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SITUATED MULTIPLYING IN PRIMARY SCHOOL: COUNTING INSTEAD OF ADDING!

Abstract:

The paper describes an empirical investigation about the arithmetical strategies of primary school children dealing with multiplicative real-world situations. We constructed three multiplicative settings, similar to each other in their arithmetical structure but different in the situational contexts. 66 second-graders – age 7 to 8 – and 56 third-graders – age 8 to 9 – solved in pairs two of these problems.
The results show that children did not use addition as their main strategy but preferred counting and number patterns. Several hypotheses are discussed explaining these results. The conclusions drawn depend on the preferred hypothesis.

1 Multiplicative Reasoning

Multiplicative understanding demands for two different aspects: Firstly, children need to know which situations can be modelled by multiplication or division. Secondly, they need strategies to solve these multiplicative modelled problems.

FISCHBEIN et. al. (1985) identified equal grouping, the union of equivalent, disjoint and finite sets, as the „intuitive model“ of multiplication. In this model, multiplication is defined as iterated addition of the same number. Although this model is restricted to integers and implies certain misunderstanding, e.g. “multiplication makes bigger” or “the dividend is always bigger than the divisor”, it is the main model in primary school, but not the only one (for details see RUWISCH 1999 a). In Germany, this model is subdivided into two aspects: dynamic situations consisting of an iteration of actions with the same number of elements and static situations which present simultaneously sets with the same number of elements.

In the literature, the strategies to solve multiplicative tasks are usually divided into levels of different complexity, efficiency and elegance. ‘Counting strategies‘, ‘iterated addition and subtraction‘ and ‘use of multiplication and division facts‘ are three strategies, which are seen as an increasing sophistication and can be found in all taxonomies (see e.g. ANGHILERI 1989, BÖNIG 1995, HEIRDSFIELD; COOPER; MULLIGAN & IRONS 1999, KOUBA 1989, MULLIGAN 1992, MULLIGAN & MITCHELMORE 1996, SELTER 1994). Other strategies are not mentioned by all researcher, depending on their distinctions between them. Whereas ANGHILERI (1989) differentiated between six strategies for solving multiplication tasks (direct modelling with materials, unitary counting, rhythmic counting, use of number patterns, use of repeated addition and use of multiplication facts), others only state four (e.g. SELTER 1994: counting, repeated addition, calculation by using decompositions, known facts). As the two examples already show, the researcher also differ in the range of the proposed taxonomy. Whereas SELTER stresses calculation strategies like using the commutative, the associative and the distributive law by forming an own group, ANGHILERI and others include these strategies into “use of multiplication fact”. On the other hand “direct modelling with material”, a category which can be found in the papers of ANGHILERI (1989), KOUBA (1989), MULLIGAN (1992) and others, may be seen as somehow crosswise to the other strategies (see RUWISCH 1999 a, SELTER 1994), because children using number patterns may also use fingers, drawings etc. to represent aspects of the given problems.

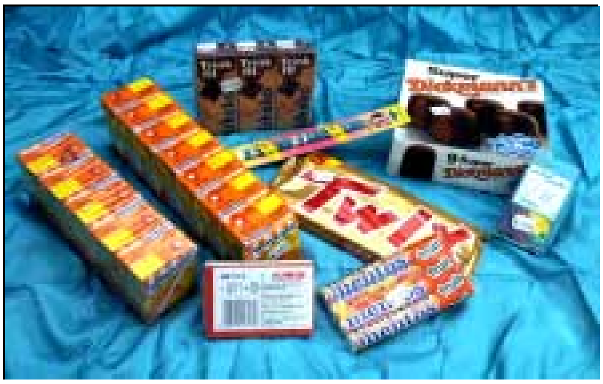
Although all taxonomies provide categories to record and to classify arithmetical solving processes, there are only few statements about the transition from one level to another (e. g. STEFFE & COBB 1984) and no results concerning complex real world situations with multiplicative structure. Whereas you will find papers about the progression in schematising additive strategies, there is little literature about the development of multiplicative solving processes.

2 Multiplicative problems in a situational context

At the university of Giessen we constructed three multiplicative settings, which German primary school children should be familiar with: classroom party, juice punch and doll's house. The children had to determine the multiplier of several tasks, which were embedded in one of the contexts mentioned above (for details see RUWISCH 1999 a).

Classroom party

In the setting „classroom party" the children were given a shopping list with seven goods. They were asked to determine the number of packs necessary for a classroom party with 18 children, under the condition that every child will get one piece of every article. To look for more information and to solve the tasks the children could use a so-called supermarket, where all the goods were presented in their genuine packs (see picture 1).



Picture 1: Goods in the setting „classroom party"

Juice punch

In the setting „juice punch" the children were presented a situational context in which a child liked to have a punch for its birthday party (for details see RUWISCH 1999 b). The recipe for the punch gave the necessary amount as numbers of glasses. The children's task was to determine the number of bottles necessary for every juice. Therefore the children were offered bottles of juice of three different sizes in the "supermarket" (see Figure 1): bottles of two glasses, bottles of five and bottles of seven glasses.

apple	banana	orange	pineapple	cherry	sparkling-water	peach	multi-vitamin
	❷	❼❺❷❷	❺❷	❷	❼❷❷	❷	
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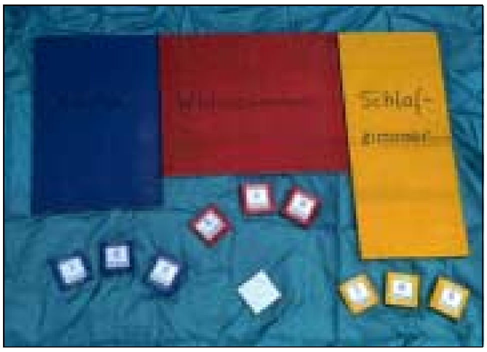
Figure 1: Tables with bottles in the setting „juice punch“

Doll's house

In the third setting, “**doll's house**”, the children were presented a doll's house. This house existed of three different rooms, which should be tiled in different colours (see picture 2). The children's task was to determine the number of packs of tiles. The tiles were again presented in a “supermarket” in three different sizes: packs of three, of six and of eight tiles (see figure 2).



Picture 2: Materials in the setting „doll's house“

blue tiles	red tiles	yellow tiles
	6 3 3	6 3 3
6 3 3	8 6 3 3	8 6 3 3
8 6 3 3	8 6 3 3	8 6 3 3
8 6 3 3	8 6 3 3	8 6 3 3
8 6 3 3	8 6 3 3	8 6 3 3
8 6 3 3	8 6 3 3	8 6 3 3

Figure 2: Tables with the packages of tiles in the setting „doll's house“

Similarities and differences

Comparing the three settings (see table 1, following page), one can stress the following similarities:

- all settings belong to the children's experience and present a complex situation, which can be divided into several sub-tasks;

situational context	given materials	situational model of multiplication	arithmetical structure
numbers „classroom party“	goods with different numbers of elements (*b*) per pack	*equal groups* number of packs number of goods per pack total number of goods	*x* ### *b* ### *18* packs with *b* given as 2, 3, 4, 5, 6, 7 or 9
volume „juice punch“	bottles of juice with different volumes (*b*)	*equal measures* number of bottles number of glasses per bottle total number of glasses	*x* ### *b* ### *a* *a* given in the instructions as 15, 12, 8, 5 or 20 *b*###{2,5,7}
area „doll’s house“	three rooms of different area (*a*) packs with a different number of tiles (*b*)	*rectangular array* number of rows number of columns number of tiles *equal groups* number of packs number of tiles per pack total number of tiles	*x* ### *b* ### *a* *a* to be determined *b*###{3,6,8}

Table 1: Similarities and differences of the designed problems

- all settings are realistic in the instructional story, in the materials used and in the possibilities of handling these materials;
- all settings are open for different solutions and different solving-strategies;
- all settings include multiplicative structures, which are based on the situational model of equal measures;
- all settings demand for the determination of the multiplier, some sub-tasks in every situation also asking for solving „divisions with remainders“.

These similarities were constitutive for our construction. Although the three settings are similar in the whole, they also differ in details:

- the settings differ in complexity: buying goods for a classroom party is less complex than measuring out rooms and determining the number of tiles and packs;
- the settings differ in familiarity: children are more familiar in buying goods than in tiling rooms;
- the settings differ in their mathematical contents: numbers in the classroom party, volume in the juice punch and area in the doll’s house;
- the settings differ in the subtype of the situational model: classroom party may be seen as equal grouping, because the packs are countable, whereas the doll’s house also requires the interpretation of the rooms as rectangular arrays of square tiles;
- the settings differ in the arithmetical demands: in the first setting the total number of goods is given and constant, in the second setting the total number of glasses is also given but

differs with the different juices, and in the third situation, the total number of tiles needed is given indirectly by the area of the rooms and must be determined first by measuring out the three rooms.

3 Data and methodology

Subjects: The subjects of this study were 122 children of seven German public primary school-classes. 66 children – age 7 to 8 – from second grade participated before any formal instruction in multiplication or division. 56 children – age 8 to 9 – from third grade had already learnt their multiplication tables when they participated in our investigation.

Procedure: All children worked in pairs and solved two of the three problems within two weeks. They were withdrawn from their classrooms and confronted with the materials in a separate room inside their schools. There, the interviewer gave them a short introduction into one of the three situations. Then they worked by themselves until they indicated to us, that they had finished. On average the working-phase – which was videotaped – lasted 20 to 30 minutes. This working-phase was followed by a short re-interview about the actions and solution-strategies we had observed.

Analysis: The videotapes were analysed in different forms. We mainly worked with categories directly on the tapes to analyse the used arithmetic strategies. A first transcription of the verbal comments was necessary to analyse the used heuristics and shown actions. Additionally, some videotapes were transcribed very specifically and carefully in the form of interactional case studies.

4 Results

Concerning the arithmetical solving strategies it was striking, that addition did not play any important role. Besides, only few children used mental strategies like distributive or associative decompositions.

strategy	2nd grade	3rd grade
counting	21	3
number patterns	–	14
mixed	6	4
addition	2	–

Tab. 2: Arithmetical strategies in the setting „classroom party“

In the setting „**classroom party**“ two third of the second graders counted one by one, tapping on the goods or using their fingers. Every pack, which elements were counted, was put on another table, so that the packs could be counted in the end. Only two pairs (one eighth) of second graders added up to 18 or more by taking the packs, counting them in the end as well. Most of the third graders (more than 60 %) used number patterns to solve these problems, knowing in the end as well the number of packs. None of the third graders used addition, whereas some mixed number patterns and counting one by one.

strategy	2nd grade	3rd grade
estimating	–	4
measuring	4	8
calculating by		
counting	4	–
number patterns	–	6
mixed	2	2
addition	2	8

Tab. 3: Arithmetical strategies in the setting „juice punch“

In the setting „**juice punch**“ the children used three different solving processes to determine the number of bottles: four third-graders *estimated* the number by comparing the bottles and the glass, four second-graders and eight third-graders *mixed the punch* by measuring with the glass and counting one by one, and eight second-graders and 16 third-graders *calculated the results* by different arithmetical strategies using the numbers written on the bottles. Altogether second graders again counted mainly (two third of them) whereas only two children added (one sixth). However, half of the calculating third graders added to solve the tasks in this setting. Nearly all of them connected adding with one of the other solution strategies. Since nearly one half of the third graders did not calculate at all, there were overall only a good quarter of third graders adding.

To solve the tasks in the third setting „**doll's house**“ the children were required to connect the given packs of tiles with the three floors.[1] One quarter of children (ten second graders and six third graders) produced this connection directly in one step by double counting or multiplicative assignments. Three forth of the children (26 second and 26 third graders) divided their solution into two steps: They determined the total number of needed tiles for every room first, before connecting this information to the given packs. The first step was carried out either by counting (¾ of the second graders) or by number pattern or multiplication facts (more than the half of the third graders). No child used addition in this first solving step. Only four of the 60 children used addition to solve the question about the number of packs. One half of the second graders used also counting strategies during this second solving step, whereas the others showed – like 60 % of the third graders did as well – multiplicative assignments based on the material.

Altogether only seven pairs out of 61 used addition as their main solution strategy.

5 Discussion

How can this result be interpreted? In the following, different hypotheses are discussed. Each of them may explain some of the results very well, whereas other details are not elucidated convincingly.

- *The preference of a solving strategy is influenced by former instructions.*
  It is not amazing, that third graders used mainly number patterns, because in Germany a lot of time is spent to learn your number patterns by heart, before knowing multiplication facts. So, their multiplicative knowledge is mostly linked to those patterns. This hypothesis is emphasised by the observation of very few distributive decompositions. So, children did not use the connections between the tasks. But – and this is an interesting point – under these conditions the second graders should have used addition as their preferred strategy, because up to that point they had learnt addition and subtraction as main issues of mathematics.
- *In new situations, children fall back on older, but more improved strategies, even if they know new and more effective ones.*

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[1] Since the table of results in this setting is very complex, we only describe the main aspects here. For details see RUWISCH 1998 or RUWISCH 1999 a, p. 226.

Psychological investigations (e.g. SIEGLER 1988; STERN 1992) showed, that older and better known strategies are much more improved concerning the situations in which they are useful as well as concerning the procedure of doing it. Since they are routinised, they are preferred in new situations. Therefore, it is not surprising, that especially second graders showed so often counting strategies. These are more routinized than the new strategies adding and subtracting. But in this hypothesised context it is very surprising that third graders liked number patterns more than addition. For them, number patterns are very new whereas adding is the older and hopefully more improved strategy.

- *Second graders don't interpret multiplicative situations on their knowledge about additive operations.*

  Second graders, as mentioned before, solved most tasks by counting. If the children interpreted the problems as additive ones which they then solved by counting, we could not decide on the basis of the videos. It is also possible, that the children did not even think of additive structures, but modelled the situation as a counting one. This hypothesis is stressed by the fact, that none of the counting children even expressed an additive task like „six plus six", whereas in other investigations we could observe, that children tend to express their elaborated knowledge. So the second graders in an investigation by KOUBA (1989) wanted to express verbally the addition and subtraction tasks they see in it, even if they have to go back on older and deeper strategies like counting to solve these tasks.
- *Spontaneous and informal strategies to solve multiplicative problems are not based on a deep and improved additive concept, but form a specific extension of the counting concept.*

  Those second graders, which did not count, but used elaborated strategies, did not add as well, but used the first numbers of the number patterns, before they changed to rhythmic counting. These spontaneous attempts to build number patterns could indicate, that multiplicative understanding in the beginning is not linked to additive concepts but to interiorised rhythmic counting processes (e.g. ANGHILERI 1989). The counting concept is extended by chunking: the countable unit is not any more restricted to one. The child chunks several elements to units of three, five or ten. These units can also be counted like ones. STEFFE & COBB (1984) named this chunking "interative schema with iterable units", which they see as fundamental for multiplicative tasks: "We cannot stress too strongly that iterative schemes are 'natural' in that they are elaborations of children's own counting schemes." (p. 22).
- *The solving strategy is determined by the possibilities of actions which can be undertaken with the material presented.*

  Situated approaches emphasise that the strategies which are used to solve problems in a situational context are influenced or determined by the possiblities of actions in the specific situation (NUNES, SCHLIEMANN & CARRAHER 1993). Through the action of putting packs or bottles onto another table the children could on the one hand reduce the complexity of the task, on the other hand they could approximate and in the end realise the goal of determining the number of packs or bottles. Counting strategies and number patterns as well are more practicable in accompanying this action, because they could be paralleled to the action.

6 Consequences

Before drawing any consequences, it must be stated that further investigations are necessary to answer the questions about the development of multiplicative reasoning. If the results presented here will be confirmed and the interpretations will be agreed to, then the following consequences may be drawn:

- If the actions with the materials in a situational context lead to (undesirable) counting concepts, teachers may avoid or disapprove real-world contexts in their lessons. At least they must know precisely the situated circumstances and the provoked concepts, if they want to work with real-world contexts in their mathematics lessons.
- If multiplicative reasoning is based upon an improved and flexible additive understanding, children should be given more time for learning these basics. The introduction into multiplication and division ought to be postponed, maybe into third grade.
- If multiplicative understanding is based on an intuitive extension of the counting scheme which is combined later on with additive concepts, then children should work with additive situations as well as with multiplicative ones from the beginning of schooling. The relations and connections between both concepts ought to be picked up later in detail.

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