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SOME PARADIGMS OF MATHEMATICS EDUCATION — AND HOW THE WORK WITH COMPUTERS IS RELATED TO THEM (WITH PARTICULAR CONSIDERATION OF GEOMETRY TEACHING)

Abstract:

Mathematics didactics are influenced by many other disciplines. Adopting their results and methods without adapting them to the concerns of mathematics teaching often entails only short term progress, if any at all. In this paper several questionable paradigms originating from other disciplines are discussed together with how they are modified or reinforced by the inclusion of the work with the computer. The considerations are put in practical terms for the case of teaching geometry using a so called Dynamic Geometry Software (DGS). Careful investigations of the positive and negative effects are still needed.
The paper is based on lectures given by the author at the 8th International Symposium on Mathematics Education at Klagenfurt, Austria, (see Kadunz et al. 1998) and at the annual meeting of the GDM 1999 at Bern, Switzerland, and reflects mainly the situation in German mathematics didactics. Thanks to John Searl from Edinburgh for his valuable comments and his help with the translation.

Mathematics didactics are influenced by many other disciplines (pedagogy, general didactics, philosophy, psychology, social sciences, mathematics, etc.) and has to take into account, and even incorporate, their results and methods. But at the same time it is a scientific discipline of its own with its own methods and results, comprising (at least) two different traits, namely a descriptive and a normative one, both directed towards theory and practice at the same time. Didactical research includes not only empirical, but also 'engineering' methods (as Wittmann, 1995, puts it). Of course, no mathematics didactician can cover the whole field, but she or he should take notice of the work of others within the field, possibly grounded on different scientific paradigms, e.g. the experience of mathematics teachers (Wittmann 2000), and not reject others' ideas from the beginning. In particular, she or he should not only accept those outcomes from research which fit her or his own theories (assumptions, beliefs), but also those, which do not support them or even contradict them. On the other hand, if methodologies from other disciplines are used puristically, i.e. without adapting them to mathematics didactics, for example ignoring the mathematical, epistemological and/or cognitive structure of the involved specific subject matter, they may well result in surprising short-term findings, but in the long run they will be of little use for really improving **mathematics** teaching. Such extraneous paradigms are, for instance:

(i) Taking over statistical methods from medical, psychological, economical, social research, and often applying them to (the outcomes of) teaching-learning-processes without controlling, let alone establishing, the standards of representativity of the sample, independence of the investigated attributes, and in particular validity of the questions.

(ii) Imposing a formalistic mathematical-logical structure on the collection of objects which are involved in the teaching and learning of mathematics (subject matter, cognitive operations, social actions etc.).

(iii) Reduction of human beings to information-processing beings, thus forcing a close relationship between artificial and natural intelligence.
(iv) Neglecting social and other non-cognitive effects on mental processes.
(v) Transporting philosophical and political theories and ideologies (for instance 'constructivism' or 'situated' or 'social' or 'collective learning') into the field of education, and claiming pedagogical and didactical conclusions which are based on minimal corpus of experience, if any at all (see Anderson, Reder & Simon 1995, 1996, 1997). (Of course, such descriptions of learning and understanding can provide insight in the classroom situation and in learning processes, but they are only partial understandings.)
(vi) In close connection with (v): over-estimation of students' and under-estimation of teachers' contributions to the learning processes.
(vii) Again in close connection with (v): disregard for the subject matter as a major influencing variable in the teaching and learning of that very subject matter.

These shortcomings have been the daily bread in the mathematics didactics community since the 1960s, and the quarrel about the so called New Math was one of the first considerable examples. It goes without saying that corresponding deficiencies adhere to other scientific disciplines as well, perhaps with the exception of mathematics.

With the promotion and propagandisation of the computer as tutor, 'tutee', a means, or a medium for mathematics teaching (and as a device for the science of mathematics teaching) the psycho-socio-scientific structure of mathematics didactics has been coated with a new quality, but was not changed in its essence. One can go through all the items listed above, and one will find a computer based variant or reinforcement. I will just indicate a few:

(ii) with (iii) and (iv) There is nothing wrong about modelling human intelligence, the human brain and neuronal system with a computer language (i.e. mathematically, in the end); the fundamental fallacy consists in confusing the computer model with human cognition (as an abstract concept!). This identification may be a paradigm of the artificial intelligence movement, but I cannot find evidence that it is of use in the education of children and adolescents, in particular, if one strives to avoid (iv).

(v) with (vi) and (vii): Surprisingly, parts of the computer-into-education camp (CIEC) claim to benefit from those 'soft' pedagogical paradigms mentioned above, based on simple lines of reasoning like:

(a) From a constructivist point of view, the person who is originally called 'teacher' has to confine herself or himself to the function of an organizer and moderator, so that the students take responsibility for their work, and the computer is the ideal gadget to make the students independent from the teacher's direction.

This argument misses several hard facts. As the teacher is replaced by the computer, the students' actions are still determined from outside, and that not only by an electronical appliance with all its well known technical, psychological and social shortcomings, but also by programmers who, when programing either Logo microworlds (in Papert's, 1980, language), or computer algebra systems or 'realistic' scenarios, did not have in mind that special student and that special situation, where their computer programs are used. And in the end students' actions are still determined by the society, the school administration and the teachers as well, who will stay in control of what the students are offered, even if schools were abolished and learning was to happen at home.

The willingness and ability of young people to take it upon themselves to learn something independently is very restricted, as they lack experience, knowledge (as a requirement for acquiring new knowledge) and, last but not least, insight in the necessity of learning this particular thing or of learning anything at all. Whether human teachers in general deal well with this basic pedagogical challenge can be doubted, but that computers do not has been appreciated even by the inner circle of the Logo community (see e.g. Hoyles & Sutherland 1990).

Mankind has not only biologically the longest time of adolescence among all creatures, but in our complex modern societies there is also need for an intensive social, intellectual, emotional etc. maturation of every individual. For the overwhelming majority, education by the parents alone would be grossly insufficient (and impossible). Schools are an important part of the educational system and, as such, part of the real world. On the other hand, they themselves (as well as the knowledge, ways of thinking, attitudes etc. which they impart) are only models of the real world. Maybe they are and deliver very poor models, but obviously these models would be immensely more impoverished, if a leading role in education were assigned to the computer (in its actual application, even including virtual reality etc.).

(b) When students are to work with computers in the everyday classroom (which happens rather rarely in Germany, and more frequently in Austria), quite often two or more students have to share one keyboard and one screen, because there are less computers than students and/or there is a lack of space. Regularly there is made a virtue of this necessity by claiming a promotion of 'social learning', of 'key qualifications' (to communicate, to work in a team etc.), of responsibility for others, of better learning, etc.

Even if one agrees to these rather vague ideas, there is no evidence of great success. The actual work often concentrates on a small subset of the students. If better students 'help' the weaker ones, they often tend to execute the task completely, thus preventing the weaker ones from grasping the subject matter in question (cf. Vollmer 1997). When trade and industry urge the schools to promote teamwork, they have in mind above all that school-leavers should be able to fit themselves into a team, and this is quite the opposite of how that qualification is understood in the pedagogical paradise. Working collectively can result in a loss of concentration; this drawback can be intensified by the computer screen with its bully effect (i.e. a permanent call for action); and thus the learning of some subject matter, in particular of mathematics whose concepts demand mental effort can be impaired.

It must be questioned whether (even older!) pupils have sufficient mastery of subject matter, communication, educational goals etc. for really successful long-term work independent from a teacher. In an extensive study, Kaiser & Blum (e.g. 1985), together with D. Burghes and N. Green from Exeter and Evesham, England, compared thoroughly mathematics instruction in England and in Germany. In those areas where the German pupils did better, they identified as one cause the lower degree of direction among English teachers, which entails a lower amount of learning time for the weaker ones among the English pupils. (Whether the work with computers can meet this drawback, as is hoped by the CIEC, is not clear.)

(c) There is nothing wrong about social learning, learning to communicate, or taking responsibility, and at all times all over the world schools endeavoured to promote these and similar virtues in the pupils. But only in the last few decades in several Western countries has this been done at the expense of subject matter. This trend seems to be a pedagogical left-over of the 1960s student movement (which itself was part of a great societal upheavel in many Western countries at that time), where the institution of school, the role of the teachers and the structure of subject matter were denounced as parts of the authoritarian capitalistic system (Marcuse).

In spite of the computer's military origin, its incessant and ubiquitous use by the military, administration and business worlds and its dominant character towards the users, its potential as a pillar of that authoritarian system was ignored at that time, and despite some deep criticism (mainly not in terms of politics, but of psychology and sociology) many positive qualities were attributed to it, like opening the world of knowledge with its large memory, making accessible complicated situations and deep concepts with its multi media potential (in particular: visualizing), or making feasible worldwide communication with its net-like linking structure (internet). And today, in Western societies and school systems the computer is imparted the image of relieving the users from old-fashioned restrictions of any kind, as if students cannot help but 'learn', if only the product to be learned is wrapped up attractively enough (based on the assumption that 'knowledge' is a product to be 'delivered' rather than a process).

One of these restrictions seems to be the necessary time-consuming hard work on mathematics (more general: with effort-demanding subject matter) at school (and in many university disciplines!), which is recognized as a fundamental way of acquiring and securing knowledge, as a prerequisite for autonomously coping with everyday life as well as for many vocational careers, and as an essential part of general education. In particular in informatics didactics (= computer science education) there can be observed a clear trend away from mathematics, not only reflecting the ongoing rapid change of paradigms in computer science itself, but also as a specialization of modern pedagogy emphasizing general qualifications and neglecting 'hard' subject matter.

The informatics didactician would argue that working on projects, modelling some reality with the computer, experimenting with the model, describing the effects etc. is the subject matter of informatics; and many mathematics didactians and teachers would subscribe to this point of view for mathematics as well, at least in a moderate manner. But however the students' activities may be defined, there is a trend to spare them some effort with a lot of 'hard' mathematical concepts and with troublesome teaching-learning-methods. There is at least one good reason for this trend. In all countries over the world the classical teaching of (advanced) mathematics is quite ineffective (even though at the TIMSS there were countries with rather high scores). But there is no evidence that those pleasant looking scenarios with students independently exploring computer microworlds with some mathematical content would meet the demands of the society and the concerns of the indivdual better. The improvement of mathematics teaching must proceed from the subject matter (cf. Bender 1998, Wittmann 2000), and the computer can assist in this, as I will point out in the part about geometry teaching.

The CIEC has added one more type of fallacies to those listed above. Overwhelmed by the technical power of the device and by the societal change it has already caused directly and indirectly, and neglecting a lot of other variables, some proponents were (and still are) inveigled into

(viii) Making far reaching predictions of a psychological, social or pedagogical nature about how computers will change education.

(a) In the year 1955 the Noble prize holder H.A. Simon predicted that within the next ten years chess computers would beat the best human chess players. Only now at the end of the millenium the computers' efficacy has reached that of the human top group. In the meantime, by adjusting themselves to the computers' strategies, the best human players managed to obtain again better results, thus postponing the computers' final victory a few more years. It is not the false estimation of the time passing until computers play better chess than humans by the factor 5, but of the amount of computer memory and elaborated man based strategies actually needed, which proves Simon's prophecy to be a mere speculation at that time. And all this applies to a simple game with two dozen of rules like chess.

(b) In his Logo based educational utopia, the computer scientist S. Papert (1980) predicted the abolition of public schools and their replacement by computers which will be available in every private home. In an interview in 1998 he committed himself to a period "within the next 20 years" (in America? Worldwide?). This prophecy can be taken as a striking example for a computer based one sided view on present and future political and societal reality (see Bender 1987).

(c) Similarly the German computer educationist K. Haefner (1982), resuming a catchword from the early 1960s (Picht), predicted a deep educational crisis for the mid 1980s (in Germany?), caused by insufficient use of computers in the educational system, and only to be surmounted by an immense intensification of this use. Every generation of educationists seems to determine a crisis of the educational system. Possibly it really is in a permanent crisis. But there is no evidence that this is due to and can be overcome by the computer.

(d) With respect to timing and scope of subject matter, the prophecies for mathematics instruction are more moderate, but within the frame of this discipline they sometimes also sound rather radical. Here it is the working mathematician or the fancier of geometry who is pleased with the possibilities of a Computer Algebra System (CAS) or a so called Dynamic Geometry Software (DGS) and believes that this can and will be adopted more or less literally by the schools and will dominate mathematics teaching (e.g. Hanisch 1992).

Meanwhile, in the computer branch of the German speaking community of mathematics didacticians the early enthusiasm of a few has given place to a more cautious approach of a majority (cf. the annual reports of the group 'Mathematics Teaching and Informatics' in the GDM, Hischer 1992ff, and Herget, Weigand & Weth 2000f):

Now as ever, mathematics is an indispensable element of general education. Traditional mathematics teaching, more or less similar all over the world, has several approved essentials of which the most important is some guidance of the learning process by a teacher. It is true that traditional mathematics instruction is not very effective and that teachers do not really have the learning processes under control. But there is neither evidence nor experience under everyday conditions that autonomous learning in general, and also if based on the work with computers, would be at least equally (let alone more) successful. (The phrase of autonomous learning is actually a contradiction in terms, as long as there is some specific support, and I question whether it is desirable as such, not to mention achievable.)

There have been attempts to re-define mathematics in order to adjust it to the work with computers (e.g. Horgan, 1993, on the university level, or Papert, 1980, on the 'school' level), but they gained little success. Indeed, new branches emerged (cryptography), old problems were solved ("four colours suffice"), examples and counter-examples can be found and calculated more easily, each mathematician has become her or his own typesetter, and at school several skills lost importance (e.g. doing complicated elementary calculations in an automated manner, because of the existence of the pocket calculator) or gained importance (e.g. visualizing situations, concepts, relations etc., because of the potentials of CAS, DGS etc.). But in the deepest depths of its substance, mathematics, at all levels, remains the same; and this resistance is not due to the inertia of the system of established mathematicians, didacticians and teachers, as several revolutionary representatives of the CIEC suspect, it comes from subject matters itself. Still, the computer is a valuable tool for doing mathematics at all levels, and neither the mathematics researcher nor the mathematics teacher can renounce it with a clear conscience.

One of the main desiderata of contemporary mathematics didactics is the integration of the computer into 'the' mathematics curriculum. In the domain of geometry there is some special research on the stimulation of formal talents like working creatively, independently, collectively etc. (Hölzl 1994, Laborde 1998, Weth 1997 and others) and some practical realization with an accent on subject matter (Elschenbroich 1997, verbal communication by Kadunz, and others), both types with more or less positive and negative results. In the meantime it is common knowledge that utilizing a spreadsheet, a CAS or a DGS for doing or for learning (by doing) mathematics requires an elaborated body of mathematical understanding, knowledge and skills (which may have and will be further developed with the help of the computer). There are also some pieces of patterns for medium-term mathematics instruction integrating computer and non-computer work (Baumann 1998 and Hole 1998). In Austrian high schools a lot of mathematics teaching is actually done with the use of computers (Wilding 1998, Wurning 1997 and many others), and also in Germany there have been (and still are) several more or less isolated projects, often with outstanding teachers and high achievers, with an intensive use of the computer in mathematics teaching (Lehmann, 2001 and many other papers, Thode 2001, and others).

But there is still missing a long term curriculum, say, for geometry teaching, from the first to the tenth (or 13th) grade. I do not call for a direct realization in school, but for a basis on which, at first, a broad academic discussion and then concrete syllabi, school books, mid- an short-term lessons, methods etc. can be founded. Geometry has close connections to the rest of mathematics, to many other disciplines and to the real world (including the computer with its many applications), and at the same time can be treated rather separately. This work would be much more extensive and harder than writing a school book in the conventional way, where one has other school books as a stimulus for good ideas and a long and elaborated tradition of goals, contents and methods (and it would not be pecuniarily lucrative). Of course, there have to be the customary differentiations, but now there is one more differentiating element, namely the computer, say in the appearance of a DGS like Cabri Geomètre (with the permanent 'danger' and chance of a revolutionary further development, so that one should not cling too narrowly to the specialities of one software). There have to be alternatives with more or less intensive use of the computer. But first of all the goals and then the contents (also mathematically, but mainly epistemologically and psychologically) and the methods have to be re-analyzed in the light of the new possibilities and restrictions. For each teaching unit there has to be considered carefully which understandings of concepts, knowledge and skills to which extent have to be provided, and which are to be developed; how the computer channels this development and how its influence can be made use of and/or must be modified. The whole analysis is primarily a matter of subject matter (I repeat) under epistemological, psychological and sociological aspects, and as an ideal scenario (ignoring all secondary and tertiary problems and paradigms) one should visualize a classroom with the students being active under the general, and if necessary also under detailed, guidance by the teacher.

There are rather early approaches by Schumann (e.g. 1991, 251ff) connecting traditional and computer oriented geometry and analyzing didactical and methodological questions in detail. But there was not given proper attention to this work. Instead of exploring the basic ways of understanding and imagining geometrical concepts (cf. Bender 1998) which the students should and/or would adopt, the students were often confronted with rather sophisticated tasks, and their behaviour towards these tasks was investigated (Hölzl 1994, 2000, Laborde 1998 etc.). The analyses of classical geometry didactics, in particular of transformation geometry and the role of (continuous) motions (based on a tradition which is more than 100 years old) (e.g. Bender 1982, Bender 1989, Schwartze 1990), were often not taken into account, although they prove to be highly relevant for the didactics of DGS with its drag mode. Here are some of the old (and a few newer) arguments:

1. Transformation geometry is motionless geometry, as transformations are one-to-one mappings from the plane onto itself without moving any point of the plane. Unfortunately, the notions (e.g. rotation) as well as the usual way of introducing them, namely by moving pieces of paper on a plane or suggesting the imagination of such a motion, leads the learners unavoidably to misconceptions, which, as all experience shows, nearly never can be repaired. Here we have a striking example for what Sierpinska (1990) calls 'mental obstacle' and thinks to be insurmountable.

   But, if we leave aside the question whether students should form a rather abstract algebraic concept of geometric transformation at all or just use transformations as naive tools for investigating geometric figures, the computer can support that formation of the algebraic concept in at least two aspects: the pixels which bring about the picture on the computer screen can be viewed as realizations of the points of the plane. It can be made comprehensible for (nearly) everybody that they do not move. By supplying them with colours, parts of them can be united so that they form some (meaningful) shape (e.g. a triangle); and by changing the colours in the right way, the illusion can be evoked that the shape moves without the points moving! It is like a wave in the ocean: It is not the water molecules which move from Hawaii to Japan, but their stimulated state of bouncing up and down. Of course, the metaphor of the computer screen can be made use of without its physical presence, but the students should have some experience with it.

The next useful attribute of the computer (which has nothing to do with the drag mode of DGS!) is to produce copies of a given shape, which are either congruent with the original shape or distorted following some rule and which can be situated at any position wanted, without any continuous transformation, thus not supporting the misconception described above.

2. But a large portion of geometric activities has to do with continuous motions or, more general, with continuous changes, and here the DGS can unfold its talents. These activities ca be subsumed under the most important general didactical principle of functional thinking, which can be formulated as: To aim at structuring problem situations with (mathematical) functions (mappings, transformations) mapping some domain into some range; to cruise deliberately through the domain and to observe the effects of this cruising in the range. For example: let F be the function which assigns to each polygon its area, and consider all triangles with two fixed vertices A and B. How does the area depend on the third vertex X? Or: let R be the composition mapping of two reflections. As one moves from one triangle to another ('move' the triangle), what is the resulting motion of its image (after one, after both reflections)?

   Although continuity of the changes is not necessary, it is very useful in order to recognize the underlying mathematical laws, because it is an important means to bring about relations between the covered positions and thus to objectivize the resulting changes. Before the advent of DGS, one had to make do with discrete sequences of pictures, and the students had to supply them with continuous transitions (if the teacher wanted them to produce continuous interpretations and mental images).

   This is quite different from the idea of the early Geometric Supposer(s) (Schwartz & Yerushalmy 1985), where the computer produced triangles of a kind (say, isosceles) one after the other with the help of a chance generator and there was no claim to any transition between the triangles at all.

   Of course, the motion of a triangle, produced in a DGS with the drag mode, is also a discrete sequence of still pictures, but it is to be regarded as continuous, and it usually is regarded as continuous.

3. Seen idealistically, there are mainly two phases of the learning process, where the drag mode can be brought into play. When approaching a new domain, problem, concept, theorem etc., in order to get acquainted with it and to discover, reject or reinforce conjec-

tures. Alternatively, after the treatment of a certain domain, problem etc., in order to get an overview, to deepen and secure the insights, to make new discoveries and to make plain the meaning of the new knowledge.

In my opinion the second phase is the major application of the drag mode by the students (without excluding the first phase or the phase(s) in between). Having themselves a much higher level of knowledge, experience and love for their subject, computer geometry didacticians and teachers tend to over-estimate the students' capability and motivation to put their own questions and answer them self-reliantly with the help of the computer. As nearly all studies show, learners need ample support in the initiating phase, and the phrase of discovery learning, allegedly one of the benefits of the work with computers, seems not to be suitable in general. Things look different, when the learners have some mathematical experience, understanding, knowledge and skills at their disposal.

4. Two major problems of classical geometry didactics and teaching are connected with the concept of mathematical proof. First: a great many assertions in the field of geometry seem to be evident to novices, whereas the expert knows that they have to be proved. Second: how can a novice tell, when a 'proof' is a proof? Both problems are exacerbated in computer geometry teaching.

The computer's image of infallibility, its graphical talents and the reproducibility of some peculiarity (if it is based on a rule) can even more diminish the students' insight in the necessity of proving this peculiarity. In particular, the teacher who stresses the making of discoveries can evoke the idea in the students that the work is done as soon as the discovery is made.

The question, whether in a geometric situation there is something to be proved, can be regarded as an extreme case of the second question "when is a 'proof' a proof?". — I want to illustrate the extra difficulties connected with DGS with the following examples:

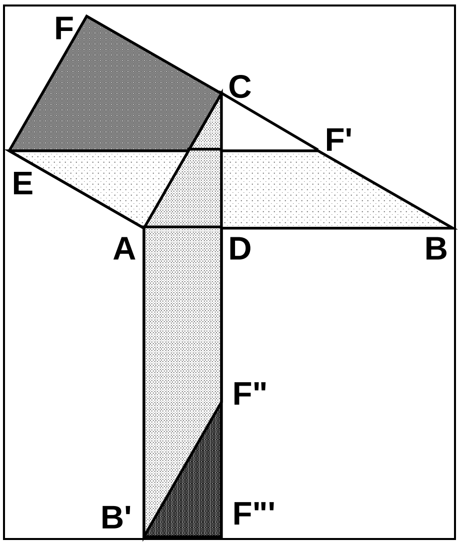

Fig. 1: Euclid's Theorem

Fig. 1 illustrates Euclid's theorem. The square AEFC can be mapped onto the parallelogram AEF'B by a shearing along EA, then onto the parallelogram ACF"B' by a rotation by 90° with center A, and finally onto the rectangle ADF"'B' by a shearing along AB'. As all these mappings are congruent mappings, the original square has the same area as the final rectangle. The three mappings can be represented as continuous motions (respectively deformations) and thus made more vivid, but this does not prove anything. One must know from other conclusions that shearings and rotations preserve the area. For shearings there are plausible arguments (for rotations anyway): During the (continuous) shearing the form gains at the head of the deformation the same amount of area as it loses at the tail. If one considers finite time intervals, this is basically the argument of classical geometry, and it applies immediately to the whole interval, i.e. to the transition from the starting state to the ending state, and there is no motion (needed) at all. Introducing the idea of infinitesimal changes either requires very elaborated concepts from calculus which are usually not available at this stage of curriculum and in addition are not adequate here, or lead to an extremely vague, if not faulty, argument.

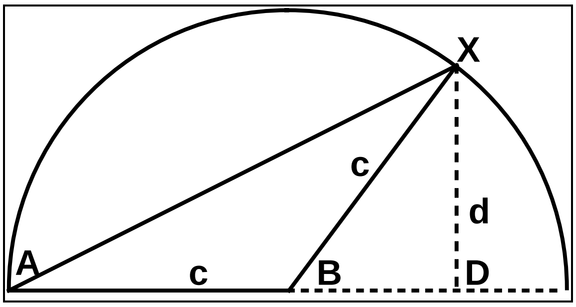

Fig. 2: The largest isosceles triangle

Fig. 2 (example by T. Weth) shows (up to congruence classes) all isosceles triangles ABX with their legs AB and BX of fixed length c , the angle at B variing from 0° to 180° (both excluded), i.e. the vertex X moving on a half circle with center B and radius c . At each position of X (with distance d from the straight line through A and B) the triangle has the area $\frac{c}{2} \cdot d$, and as c is fixed, this value is maximal, if and only if d is maximal. Either one accepts as being obvious that d is maximal, when X is on the 'top' of the circle (B=D), or one must go on with the proof: If D?B , then there is a triangle BDX with the side BX lying opposite the largest angle (namely the right angle BDX), thus being the largest side, whence d<c . Whereas d=c , if D=B .

The assertions used here ('if one angle of a triangle is a right angle, it is the largest' and 'the largest side lies opposite the largest angle') can be taken as obvious again, or have to be deduced from a system of axioms in the end. Of course, in modern geometry teaching nobody works really axiomatically. But even if several obvious 'facts' are taken as starting points for geometrical reasoning, it is not only a matter of the students' deficiencies, but a consequence of the curriculum, if it is often not clear, when a 'proof' is a proof, because it is often not clear on principle, whether a fact is obvious.

Again, on close inspection, the continuous motion of the vertex X does not deliver the proof directly, but only indirectly: In order to create a continous motion, one looks for a locus of a point, thus reducing the space of solutions from the plane to a line. In the end one has a one-dimensional set of points, of which one (several, or none) has the quality in question, thus representing the solution. There remains, of course, the task of finding the right point whose locus shall be considered, which can make many a geometric problem a difficult one even for experts.

In order to structure the situation, and, in doing so, to get an idea of the position of the solution and for a proof, it is obvious and reasonable to pass through the locus continuously and directedly, i.e. to use the drag mode, either in one's imagination or with a DGS. This distinction between the (indirect) mathematical and the (direct) didactical function of the drag mode seems to be not always clear to didacticians and teachers, let alone to students.

Anyways, there remains the crucial problem to translate the conditions of the task into a system of objects of the DGS and to omit cleverly one of those conditions in order to create a suitable locus as cognitive basis for the proof.

For more arguments and more examples see (Bender 1989).

5. In spite of all the assumed and actual visualizing and stimulating benefits associated with using the drag mode, there will be problems, where students could do better on the base of discrete sequences of pictures or of just one picture (cf. Lewalter 1997), whether their attention can be better drawn to the essential features (if the problem does not consist of finding them), whether they are not distracted by the impressive visual means of the DGS, whether they do not feel urged by the drag mode to be active, whether they are seemingly granted more freedom, whether they have no choice but do mental geometry, or geometry teaching includes more contents than just pencil-and-paper geometry (PPG) (on the computer screen or on real paper).

6. One of the weak points of DGS is their extensive disconnection from real world geometry. One of the main causes of the worldwide decay of geometry teaching in the last century was the treatment of geometry as axiomatic theory of the Euclidean plane, as range of application for algebra, as theory of the relations of point sets in the plane, as collection of sophisticated constructions and proofs, in short: as pure mathematics. All along, there has been a broad movement in geometry didactics, striving for a closer connection of geometry teaching to the real world, thus making it more vivid, more application oriented, including more intensively real actions (drawings, handicrafts etc.) and more directed towards the needs of the students. In contrast to this, following nearly all papers relevant to the subject, DGS just transfers PPG (with many more features, but still disconnected from the real physical world) to the computer screen, thus laying again a much stronger accent on PPG. Because of the old reasons (which I cannot discuss here in depth) it is doubtful, whether this will re-animate school geometry.

7. There is no doubt that today DGS have to be a part of the geometry curriculum. With their features like the drag mode, production of loci, the technique of subprograms (modularity), the possibility of continual accurate measuring, arrangement of colours etc., they cover an essential part of that curriculum, namely PPG. At the same time they give rise to basic experiences with New Media, and so they create some distinguished connection to the real world, not to the physical one, but to the visual one, which becomes more and more important to all of us.

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