by Schmalz (1993)), e.g. mathematics as an art, however the aspects mentioned above seem to be the leading ones in school mathematics.

In the following sections we define the traditional view (T) as the toolbox view of mathematics, the formalistic perspective is interpreted as the system dimension (S) of mathematics, and the constructivist perspective is understood by us as the process aspect (P) of mathematics.

Finally, it is relevant for us to point out that in the present survey it appears problematic to distinguish between beliefs on mathematics and beliefs on the teaching of mathematics. In the university context of mathematics this may be easier. There mathematics is conveyed to the students in lectures and seminars and there is applied mathematics, there is mathematics in research literature, etc. There are good arguments for assuming that mathematics in distinct contexts bears witness to various aspects of mathematics. In this view beliefs on the teaching of mathematics are not automatically linked to fundamental beliefs on mathematics.

The situation of mathematics in schools appears to us, however, to be different. In school, mathematics appears solely in the form of mathematics taught by teachers to pupils. Further aspects for both groups lie to a great degree beyond the scope of mathematics in schools. We, therefore, see no possibility of distinguishing between beliefs on mathematics and beliefs on the teaching of mathematics. Incidentally, this conditional inseparability is underlined by the well-known quotation[2] of René Thom (1973).

2. The Design of the Research

2.1. The General Setting

The subject of our survey, from which we report some aspects here, is to investigate views of mathematics of teachers from different school forms in North Rhine-Westphalia (Germany). Theme-centered, open interviews were conducted with 13 experienced teachers who additionally filled out a questionnaire in our presence. Both situations were recorded on video. In an exchange of letters following the interviews the teachers were requested to give a twofold self-assessment

[2] In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics. (Thom, R. 1973)

of their mathematics lessons. First they were ask to distribute 30 points to the factors (T), (S) and (P), (see Appendix 1), and second to mark their own individual self-assessment on real mathematics lessons verses ideal mathematics lessons into an equilateral triangle with the three corner points representing (T), (S) and (P). This form of data representation, which develops the so-called *Dionne method* a step further, had previously not been described in the literature of mathematics education.

2.2. The Dionne Method

In a survey of elementary school teachers, Dionne (1984) sees himself confronted with the problem of characterizing the beliefs of the test persons numerically. He reduces and quantifies the beliefs of the teachers by means of vectors with a standardized list of weightings and by letting the teachers name their subjective numerical evaluations of three aspects of mathematics. More exactly the teachers have to assign natural numbers to the three aspects (T), (S) and (P) (see above), whereby the sum of the three numbers has to amount to 30. For Dionne, this triple number characterizes the attitude of the teachers. Incidentally, by this method he views the field of beliefs as being two-dimensional. One should not forget that amidst all criticism of this admittedly rather crude method, it can be considered appropriate for a survey of mathematics at elementary level.

A characteristic of this method is that it is also a self-assessment of the test persons, whereby it is relatively easy to determine the values.

The following table shows some mean values from diverse surveys, also including the Dionne survey (1984). It underlines that mathematics is regarded differently by different groups. We omit any detailed discussion.

	T	S	P
Elementary Teachers (Dionne (84), N = 18) (Pre-test)	10.8	6.3	13.7
Elementary Teachers (Dionne (84), N = 18) (Post-test)	9.5	7.5	13.4
Elementary Teachers (Dionne (84), N = 15) (Control group, Pre-test)	7.5	9.5	13.0
Elementary Teachers (Dionne (84), N = 15) (Control group, Post-test)	10.4	9.8	9.8
Teachers at Comprehensive Schools (Törner, 1995; N = 19)	10.8	11.2	8.0
Teachers at Gymnasium (Törner, 1994; N = 14)	12.8	10.1	7.1
Students at University (Törner, 1997; N = 15)	6.4	11.8	11.8

However, if one examines the individual differences, it is obvious that the self-assessments of the various groups on the aspects of mathematics diverge significantly.

3. Research Questions

At earlier stages authors have already employed the Dionne method. When giving self-assessments, it quickly became apparent to us that the teachers like to distinguish between the reality of mathematics teaching on the one hand and ideal mathematics lessons on the other hand. We have taken this into consideration by taking into account **two** self-assessments for each individual test person, namely their view of their mathematics lessons in reality and their view of ideal mathematics lessons (Appendix 1).

In the end we are conscious of the fact that the weighting allocation given to the components can only be conditionally quantified for the test persons. Beyond this, the Dionne method demands the nomination of three numbers of which the sum underlies a marginal condition, namely it must amount to 30. Thus an "in-principal-equivalent" but in detail completely different graphic data allocation method appeared to us to be appropriate.

When one graphically represents the solution space of all possible answers by means of an equilateral triangle with the three corner points representing the dimensions (T), (S) and (P), all non-negative number triples (with the sum of 30 of the entities) can be converted into barycentric coordinates, which define points inside the triangle. Inversely, all the points in the triangle can be interpreted as a triple; the corner points (T), (S) and (P) then correspond to the points (30, 0, 0), (0, 30, 0), (0, 0, 30) respectively.

In view of the initial situation described above we intend to deal with the following research question:

To what extent is the principally redundant information from both self-assessments of the test persons on mathematics lessons compatible?

By this approach we hope to indirectly reach a conclusion on the usefulness of the graphic method presented here. We also expect illumination on the processes leading to the distribution of points within the self-assessment tasks.

4. Results

Before we discuss the results of the self-assessments of the test persons, the teachers involved will be introduced briefly here in a gender-neutral fashion by employing quotations recorded during the interviews with them. We limit ourselves to the discussion of six teachers from the group of thirteen teachers to keep the case study clear and concise. We furthermore consider the six persons to be typical. They are also representative of teacher profiles and school forms according to formal teacher certifications. For reasons of conciseness we do not offer an overview of the German school system here but refer instead to Robitaille (1997).

4.1. Quotations from the interviews

Dylan (Dy) is a teacher in a Gesamtschule; he teaches mathematics in both the lower and the upper secondary classes. Dylan's view of mathematics and its teaching are revealed in the following quotation: *"I do not regard mathematics as dry; I find it fascinating. To me, mathematics is alive and I derive pleasure from it."* And furthermore: *"It is always of importance to me that the teacher makes the textbook understandable to the students ... That forms a visible line throughout my teaching, up to the maturity examination."* The next quotation shows that formalism is not highly important for him: *"Proof just for the sake of proof I regard as arrogant abundance. Pillars could support mathematics just as well as solid walls."*

Harry (Ha) is a teacher at a Hauptschule; his pupils are aged 10-16. Harry's mathematical view is strongly process-oriented: *"I disapprove of any product-orientation, I regard the process as being too important"*, a statement he mentioned several times. The core idea of his teaching is described by *"We have to find a way to meet the demands of the labour market"*. Harry and his students have fun and derive pleasure from mathematics, *"It is a decisive factor that education should be enjoyed by my students as well as by me.*"

Henry (He) has the same teacher's qualification as Dylan, i.e. he teaches in both the lower and the upper secondary classes. He also teaches at a Gesamtschule. Henry's view of mathematics is toolbox-centered: *"What works well is giving formulae. When you give the formulae which are present in the classwork, you get the results"*. His statements are reminiscent of a factory worker who manufactures products. Another notable point is his frequent use of the word "thing" when referring to mathematical contents. Furthermore, he himself offers another metaphor, namely that of a nursery school teacher who, *"... leads small children by the hand through a garden without leaving the path, in order not to confront unexpected things"*. This teacher makes a tired, uninvolved impression.

Joseph (Jo) is the only teacher who teaches at a Realschule. Joseph's view on mathematics could only be indirectly considered as belonging to teaching mathematics. He understands his teaching as a continuous use of worksheets, which points to the toolbox aspect. His principle of teaching is reflected through the following quotation: *"The teacher-centered lesson has proven to be successful, because questions could be answered and problems cleared with regard to the whole class."* Joseph makes a worn-out, resigned impression, and seems to be often preoccupied with his own thoughts.

The following teachers, Ken (Ke) and Larry (La) have completed a full academic course on mathematics and are qualified to teach in all classes of the Gymnasium. They are also both employed as student teacher trainers.

Ken is always keeping his mathematical knowledge up-to-date on an explanatory level. His view can be considered as well thought out, detailed and balanced. This interpretation is supported by the following quotation: *"In the beginning, I was strongly structurally-oriented; fractions were dealt with rather formalistically... and Freudenthal was not much of a help either because he, too, is very rule-oriented in this regard"*. He continues: *"My openness implies that I try to involve more visual elements than formalism, which compels...*" and further *"I came to realize that visuality stays in the memory longer, as well as association..."* As a teacher, he is realistic, pragmatic and at the same time lacking illusions: "*I suspect that my teaching was teacher-oriented in the beginning, and I suppose it still is today"*.

Larry regards mathematics as *"a colourful structure that allows the formal contents to be dealt with abstractly and systematically*". He is still in contact with his former university and would like to obtain some new ideas from there. His teaching is moderately teacher-oriented, but his starting point, in general, is mathematics-oriented. His knowledge on mathematical topics is convincing. Furthermore, during the interview there is no mention of teaching in groups. In addition, he in fact expresses several objections against project work, for example.

Further quotations are integrated into the discussion following in the next subsections. Finally, in Pehkonen & Törner (1999), which focus on the change aspect one can find additional interview results.

4.2. Self-Estimations

4.2.1 The Numerical Self-Estimation

In Table 4.1 we give the scores which teachers attributed to the three components of the view of mathematics (see Section 1.2.), and which were asked for in the letter (see Appendix 1) sent to the teachers during the second round of the study.

Teacher	Tool	System	Process	Tool	System	Process
	REAL			IDEAL		
Dylan	**15**	5	10	5	5	**20**
Harry	9	1	**20**	4	1	**25**
Henry	**14**	8	8	6	**12**	**12**
Joseph	**15**	3	12	10	5	**15**
Ken	8	10	**12**	10	8	**12**
Larry	9	9	**12**	6	9	**15**

Table. 4.1. Scoring of the self-estimation; Tool = mathematics as a tool box, System = system aspect of mathematics, Process = process aspect of mathematics. The leading positions are in bold.

Each teacher, with the exception of Ken, wants to change, more or less, towards "process"; also, Harry and Larry want to emphasize the process aspect even more.

At first glance, the following statement is not surprising and becomes obvious from the table. None of the teachers choose an extreme position, either in their real view or in their ideal view. Thus Dionne's polarization is, in a particular way, balancing.

Each teacher regards the process aspect as the most important factor in his ideal view. Regarding real teaching Harry gives the process aspect the highest rating, which corresponds with his quotations in the interview.

It is noteworthy that the estimations of Ken and Larry are about the same regarding their real teaching, although they have not met each other before. The interviews depict both teachers as highly qualified in mathematics (see Section 4.1). Also, Table 4.1 shows the teachers as being close in their estimations of mathematics. They are more or less satisfied with the current classroom situation. Larry points out that mathematics lessons are generally not very encouraging and motivating because of the subject. And therefore, Larry "*... is continuously looking for external stimulation*". Larry hereby underlines his need for external stimulation for variation of lessons.

The interviews also support the entries in Table 4.1, in which Dylan, Joseph and Henry share the highest score with respect to the toolbox aspect, namely "doing mathematics means working with figures, applying rules and procedures and using formulae".

Of course, there is no uniform meaning and implication of what the toolbox aspect means. However, the teachers' description in the interview is fitting into this schema: Dylan points out the importance of a students' ability to handle textbooks, whereas Joseph and Henry believe that routine exercises are the only possible way to motivate the majority of the students within a class. Again, the three persons mentioned are precisely those teachers who are not quite satisfied with their own teaching of mathematics and would like to change their situation, using quite different methods however.

On the basis of the interview, we may classify Dylan and Harry as the most innovative among these six teachers. The tendencies in their ideal view of mathematics are the same, however there are small differences concerning the role of systems and structures in mathematics (Dylan, System = 5; Harry, System = 1). Minor differences may originate in their different academic careers. However, on the basis of the figures for the real classroom lesson, the assessment of Dylan (Process = 10) is considerably more rational than Harry's (Process = 20). Perhaps this discrepency is explained by the fact that Dylan, in contrast to Harry, has completed a higher university degree, so Dylan's mathematical horizon can be regarded as broader and his estimation of what is happening in school appears more modest.

Since there is no objective scale for the three mentioned aspects, the absolute numbers should not be overestimated. Moreover, it seems natural to us that primarily the weights set by the teachers, not the exact scoring, indicate their understanding of mathematics teaching, which leads to a linear ordering of the components. Thus we derive Table 4.2:

Teacher	real	ideal
Dylan	T > P > S	P > T = S
Harry	P > T > S	P > T > S
Henry	T > P = S	P = S > T
Joseph	T > P > S	P > T > S
Ken	P > S > T	P > T > S
Larry	P > T = S	P > S > T

Table 4.2. The ranking of the components derived from Table 4.1
with T = tool, S = system and P = process aspect

Apparently, with the exception of Ken, all teachers rank the system aspect at least second in their real teaching and this can be clarified through the interviews. In the past, Ken was extremely in favour of the formalism aspect; this conception may not have disappeared completely. It is remarkable that Larry gives formalism, together with toolbox, an equal ranking. This fact may most likely be explained through Ken's and Larry's teaching career in a *Gymnasium* and its mathematics curriculum.

On the other hand, with respect to ideal teaching, the process aspect is ranked first by all of the teachers, and, in Henry's case, on the same level as the system aspect. The interview data confirm these observations where Henry favours a stronger dominance by formalistic aspects. This teacher seems to be obligated to mathematics, which he has been taught at university to be of structural importance. In the interview, the authors get the impression that he feels somewhat guilty since his current situation makes it, in his opinion, impossible to present this subject in an adequate manner.

4.2.2 Triangular approach

In Figure 4.3 we illustrate the marks within the equilateral triangle which were taken from the teachers' original responses (Appendix 1).

There are three features which come to mind at first glance: (1) the distribution of the respective positionings, (2) the tendencies of change which are represented as vector arrows and (3) the magnitude of assumed change.

Firstly, the outer distribution underlines the observations in 4.2.1. Dylan, Joseph and Henry are found, with their implemented lessons, in the toolbox corner. We have already listed some corresponding quotes in Section 4.1.

Secondly, the predominant tendency of change for the teachers underlines the importance of the process aspect. The two *Gymnasium* teachers Ken and Larry show some slight differences, in particular Larry. It should be noted that Larry does not estimate the necessity of change in his own classes as very high. Furthermore, the interview reveals that he does not believe in the possibility of fundamental change in the present system. Possibly the daily disturbances that take place in his classes lead only to marginal frustration, because for him (as for Ken) mathematics exists outside the classroom and, as a result, is a pure and philosophical discipline worthy of respect. Equally, he has adapted to the system.

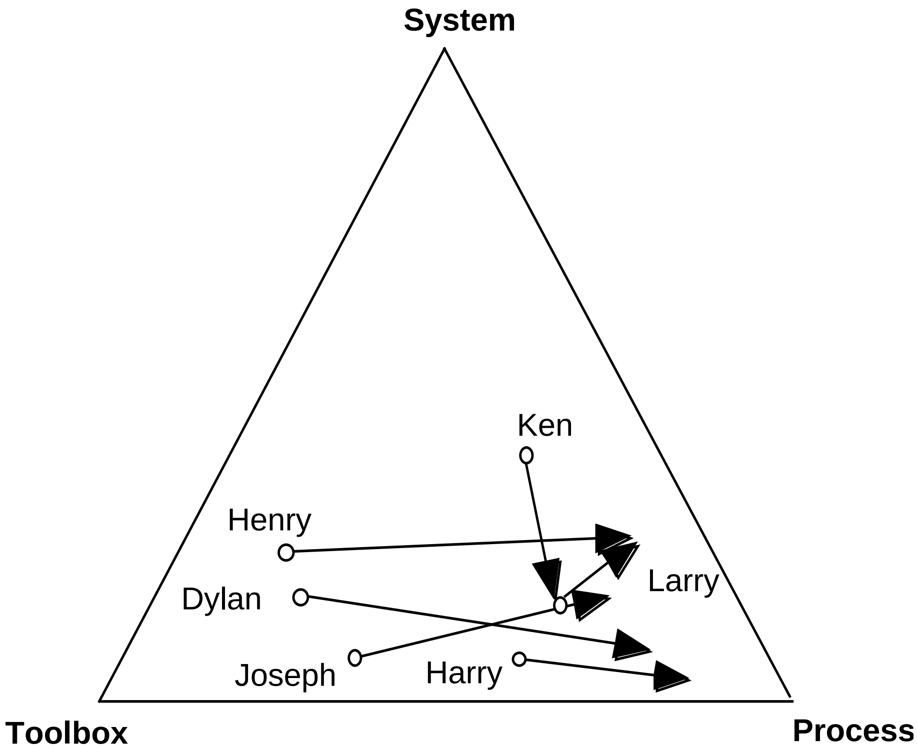

Figure 4.3. The self-estimation data in graphical form as given by the teachers. Arrows (real to ideal) indicating tendencies were drawn by the authors[3].

Thirdly, the diagram points on the whole to three classes in view of the arrow lengths: Dylan, Joseph and Henry; then Ken and Larry, with Harry taking on a middle position. These observations do not contradict the interview data when intervening feelings are included as a measure for change. In the cases of Ken and Larry we have already given explanations above. The numerically determined Euclidean distance on the basis of the Table 4.1 (see in contrast Figure 4.3) is $\sqrt{8}$ for Ken and $\sqrt{18}$ for Larry.

4.2.3. Comparison of Both Self-Estimations

The question arises as to which data should be more seriously regarded: the graphical or the numerical data, or whether both 'messages' are equivalent. Of course, the representation modes are equivalent in a mathematical sense as each three-element distribution can be calculated as a vector in the triangle and vice versa.

Next, we believe that the teachers are not trained to transform numerical data into barycentrial coordinates, however since they are trained in mathematics they should be able to handle graph-

[3] The idea to show the tendency with arrows is due to Peter Berger (1995).

ics. Furthermore, we do not believe that the teachers attempted to present direct translations of both data sources. However there was no chance to interview the teachers about that assumption.

Our hypothesis is that both representations have their own messages and may cover partly different aspects. Arguments and quotations from the interviews support many entries in the graph as well as in the table. Thus the fact that corresponding data are not exactly coincident should not be overemphasized.

For example, Henry estimates his mathematics view by Toolbox = 14, System = 8, Process = 8, thus the aspects of system and process play an equivalent but subordinate role. In the ideal teaching, his scores are toolbox = 6, system = 12, process = 12. This feature is not reflected in the graphical representation, where the system aspect remains unchanged. However, the length of the vector indicates his feeling that his real teaching differs greatly from ideal teaching.

Apparently there are some inconsistencies in the numerical and graphical data of Ken. His estimations of real and ideal teaching show some interchange of the roles of toolbox and system, which should be represented by a reflection of positions within the equilateral triangle. On the other hand, Ken's arrow in the graphical mode calls for some change, in particular, towards more process aspect and less system.

Joseph's data in both surveys also deviate considerably; the Euclidean length of the alteration vector is $\sqrt{5^2+2^2+3^2}$ i.e. $\sqrt{38}$, whereas for Dylan it is $\sqrt{200}$. However, the arrow length in the graphic representation is in the same length region. When considering the presentations given in the interviews, these graphic representations reflect the situations more adequately.

Whereas it is easier to realize the tendencies and the direction of the changes in the graphical mode, the table may show some clues or patterns as to how the changes should take place. Note that all three, Dylan (system = 5), Harry (system = 1), and Larry (system = 9), would not be likely to change the absolute value of the factor system; they only prefer an exchange between the tool aspect in favour of the process aspect. It must remain an unanswered question whether or not it is an intentional exchange, or perhaps whether it is just merely a strategy to treat the data which is to follow the Dionne categorization and to distribute 30 points twice among three entries. These arguments again support our hypothesis that the derived Table 4.2 is no less important than the absolute values.

4.3 Evaluation of the Information from the Different Sources

It is indisputable that Dionne´s three-pole polarization, corresponding to the Ernestian categorization, must be seen only as a primitive model, particularly as through this approach itself the dimensionality of the views is limited to three. Nevertheless, in spite of its simplicity, it still possesses a high degree of clarification power, especially when related to a first approach to the problem of becoming aware of, and identifying different views, on mathematics. The detailed comparision of the partly different self-estimations on the part of the teachers, in view of the Dionne components, makes clear that the numerical data are in need of commentary. They could never have been derived only from the interview data! The ranking table 4.3 derived from the information would also be too coarse to allow detailed conclusions, as in our survey it relates considerably different persons to the same ranking lists. In other words, determining views is of little evaluative use when one asks the interviewees for rankings.

It is vital to highlight such examinations, whether or not they concern the real teaching of mathematics or are valued as the ideal teaching of mathematics. In turn, one should determine the weight distributions of both teaching situations and their scores, using the Dionne components. Since it is not evident how to choose 'objective' figures, the two respective point distributions stabilize themselves reciprocally.

Next, we highly recommend use of the tabular as well as the graphical mode of investigation in the sense of Dionne (1984). However, the noticeable inconsistency should not be overvalued because both sources of data make allowances for varying emphases.

Metrical aspects play an especially strengthened role in pictorial illustrations. The examinee can highlight his basic discrepencies, and, finally, a direction of change will become evident in relation to the three components represented by the three corners of the triangle. Whereas the graph informs one on the magnitude of a change, the table may show the kind of change.

In our experience it seems to be essential to address both assessments in a parallel fashion in order to give the examinee the possibility of internal self-comparison. The respective point distributions stabilize themselves reciprocally. Corresponding numerical data also possibly provide an additional indication of which changes the responder has in mind.

For these reasons we urge that, when the relatively coarse Dionne model of catagorization is used, attention should be paid to the clarity between the actual and the ideal mathematics classroom situation. Both three-dimensional figures should be examined simultaneously, and the test person should be allowed the possibility of a graphic representation by the corresponding self-estimation.

5. Conclusion

The main conclusion to the research question examined through our observations is in a sense trivial: related methods will mainly produce both partly redundant and additionally confirming information, thus proving the consistency of the data. Furthermore, there were no *global* contradictions between the information of the teachers (obtained from different data sources), particularly with respect to the Dionne parameter (numerical as well graphical). This is of great importance, especially when employing and mixing qualitative as well as more quantitative methods. Individual inconsistencies, however, could be accounted for, i.e. made plausible by the statements given in the interviews.

Further, we can state that there is no best (indirect) method by which teachers' views can be investigated. Firstly the open interview, as well as the open discussion of the questionnaire, shows once more that closed questionnaires are of limited use. Since we began the process, we had the possibility of correcting some formulation of the questions, e.g. we had to omit one item whose discussion showed the inadequacy of the questioning.

Secondly, the interviews lead to some interesting central quotations describing the main features of the teachers' views. It is not easy however to condense the verbal profile and to 'coordinatize' the teachers' positions in order to compare them objectively and quantitatively.

Thirdly, although the Dionne method appears to be a very rough quantitative tool which ignores some details and seems to be unaware of other details, it leads to some numbers which contain worthwhile information. Of course one cannot disregard that the fact that the Dionne approach (in both versions employed here) projects a highly dimensioned world of beliefs into 2- or 3-dimensional versions. Such a procedure leads inevitably to serious reductions. Individual conclusions about the persons, even when based on Dionne parameters, still contain many uncertainties, unless one couples these results with the statements in the interviews. The interview statements in this survey thus fulfill the central function of being explanatory and authoritative. As expressed before, one should clearly distinguish between real and ideal teaching. It is then the pair of vectors which tells the true story. Finally, we believe we have proven that additional collection of graphical data is not unduly costly and is of important explanatory value. Intervening feelings and effects are revealed, e.g. in the length of the arrows, more clearly than in the numerical data.

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Appendix 1 The letter to the teachers

Starting point: a rough classification of mathematical views consists of the following three perspectives, which are part of every view of mathematics and the teaching of mathematics:

T Mathematics is a large toolbox: doing mathematics means working with figures, applying rules and procedures and using formulas.

S Mathematics is a formal, rigorous system: doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts.

P Mathematics is a constructive process: doing mathematics means learning to think, deriving formulas, applying reality to Mathematics and working with concrete problems.

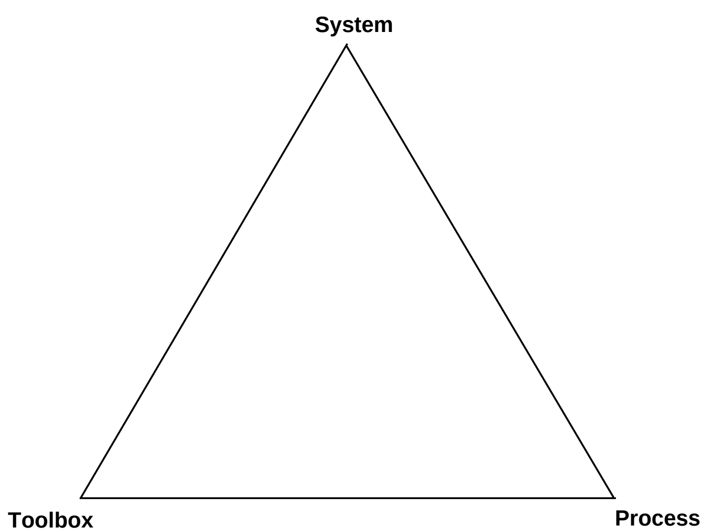
Question 1: Distribute a total of 30 points corresponding to your estimation of the factors, T, S, and P in which you value **your**

	T	**S**	**P**
real teaching of mathematics			
ideal teaching of mathematics			

For additional comments please use the reverse side of this page.

Question 2: Acknowledge your position on the three factors mentioned above by marking points within the equilateral triangle below.

x = real teaching of mathematics
o = ideal teaching of mathematics



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Thank you very much!

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