Ralph Schwarzkopf

ARGUMENTATION PROCESSES IN MATHEMATICS CLASSROOMS

FUNCTIONAL ARGUMENTATION ANALYSIS: A METHOD TO DESCRIBE ORALLY DEVELOPED ARGUMENTS

Abstract

This article focuses on argumentation processes that have been observed in mathematics classroom interaction of fourth and fifth graders. Argumentation processes are understood as a special kind of interaction process between the teacher and the pupils, in which the participants explicitly request and give reasons for expressed statements. In the underlying research project, video transcripts of interaction processes like this have been analysed following the interpretative paradigm.

In this paper some results of this research project will be presented. First, a short description will be given of how argumentation processes start in mathematics classroom interaction. The main focus of this paper is on the functional analysis of arguments; i.e. a method to analyse orally produced reasons regarding their functions of argument. This method will be presented and used in this paper to interpret an argumentation that has been observed in a fifth grade class. Finally, some results of the presented interpretation will be discussed.

Introduction

In the research project ”argumentation processes in mathematics classrooms” (Schwarzkopf 2000) the focus is on a special type of interaction processes between teacher and pupils (namely ”argumentation processes”). This particular interaction type is identified by the following characteristics:

1) a reason for an expressed statement is explicitly requested,
2) reasons are given in response.

Argumentation processes are analysed in a descriptive way regarding their social regularities (argumentation) and the orally produced mathematical reasons (arguments); in other words ”argumentation” is understood as a social process in which special contentrelated structures, ”arguments”, are developed.

The theoretical and methodological framework employed in the study comprises on the one hand, theories of symbolic interactionism and microethnography as modified for qualitative research in mathematics education by Bauersfeld, Krummheuer and Voigt (e.g. Bauersfeld / Krummheuer / Voigt 1988, Krummheuer / Voigt 1991, Voigt 1984). On the other hand, theories of argumentation which are developed in pragmalinguistic (e.g. Klein 1980) and sciencephilosophy (Toulmin 1969) are consulted and specifically modified for the analysis of mathematics classroom interaction.

The data collection was conducted in four fourth grade (final year of primary school) and four fifth grade (first year of grammar school) mathematics classes. The teachers were informed about the goals of the research project but the researcher was neither involved in the preparation nor in the instruction of the observed lessons. Five mathematics lessons were videotaped in every class. Subsequently, those episodes, in which argumentation processes as described above were identified, have been transcribed and analysed.

The following research questions guided the analysis of argumentation processes identified in the classroom interaction:

1. How does a need for argumentation processes develop in mathematics classroom interaction between teacher and students?
2. Which kind of functional structure of arguments can be reconstructed in the orally produced reasons?
3. Which potential influences of argumentations on the mathematical teaching–learning processes can be reconstructed?

In the first section of this paper the development of a need for argumentation processes is briefly discussed using pragmalinguistical theories, which are then related to respective results from this study with respect to mathematics classroom interaction.

The second section is concerned with the functional analysis of arguments. A scheme, which differentiates arguments into several functional components is presented. This scheme was developed by the science philosopher Toulmin in order to analyse ”everyday arguments” regarding their functional components. Toulmin's scheme was used in the study reported here to analyse arguments that are produced in mathematics classroom interaction.

In the third section, an argumentation between the teacher and pupils of a fifth grade class is interpreted on the basis of a functional analysis of arguments. The paper concludes discussing some potential influences of the presented argumentation on the mathematical teaching–learning process.

1. How does a need for argumentation processes develop in mathematics classroom interaction?

In pragmalinguistical theories the researchers assume that an argumentation process is a special kind of communication process in which the members of a social group try to solve a problem (in a more or less cooperative way) in a rational way. This process starts in an ”everyday interaction” of a social group, when the group is confronted with a ”quaestio” – that means a question for that none of the members has an answer that would be accepted by the group. The participants try together to find an answer for the quaestio and make sure, that the whole group will accept this answer due to rational reasons (e.g. Klein 1980). Weingarten and Pansegrau (1993) investigated communications in the institution school. From their point of view, processes of reasoning between teacher and pupils do not start due to a confrontation with a quaestio, but *are initiated* by the teacher: It is him who decides due to his teaching goals whether a statement has to be reasoned and whether the reasons may be accepted or not. Hence, Weingarten and Pansegrau assume that members of classroom interaction only ”simulate” an argumentation on the surface of the language - processes of reasoning in school only have some keywords (e.g. “why”, “I would like to know”) in common with “everyday argumentation” as they are understood in pragmalinguistical theories. Hence, these authors understand argumentation in school only as a language style of instruction and term this kind of process ”as-if-argumentation” (”Als-ob-Argumentation”).

In the observed classroom interaction of the presented research project it was always the teacher who asked for reasons at the beginning of an argumentation (understood as cleared in the introduction). Often there was no sign for an uncertainty about the correctness of a statement for which reasons were called in and produced. In many cases, the teacher explicitly made sure of the correctness of a statement, *before* he asked the pupils to reason it and the argumentation began. But even when the teacher did not do this at the beginning of an argumentation, there was no development of a ”quaestio” in the meaning of pragmalinguistical theories observed: One can say, that the pupils knew that at least the teacher knew whether the statement is correct or not. It seemed to be clear that the teacher’s ”judgement” would be accepted by the whole group.

Hence, in this research project the start of an argumentation process is also understood as an ”initiation”. But in contrast to Weingarten and Pansegrau, this initiation of an argumentation process is not understood as a result of only the teacher’s action, but as an interaction process between teacher and pupils which may be described as follows: The pupils produce a statement in the classroom interaction. If the teacher thinks, that there is something unclear about this statement, he may ask the pupils to reason it and try by this to solve a potentially existing problem in understanding. The pupils then have to answer the questions of the teacher and show by this, which kind of reason they would accept and how they understand the statement. The teacher than must try to understand the pupils’ reasons and find out by this, if there are really problems in understanding and how these problems may be solved. Speaking with theories of symbolic interactionism, this ”to and fro” between the participants bases on interactional ”obligations” between routines (s. Voigt 1994, p. 286 or Voigt 1984, p. 55ff.).

2. Functional Components of Arguments

The principal interest of this research project was to find a way to analyse and describe reasons that are orally developed in mathematics classroom interaction. Therefore, a theoretical work of the science philosopher S. Toulmin (1969) was consulted. He was interested in the *microstructure of arguments*, in other words: Toulmin theoretically analysed arguments with that one can reason an assertion with the help of an undoubted predicate. Therefore, he developed a scheme that distinguishes an argument into five functional components. The following figure shows this scheme. It will be theoretically described in this paragraph. Afterwards it will be used to interpret an episode, which was observed in a fifth grade class.

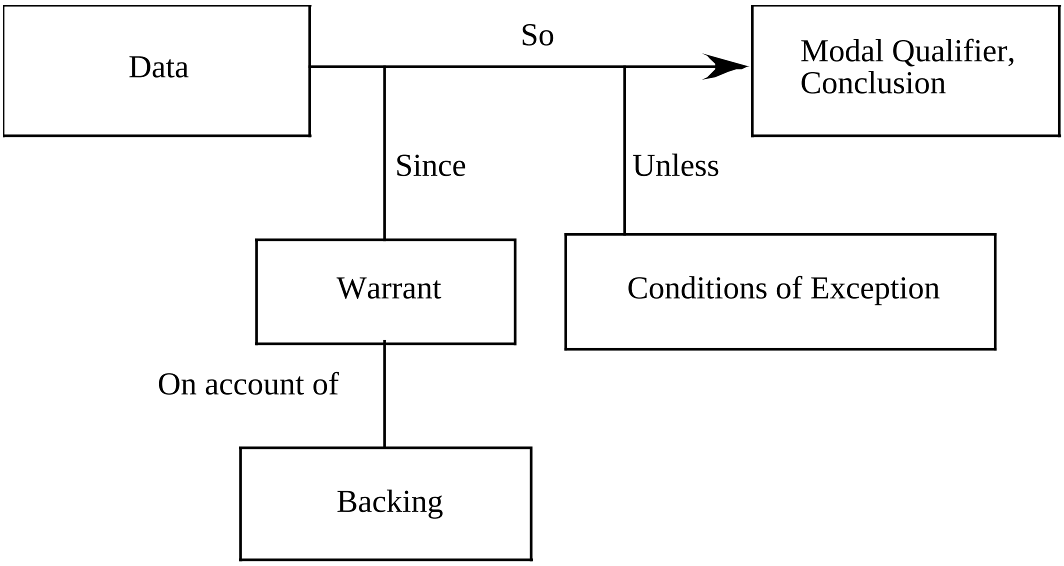


Fig. 1: Functional Components of an Argument (Toulmin 1969)

In the view of Toulmin, an argument is produced to justify an assertion. The function of this assertion in the argument is called ”conclusion”. To justify the conclusion, we first have to offer facts, i.e. predicates, that are undoubted by the participants of the argumentation. With an undoubted predicate, whose function is called ”data”, we may try to support the acceptance of the conclusion. To show how we can do this, we need a further functional component, which builds an ”argumentative bridge” between data and conclusion: This one is called ”warrant”. Hence, warrants do not justify any predicate, but they show the relevance of the data to support the acceptance of the conclusion. Warrants are general rules, which may be understood in a mathematical way of thinking as predicate forms. They can be formulated in detail like ” Data such as D entitle one to draw conclusions, or make claims, such as C (...)” (Toulmin 1969, p.98). Whereas warrants often become explicit in mathematical argumentation, they normally stay implicit in the practice of ”everyday” argumentation.

Additionally, an argument may be doubted regarding its warrant in two different ways: First one can doubt that the rule in *general should be accepted as having authority* (s. Toulmin 1969, p. 103). Second one can doubt the *applicability of the rule on the data* (even if the rule in general might be accepted). So, to complete the argument, one has to consider two more functional components, which are described as follows.

If the warrant is doubted *regarding its general authority*, one has to indicate further undoubted predicates to support the rule in general. This function of the argument is called ”backing” and binds the argument to the context of the argumentation. From Toulmin's point of view, the possibility to support a warrant with a special backing is depending on the *organisational* context of the argumentation. For example, the backings that may be accepted in the context of a beer party are different from backings that would be used arguing in a law court. Toulmin calls this context the ”field” of the argumentation and assumes that backings are ”field-depending”.

Krummheuer (1995, p. 249ff[1]) modified the term ”field” for research basing on theories of symbolic interactionism. The participants of an interaction judge about sense and nonsense within their ”frames”, i.e. their individual understanding of a situation. Hence, it is in particular dependent on the frame of an individual, whether he accepts the general validity of a given rule. This means that the acceptance of a warrant in a group of arguing people is depending on the frames of the participating arguers: Backings are *frame depending*. Frames and especially frame-conflicts of several participants in classroom interaction are very important in interactional theories of teaching and learning, because from this point of view, learning is understood as a ”frame-modulation” (s. Krummheuer 1992): The pupils learn by modulating their frames near to their teacher's frame. Two consequences regarding the frame-dependency of backings are important for the interpretation presented in the following paragraph:

First, the frames of the arguers provide an “argumentation base”, i.e. possibilities to support warrants by special backings. Regarding the interaction in mathematics classroom this means, that the pupils have to frame a situation so that they can produce reasons, which the teacher would like to hear. And, vice versa, the teacher must try to find out in which way the pupils are framing the situation to understand their reasons in an adequate way. In the presented investigation of fifth grade classes, it is often observed that the pupils have to modulate calculation-bounded frames into more algebraic orientated frames to satisfy the teacher's expectation – they especially learn to use more abstract argumentationbases.

[1] Krummheuer (e.g. 1995, 1997b) uses the functional analysis of argument (and especially a part of the Toulmin-scheme) in a different way than it is done in the presented investigation. For the differences of use see Schwarzkopf 2000, p. 217ff.

Second, by producing backings to support a warrant, the participants explicitly offer hints to their underlying frames. This is as much essential for the researcher as for the teacher: If we want to teach the pupils to see problems from a mathematical point of view *we* have to understand how *they* understand the problem. When backings are produced and therewith hints on frames are given, argumentations are a good chance to understand the pupils' points of view. In the opposite case they are a chance for the children to understand, how they should see mathematical problems. The teacher might support this by somehow asking for backings of arguments.

The last function of the argument has to be filled with predicates, if someone doubts *the applicability of the rule on the special used data*. Even if we accept the authority of a rule in general, so Toulmin, we can still ask whether the rule might be appropriate to fulfil the function of a warrant in the special argument (with special data and conclusion). In the practice of rational argumentation we can (nearly) never conclude free of doubts on a conclusion, but we have to insert "modal qualifiers" (such as "probably is" or "presumably is") to the argument. With these qualifiers we indicate the *"degree of force which our data confer on our claim in virtue of our warrant"* (Toulmin 1969, p. 101). To make this degree of force more concrete we have to fill the last function of argument, the "conditions of exception", with predicates. From Toulmin's point of view one can say that it is typical for mathematics argumentation to make arguments as strong as possible, i.e. to use only modal qualifiers like "with certainty". This aspect will be discussed in the last section of this article.

3. Functional Analysis of Arguments: An Example from a fifth grade Class

In this episode teacher and pupils are talking about the homework. The children had to find out the solution-sets of the following equations that are written on the blackboard:

a) (1) $8 \cdot x = 56$ (2) $6 \cdot z = 63$ (3) $x + 13 = 18$
e) (1) $5 \cdot x + 7 = 22$
g) (1) $3 \cdot x + 4 = 13 - x$

In the context of this lesson the pupils learned some new technical algebraic terms such as "predicate" and "predicate form". Maybe the teacher Mr. Moeller[2] wants to find out, whether the children have understood the difference between these two terms, when he asks the following question:

1	T	Who votes for that all of these tasks are predicate forms? (*children point up their fingers*) good, so as I can see all of you agree,
2	T	but what (*simultaneously spoken with lines 3 & 4*)
3	S	but nevertheless
4	E	no I don't

While most of the pupils seem to agree that all of the tasks are predicate forms, Erik points out, that he does not. The teacher brings this protest up to discussion:

5	T	so, please tell me Erik, why do you believe that there is one task on the blackboard, which is <u>no</u> predicate form.

[2] All names in this transcription are aliases.

Mr. Moeller asks Erik to reason that he believes one task not to be a predicate form. So, Erik made a statement and the teacher asks for a reason – because afterwards there are some reasons produced, an argumentation (in the terminology of Schwarzkopf 2000) begins. Because some of the pupils do not agree with Erik's statement (they vote for the assertion that any of the tasks is a predicate form) one now could say that the following argumentation is not initiated, but it starts due to the confrontation with a "quaestio". But even though there are several opinions shown regarding the correctness of Erik's statement, no quaestio in the sense of pragmalinguistical theories is produced: No one in the class would doubt that the teacher knows whether every task is a predicate form or not. So he could solve this problem by simply judging Erik's statement. Hence, the argumentation is not necessary to solve this problem. It begins, because the teacher decides in this situation to ask for a reason and because the students afterwards do produce reasons. From this point of view one can say, that especially Erik (who produces the first argument in the following) and the teacher together initiate an argumentation.

In the lines 6 to 11 the teacher and Erik concretise the statement that should be reasoned. Erik points out, that his protest is about the equation g(1) and that he will argue for the following conclusion:

The equation ”$3\cdot x + 4 = 13 - x$” is not a predicate form.

The student offers the first reason in line 12:

12 E because afterwards one more <u>after</u> the equal there is one more calculation

In Erik's reason it seems to be important to differentiate between the places "before and behind the equal-sign". From an algebraic point of view, this differentiation does not make sense regarding the definition of predicate forms. But it becomes meaningful from the point of view of another possible frame that might be influenced by the title of the homework ”*find the solution-set*”. For some students at home, the equations had probably the meaning of calculation-tasks and the ”equal-sign” may have been understood as ”result-sign” (the calculation on the left side, the solution on the right side. Erik's kind of reasoning, together with the thoughts about the context of the episode, are indications for that he frames the situation as a ”mechanical calculator” (”algorithmisch-mechanische Rahmung”, s. Krummheuer 1983). In this frame, equations are understood as tasks, for which one has to find a solution by calculating (often by the use of known algorithms). Nobody who frames this situation like a ”mechanical calculator” would doubt Erik's predicate (remember that everybody can see this equation on the blackboard): There is in fact another calculation on the right side. Erik uses the speech-marker ”because” and therewith he binds the produced (indisputable) predicate to the already formulated conclusion:

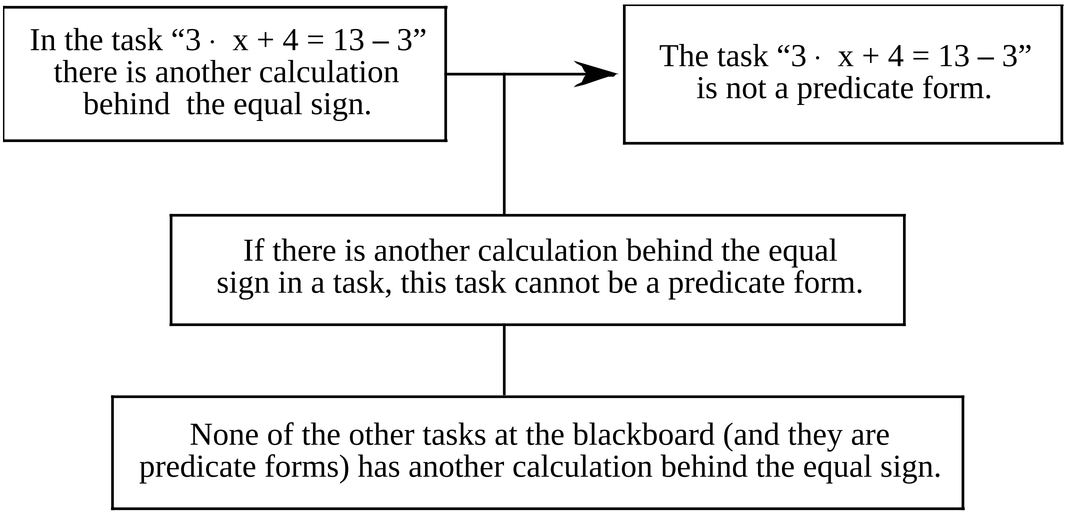
> ”g(1) is no predicate form, because there is one more calculation behind the equal-sign”.

Hence, one can say in the sense of Toulmin that the student offers a data for the conclusion above. Additionally one can say, that he binds this data with a warrant to the conclusion – this warrant may to be so clear to him, that he does not need to mention it explicitly (from Toulmin's point of view, this would be typical for ”everyday argumentations”):
”If there is another calculation behind the equal-sign in a(ny) task, this task cannot be a predicate form.”

One now could ask, why Erik thinks that this may be a sufficient reason to come to the conclusion he argues for. The student does not explicitly give us any more information about this - he does not explicitly produce a backing. But regarding his probable underlying frame one can ”complete” the argument as follows: The equation g(1) is the only one of those on the blackboard, which contains the variable ”x” two times. None of the other tasks is doubted to be a predicate form. By offering the data, maybe Erik wants to show this difference between the predicate forms (that means all of the other equations on the blackboard) and the equation g(1):
The equation g(1) is different from the other equations on the blackboard (and they *are* predicate forms), so g(1) cannot be a predicate form.

In this interpretation one can say, that Erik’s backing consists of some experiences with tasks that were recently called predicate forms. Tasks that are different form those may then be no predicate forms.

Based on these thoughts one can formulate Erik’s argument as following:



The teacher reacts by reformulating Erik’s Argument:

13 T so Erik says because here (*the teacher points at ”13-x”*) stands another 13 minus x that would be no predicate form

The teacher uses – contrary to Erik – no calculation-specific language. He replaces ”another calculation” by ”13 minus x” and stresses the term ”predicate form”. By this he modifies the mechanical looking expression of Erik into the direction of an algebraic like language. Thereby those pupils, who are framing the situation in a mechanical-calculator’s way, get the chance to modulate their frames to more algebraic ones. For them it may become clearer what this argumentation should be about (regarding the intentions of the teacher) – the focus should be on an algebraic-like definition of a predicate form. Therefore, the pupils shall not interpret the equations as concrete calculation-tasks but they shall frame the situation in a more algebraic way and understand the equations as abstract terms. Probably, the teacher wants the children to disprove Erik’s argument within this frame by using the definition of predicate forms.

One can say, that in an algebraic frame, Erik’s argument would not be accepted because the kind of backing would not be accepted. While his backing refers to experiences with equations that are called predicate forms in the past, in an algebraic frame one should refer to grounded, clearly formulated definitions. (Additionally, in this frame one would not really understand the data of Erik's argument.)

In the lines 14 to 23 the teacher goes on to help the pupils modulate their frames. In a teacher-guided dialogue the term ”predicate form” is being specified in the following way:
Predicate forms differ from predicates because there are variables in the predicate forms.
One can say, that in this passage the teacher structures the argumentation: He stresses the difference between “predicates” and “predicate forms” as they are defined in previous lessons and marks thereby implicitly aspects of calculation as unimportant for this argumentation. By this, the pupils may be oriented to frame the situation in a more algebraic way, i.e. to use the algebraic definitions, as they were formulated in the mathematics classroom, especially for the backings of their arguments.

In this term-clearing passage there is no argue produced and so the interpretation goes on at line 24, when the teacher ends this passage by reformulating the problem:

24 T Good. Now the question again. Erik says that the task g is no predicate form. Is Erik right with this or isn't he .. Katrin what do you think.
25 K I believe that he isn't.
26 T Why do you believe this?

Katrin points out that Erik was not right. The teacher asks her for a reason, i.e. for developing an argument different to the one of Erik.

27 K well
28 E Believing is something for religion.
29 T With this you are not so wrong Erik. In this point you are not so wrong. But with the predicate forms it looks different. So Katrin has the feeling that you are not right but the question is why not. Why is Erik not right please?

In this remark the teacher makes explicitly clear to the class that Erik's argument is incorrect and asks the pupils to disprove his conclusion. The teacher still structures the argumentation regarding its content by already giving the following conclusion for the next argument to be developed:

> ”The equation g(1) is a predicate form.”

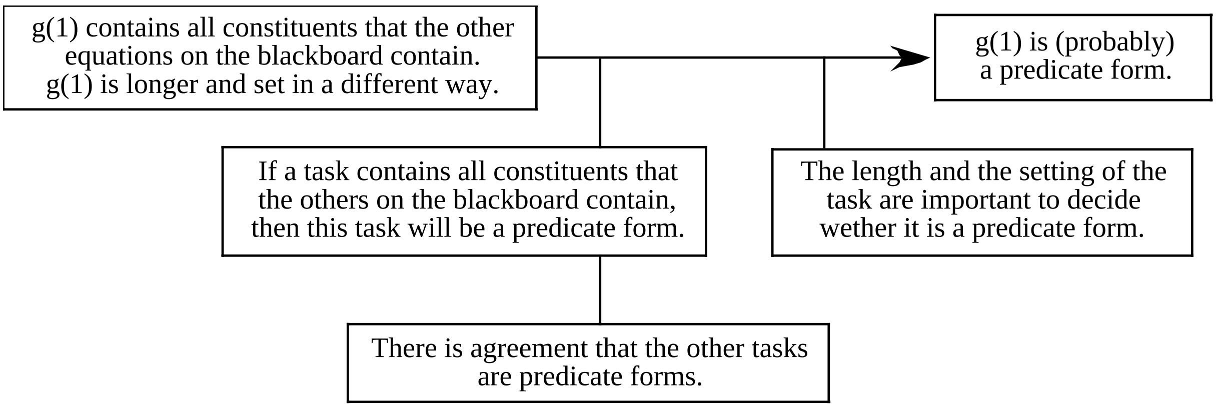
In the following, the students can be sure to argue for the ”right conclusion” (in contrast to ”everyday argumentation” from the point of pragmalinguistical theories (e.g. Klein (1980)):

30 S Because there is everything inside what is inside even in the other tasks too. There is actually no difference except that this one is a little bit longer and that it is, maybe, that it is set in a different way.

This student points out that the equation g(1) contains everything that ”the other” tasks do too. Probably he means by ”the other” tasks the other equations standing on the blackboard (they are not doubted to be predication forms). He does not explicitly use points of a definition as the teacher stressed them in the previous passage (”there are variables inside”). The child rather compares the equation g(1) with the other equations standing on the blackboard that are accepted to be predicate forms. Hence, the algebraic oriented backing, the teacher probably wishes to be used, is not mentioned. The student's formulation is more similar to the backing of Erik's argument: Both pupils refer to some kind of experience with what is called ”predicate form”. One can say, that he implicitly uses a warrant that may be formulated as following: If a task contains all constituents that the other equations standing on the blackboard do, then this task is a predicate form.

But additionally, the pupil mentions two differences between g(1) and the other equations standing on the blackboard: The task g(1) fulfils all conditions that the others fulfil, *but* its length and its kind of setting differs from the other tasks (which may be important).

On the one hand one can say, that the student mentions these aspects to differentiate between the important constituents and the unimportant form of equations regarding the question whether they are predicate forms. But on the other hand one can also say, that mentioning these differences between g(1) and the other tasks means to regard the possibility, that the form of the task may be important after all. In the view of this second interpretation, the argument seems to be weakened by these differences. An indication for this interpretation is that the student uses the words ”there is *actually* no difference”. In the sense of Toulmin these are signs for that it is necessary to test a condition of exceptions for the warrant: Maybe in this special case it is not only important to compare the constituents of the tasks, but also their forms. Following the second interpretation, one can split this argument into the following components:



The teacher reacts in an approving way:

31 T Yes, this is the only difference isn't it? So we have the possibility to choose numbers for the variables, and this possibility looks admittedly a bit different than it did before but it works. So, Erik, because of this it is very clearly a predicate form too.

Mr. Moeller shows agreement with the argument produced by the student. On the first view he ”only” seems to reformulate it. But from the view of functional analysis of arguments one can say that he offers another argument or at minimum that he refutes the relevance of the previous argument's (as interpreted before) condition of exceptions. This is indicated by to aspects: First, he again stresses the algebraic-oriented frame that he wants the pupils to modulate towards: The decision, whether an equation is a predicate form or not, depends on nothing but on the occurrence of variables. The backing of the previous argument (as interpreted here) did not explicitly point to this definition, but it referred on some agreement about that the other equations shall be predicate forms. This means that Mr. Moeller produces an argument basing on a different backing than both of the previously developed arguments.

Second, he marks the strength of the reasoning (”because of this it is very clearly a predicate form too”) whereby the previous argument could be interpreted to be depending on some conditions of exceptions. Therefore the teacher reformulates the differences between g(1) and the other tasks that were important for the last argument (and, fulfilling a different functional component, for Erik’s argument too) to sign them as absolutely irrelevant to the argument.
So the teacher uses a kind of backing which is different to those of the previous arguments. Additionally he makes clear, that in the ”correct argument” there is no condition of exceptions to be checked. Hence one can say, that the previous argument is not only reformulated, but that it is modified by another argument that especially refutes the condition of exception above and may be described in the sense of Toulmin as follows:

g(1) contains variables. → g(1) is undoubted a predicate form.

If an equation contains variables, it is a predicate form.

Some kind of definition that was negotiated in the lines 22,23,

At this point, the argumentation ends.

4. About some potential influences of this argumentation on the mathematical teaching-learning process

This argumentation is initiated when Erik produces a (mathematically incorrect) statement. The following argumentation deals with the identification of predicate forms – a basic-term for an algebraic understanding of equationproblems.

In the interpretation of this argumentation it was assumed, that Erik’s statement is rational regarding his framing of the situation: He understands the equations in the view of a mechanical calculator and does not see them as algebraic terms. The teacher tries to make the pupils (especially Erik) modulate their calculation-based frames towards a more algebraic frame. So, one can say that in this episode some kind of framing-conflict is shown - the pupils do not only have to learn ”a new word”, but they must generally change their point of view regarding equations. This frame-difference is shown in the kinds of backings that are used by the pupils (some experiences) and by the teacher (definitions). One can generally say, that the two kinds of backings may be accepted in different mathematical activities: Arguments, produced in the context of solving calculation tasks, give answers to questions of how to solve a problem in a good way: ”What shall we do to solve the problem?” To argue for the quality of a solution way, referring to experiences (e.g. in clever calculating) – like it is done in Erik’s argument – are qualified.

When considering arguments in the context of proving the solution of an arithmetical problem to be correct one has to produce arguments that give e.g. answers to questions of correct term manipulating: ”Why is the solution correct?” In these discourses, backings have to refer explicitly to underlying algebraic definitions – like the teacher’s argument does – or already somehow ”proved” conclusions.

”Backingconflicts” like these were often observed in fifth grade classes in the presented investigation: Students tried to explain by referring to experiences in calculation, that they solved a task in a good way. The teacher contrarily wanted to hear arguments for the correctness of term manipulations by mentioning algebraic laws. By negotiating which kind of backing might be successful, the pupils had the chance to modulate their frames into a more algebraical view. Hence, one can say that these argumentations (especially the one presented above) potentially influence the modulation of the often calculation-bound frames of the pupils into a more abstract, algebraic frame.

Regarding a formal aspect of argumentation, one more result of the interpretation may become interesting: The second argument, which was reconstructed as concerning a condition of exceptions, was replaced by the teacher’s argument, which proved the conclusion “without any doubt”.

In the sense of Toulmin, this is a formal aspect of mathematical argumentation: The author assumes, that the only kind of arguments that may use the modal qualifier ”without any doubt” are mathematical arguments. From this point of view one can say that another potential influence of the presented argumentation is to learn how to formally argue mathematically, namely with warrants, that do not need to check any condition of exceptions at all.

In contrast to Toulmin one can say, that conditions of exceptions may be produced also in mathematics argumentations, for example when a good way of solving a problem is discussed. ”Only” regarding to the correctness for example of term manipulating, do mathematicians try hard to make conclusions with the highest possible certainty and without condition of exceptions. So, the difference between arguments with ”conditions of exceptions” and arguments with the modal operator ”without any doubt” is not the difference between ”everyday-arguments” and ”mathematical arguments”, but both kinds of arguments are used in mathematics to argue for different goals.

Of course it is not clear whether the pupils become conscious of these substantial and formal aspects of the produced mathematical arguments. It is additionally in question, whether the teacher realises any difference between his argument and the one pointed out before. Hence, the described influences on the teaching-learning process could only be reconstructed as a potential of the argumentation. Their realisation might have become more probable if the arguments had been discussed in more detail. Maybe a higher emphasis on analysis of argumentation-processes in mathematics classroom interaction in teacher education could help to make argumentation more transparent for teaching-persons and, thereby, help to make argumentation processes in mathematics classroom interactions more effective.

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Ralph Schwarzkopf
Fachber. Mathematik
Univ. Dortmund
44221 Dortmund