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TWO CULTURES IN THE EDUCATIONAL SYSTEM — AND HOW MATHEMATICS CAN HELP PROSPECTIVE (PRIMARY SCHOOL) TEACHERS TO OVERCOME THEIR DISPARITY

Abstract:

In the German educational system one can identify two cultures, one which is oriented towards subject matter (**osm**), and one which is dissociated from subject matter (**dsm**). In this paper some cases, some characteristics and some examples of the **dsm** culture in general and in the domain of (primary school) mathematics education in particular are described. The potential of mathematics education and school mathematics to overcome the disparity between the two cultures is analysed.
The paper is based on lectures given by the author in 1999 at the Universities of Frankfurt, Paderborn, Kassel and Berlin (Humboldt) and at the annual meeting of the GDM 2000 at Potsdam. Thanks to John Searl in Edinburgh for his valuable comments and his help with the translation.

Introduction

In the spring of 1999, several persons who were involved in primary school teacher education at the university of Paderborn started a series of meetings in order to get to know each other better, to exchange ideas and to explore possibilities for co-operation. As a basis for the discussions, two members of the Faculty of Pedagogy opened the first meeting with an overview about those topics which, in their opinion, are fundamental for the education of primary school teachers. They talked a lot about how to take care of mentally or physically disabled children, how to deal with minorities, or how to cope with some students' abnormal behaviour. They did not mention subject matter oriented categories like goals, didactics or teaching methods related to specific topics. The following discussion focused on classroom communication, individualized learning, constructivistic principles etc., again without referring to any concrete subject matter. There were only a few people taking part in this discussion, because it did not meet the needs of the majority of those participants who are more subject oriented in the framework of primary school teacher education and concentrate on mathematics, German, music etc. teaching.

This situation is symptomatic of the existence of two cultures in the German educational system, in particular in the school system, especially in primary school. One which is oriented towards subject matter (**osm**), and one which is dissociated from subject matter (**dsm**). These two cultures seem to be opposed to each other within the field of education like the two cultures in western societies (science oriented vs. literature oriented), as they were described by Snow (1959). Perhaps the disparity between the **osm** and the **dsm** cultures is particularly pronounced in the German educational system but it can also be observed in other western countries. In contrast, in the second and in the third world, the **dsm** culture is more or less undeveloped and there is no such disparity.

1. Some characteristics of the dsm culture in the educational system

1.1 Manifestations

Most people all over the world, based on their own experience, consider school to be a place for the acquisition of knowledge, rote learning, sitting still etc., and, what is more, they think that school has to be like that. Ever since something like the teaching of subjects was introduced, however, there have been attempts to extend this specific kind of instruction to some general education covering the development of competencies, attitudes etc. In German pedagogy and didactics we have a long tradition of intellectual formation within the teaching of subject matter, more or less successfully. In my opinion one of the pinnacles of this pedagogical orientation was Wolfgang Klafki's famous "Didactical analysis of the subject matter as core of the preparation of lessons" (1958).

There have also always been efforts to put general qualifications, detached from specific subject matter, into the centre of the work at school. For a long time there has been a balance between these two pedagogical currents, i.e. attaching general educational goals to or detaching them from the teaching of subject matter, which I would classify (in my notion from the introduction) as part of the **osm** culture and the **dsm** culture respectively. As far as I can see, in the last decades the **dsm** culture has appropriated large parts of the educational system in Germany. This has occurred, at least in the theory-oriented teacher education at the universities and in the practice-oriented teacher education at the training colleges, more in the primary and less in the secondary sectors. It is doubtful whether the influence of the **dsm** culture is as important in actual school teaching as it is in teacher education, but it might increase.

In the early 1990s the minister president of the state of Nordrhein-Westfalen (NW, one of the 16 states of Germany, comprising about one fifth of the population) established a commission of educationalists and others, among them Klafki, to reflect on "the future of education and the school of the future". In 1995 this commission presented the outcome of their work in the form of a memorandum (NW 1995), which can be looked upon as 'the compendium' of the **dsm** culture in the German educational system. Since then, the ministry of education of NW has been trying to turn some of these ideas into legal provisions. The new teacher education law provides that prospective teachers must acquire so-called key qualifications and competencies. This does not sound bad, but students are to be assessed formally in these competencies (like the ability to work in a team, for example). When I asked the ministry how to test this, a representative told me to take two or more students at a time in the oral mathematics examination (as, of course, the universities are still structured along the disciplines) and to rate their co-operation in this situation.

There are voices who object to the expansion of the **dsm** culture with its alleged general disregard of high performance and with other negative consequences (from their point of view) in the German educational system (Maier 1996, Kraus 1998, Seifert 2000 and others). However in the main pedagogical discourse they are treated as outsiders and are largely ignored. There are also a few established educationalists opposing the pedagogical mainstream, among them Hermann Giesecke (1997) who attacked the above mentioned memorandum in particular. He does not only criticize the unsubstantiated importance which the memorandum attributes to the concept of key qualifications for the solution of so-called key problems. As a political scientist he also questions the promotion of social and political problems like "social inequality, ..., majorities and minorities, the relation between the genders, 'developed countries' and 'developing countries', Germans and foreigners in Germany, ..., the role of the trade unions, ..." (NW 1995, 113) to so-called key problems which the memorandum recommends be taught in school as canonical topics from the beginning. Giesecke does not object to their discussion in school, rather the contrary, but he points out that the promotion of a problem to a key problem as well as the judging of a problem are a matter of the individual's beliefs and cannot be part of the syllabus.

Another facet of the **dsm** culture is the over-accentuation of the social aspects of the work in the classroom together with an associated reduction of the teacher's role. In my opinion the willingness and ability of real students to work on their own initiative, independently and without assistance is over-estimated. What is more, even the most autonomous project is still an offer by the school, imposed (possibly very cautiously) on the students by the staff and connected with certain aims which do not come from the students. This is as well known by first graders as by high school leavers.

On the basis of his wide research in the field, Franz E. Weinert (1997) states: "Whereas in many new 'theories of learning' some accent is laid on intrinsic motivation and the learner's active, constructive and independent role and the teacher is ascribed the function of a stimulator, advisor and moderator, all classroom studies show the importance of 'effective' instruction controlled by the teacher and oriented towards tasks. (...) Good teachers do not make their students passive consumers of their teaching efforts, but ensure that all learners tackle the learning tasks actively and in a way which is appropriate to them." Of course, one can question what the notion of good teaching ("good teachers") means. If someone understands good teaching as doing without direction, then, of course, Weinert's statement is empty.

1.2 One cause: The social change in the last decades

At first, the contemporary importance of the **dsm** culture is a result of the social changes which our country, as well as other western countries, underwent in the last thirty-five years.

Today, in many domains of society like jurisprudence, administration, politics, economy and in particular in the educational system, we find the 'system' supporting ramifications of a movement which originally proposed to overcome the political 'system'. Many young people had started their big jump as a revolutionary tiger and had landed as a seat cover on a chair in some ministry, sublimating their old ideas. In the educational system especially, our society has assumed a much more 'friendly' character. On the whole, resistance against the authoritarian traits which were, in former times, perceived in the structures of the subject matter, of institutions like schools and universities and of the personalities of their representatives, has become obsolete in the meantime.

1.3 Some delimitations

First remark: The didactical principle of teaching and learning through examples and leaving gaps is not part of the **dsm** culture, because in this principle it is precisely the content that matters.

Second remark: Traditional schools always wanted to develop students' key qualifications (and in the last decades practically all of those which are under discussion today) and to educate the students to be responsible, critical etc. citizens. Didactics and methodologies of teaching always provided the teacher with teaching ideas and with classroom arrangements which differed very much from a teacher's solo performance, but always along a flexible canon of subject matter topics.

Third remark: Qualifications, like the ability to work in a team, and classroom arrangements, like working in teams, are usually pictured without discussion as advantageous. But they, too, have to be questioned.

For example, outside the educational system the notion of 'ability to work in a team' (as a key qualification with respect to the students' later vocational life) means something quite different from those pleasing ideas which the members of the system use to connect with this notion. In business, trade and industry there exist decidedly hierarchical structures as basic requirements for clearly delimitating domains of responsibility from each other. (In the former socialist countries it was just the same, by the way.) To be able to work in a team means, in vocational life, initially to be able to fit in well in the team and to work under its leadership.

Teamwork can cause obstructions to the thinking processes and can impair mental concentration in particular, if there is a lack of the necessary abilities, knowledge and skills or if these talents are distributed unevenly. The better performing students may feel bothered if they continuously have to 'help' the other group members, while the weaker ones may feel uncomfortable because they need aid. This assistance, again, is often given in a way to ensure that the task is fully accomplished and the weaker students have no chance to understand the ideas in question as they would have had if they could have worked at the solution on their own (Vollmer 1997).

1.4 Another cause: Over-estimation of media

It goes without saying that the contents get easily pushed into the background, if the didactical efforts are oriented too intensively towards the media. I think that, in recent years, too many investigations about new media and how they are used by students are pre-occupied with the kind of abilities and skills which are dissociated from subject matter (**dsm**), e.g., the handling of the software itself, the development of creative attitudes (whatever this means) or the acquisition of key qualifications. In the field of new media in mathematics education, the didactical aspect of the work with the computer is often reduced to programming the implementation of some content. In short: there is a great demand for studies which integrate questions of the existing curricula, possible changes in subject matter concepts, new cognitive approaches etc.

1.5 One more cause: Reliance on (philosophical) constructivism

The core of epistemological constructivism consists of the conviction that all human beings create their own cognitive reality, and most educationalists seem to agree upon its psycho-pedagogical version. Of course, the explanation or negation of phenomena like 'communication', 'shared knowledge' or 'culture' becomes more difficult, the more radical the core of constructivism is taken. On the other hand, a compromise like Paul Ernest's (e.g. 1994) draft of social constructivism makes the whole theory more or less trivial.

Currently however, I am only interested in possible didactical implications. Here we again find the motivation of an anti-authoritarian attitude (towards people and towards subject matter). On one side the 'teacher' is denied the ability to teach, while on the other the subject matter is denied the liability and the potential to be taught. Of course, the demand that the adult teacher (who is trained and paid for the job) should draw back from what the non-constructivist would call a teaching-learning process, and that the students' own ways of thinking have to be stimulated, is, like every didactical principle, justified, at least if it is meant conservatively. However if it is meant radically, it contains at least one crucial error. If one denies any direct influence of the 'teacher' on other human beings' cognitive constructions of reality and one still arranges something called teaching (organization and moderation of learning processes), then one has to prepare and carry through these activities even more carefully and must not leave the classroom situation to its own devices.

1.6 One outcome: TIMSS

The performance of the average German student in the third international mathematics and science study (TIMSS) II (about 14 year old students) and III (about 17 year old students) (Baumert & Lehmann 1997, Baumert, Bos & Lehmann 2000) was rather disappointing, even more so if one takes into account the expenditure of money on the German educational system in comparison with many poor countries. There has been some criticism of this study based on alleged shortcomings in the statistical methodology as well as on the underlying concept of mathematics education in general and of many problems in detail. This criticism is in parts justified.

It must be stressed that the results of TIMSS neither give rise to rankings in (mathematical) intelligence nor to rankings in the quality of (mathematics) teaching. In my opinion they only reflect, at least in the wealthier countries, the appreciation of mathematical education in these societies and, closely connected with that, the amount of time and energy students spend on their school work in general and on mathematics in particular.

This, again, provides no evidence for the supposition that students performing well in TIMSS use mathematics in a sensible manner or have available adequate basic ways of imagining and understanding (cf. Bender 1998). What is more, the early planning and dogged pursuit of school and vocational careers to which, e.g., Japanese young people are subjected (according to our alleged knowledge about the Japanese society and educational system), is unsound and should not be an example for our educational system.

School must not only demand effort but students must also be granted the opportunity for reflection, on a small as well as on a large scale. This means for instance that in our rapidly changing society, school has to be one constant factor and should not indulge every five years in the revision of paradigms to which computer science, media science or biological science are subject.

2. The dsm culture in mathematics education

In the community of German mathematics educationalists, a remarkable trend to disregard the influence of the subject matter on teaching-learning processes in mathematics 'lessons' can also be observed, and colleagues who concentrate in their work on subject matter are slightly scorned and are sometimes called "Stoff-Didaktiker" (didactician of subject matter).

2.1 One cause: The shock concerning the failure of the 'New Math' movement

One specific cause of the **dsm** culture in mathematics education (beyond the general ones I presented in the first chapter) is the failure of the 'New Math' movement thirty years ago. This movement had been decidedly subject matter oriented in that certain traits of university mathematics were put on the school mathematics curricula, based on arguments from cognitive psychology and, by the way, fiercely resisted by university mathematicians.

2.2 Literal adoption of extraneous research methods

With the emergence of new efficient technical devices like computers or video players, new methodologies were developed for empirical research in many disciplines. When, finally, 'ordinary' mathematics educationalists obtained access to these devices, they too could do better empirical research. They often wanted to do it in a way that was acceptable to disciplines with longer traditions and higher reputations, and, consequently, they took into consideration the research methods which had already been established in those disciplines. Whereas the data processing power of computers made available elaborate statistical methods, video techniques allowed painstaking analyses of (short passages of) classroom interaction. Although deeply opposed to each other, these two research directions, at least when taken puristically, meet in their refusal of plausible explanations and of allowing any influence of the subject matter on their outcomes. Instead, they adhere to the behaviouristic creed that there is no scientific understanding independent of empirically gathered data. (Naturally the interactionist school would reject this association with behaviourism.)

2.2.1 Over-estimation of dsm statistics

Of course, any data gained from some empirical research can be processed by statistical methods but if conclusions are to be drawn by means of classical statistics, some fundamental prerequisites have to be fulfilled. The sample has to be representative of the population for which predictions are to be made; the variables in question as well as the students' responses (which can be viewed as another set of variables) must be independent from each other; and the data which are collected must be valid for the subject of the investigation. In particular validity is a matter of the conceptual framework and not of some empirical outcome, and therefore it is closely related to subject matter.

In classical disciplines like biology, medicine, psychology, economy, social sciences etc. it is comparatively easy to meet these prerequisites because, often, either rather small sample sizes suffice or data from larger samples can be collected without too much effort. In contrast to this, if one wants to do statistic-based research on problems which are closely connected with the teaching-learning, and other social and psychological processes, in the classroom, it is not enough to choose a few classes from one or two schools in a district where the researcher happens to live. While this may produce seemingly impressive results neither the representativity of the sample nor the independence of the variables is guaranteed.

Many research designs in the field of mathematics education (and in the educational system in general) follow that pattern however, and the authors often do not become aware of the absence of some or all of the mentioned prerequisites, or of the need to remedy this fault. Sometimes, at least, they feel uneasy and they avoid the methodological demands by defining their investigation as a pilot study, but then they often do not get beyond pilot studies.

Studies of that kind do not allow profound assertions about, e.g.,

— the spatial ability of primary school children,
— the classification of people into two groups according to their mental image when they carry through the arithmetic operation of division (to detach portions with a given size from the whole set and to count the portions; or, given the number of portions, to count how many elements each of them contains in the end),
— the denial of any transfer of some (positive) achievement from mathematics lessons to the individual's everyday life.

Compared to an experiment in medicine, in the field of mathematics teaching and learning the research objects are much more complex. Besides this, they are strongly determined by norms, subject matter plays an important role and many extraneous variables like the family, television, computer games, hobbies etc. must not be neglected.

2.2.2 Over-estimation of a dsm theory of social interaction

When the ideas of the sociological theory of interaction were introduced into the field of mathematics education in Germany, this surely had to be done in a rather puristic way in order to identify typical social structures of mathematics teaching like the "Erarbeitungsprozess-Muster", i.e. a typical pattern of the mathematical discourse which is directed and dominated by the teacher (Voigt 1984) which, as we know, can be found all over the world. But at some time this research paradigm had to be accommodated within 'the' mathematics educational mode, and in the meantime it actually has been accommodated.

Example: Reinhold, a prospective primary school teacher, wants to introduce the concept of area to ten year old students. He draws a square on the blackboard, points at each of the four sides, emphasizes that they all have the same length 1 cm, and then asks how large the area is. Of course, the students all think it to be 4 cm, because Reinhold's action evoked an unsuitable basic understanding of the situation, namely one connected with the notion of circumference. For the interactionist this is a problem of social interaction. To me it has always been questionable to depreciate the role of the teacher on constructivist grounds and at the same time to overemphasize the students' utterances in the "negotiations" in the classroom about the possible meanings of the subjects.

It is true that, all over the world, students often do not acquire mathematical ideas in a way which is intended by the teacher or by the written curriculum (including books) underlying the lessons. According to the theory of basic ways of imagining and understanding (cf. Bender 1998) this 'failure' is not due to a lack of the teacher's influence in the first instance, but in most cases to some inadequate framework evoked by the way of teaching. It may be that the teacher has no adequate framework (like Sandra and Valerie in the article by Gardner, 1998), or that the teaching is done in an inadequate way (with respect to individual students).

3. The possible contribution of mathematics and mathematics education to bridging the gap between the osm and the dsm cultures

It is undisputed that, in Germany, mathematics teaching starts in the first grade with basic arithmetic and has to go on at least until the seventh grade to cover the notions of percentage and interest (in fact, it goes on until the tenth grade). I suppose that in other western countries circumstances would be similar. Of course it is not only plain subject matter which is to be acquired in mathematics lesson, there are also 'higher' goals.

3.1 Goal: To penetrate cognitive (and other) situations analytically, to view them in a holistic way at the same time, to structure them, and hence to experience and enlarge the power of one's own intellect

The situations I mean can be part of the students' everyday life, but they can also consist of mathematics applications (from the mathematics point of view), recreational mathematics, or collections of problems for drill and practice (with some inherent regularity which can be discovered and then helps to perform the work with more ease), etc. There is one special quality (among others) about mathematics: the individual does not depend on information given by, or on opinions expressed by, other people. At the same time, however, (this is another essential feature) to do mathematics means to communicate one's ideas to other people, whether in reality or only in one's imagination.

Certainly this kind of communication has to take place in other disciplines and actually in every area of thinking as well. But mathematics (including logic, as a part of philosophy) is particularly responsible for this trait of human nature. It has the advantage over general philosophy that solid, well defined, simple notions which are already accessible to young children can be provided. School mathematics cannot merely be a lower version of university mathematics, thinned out for the 'limited intelligence' of pupils (Freudenthal 1986). Formalism with symbols must be superseded by intuition and meaning, and the rigour of logical deduction must be replaced by the consistency of common sense (as was always claimed by Arnold Kirsch).

A specific aim of any mathematical work is to obtain a complete overview of some situation, to relate new ideas to well known structures, to complete a theory etc. All the history of mathematics can be studied within these categories. Here are some examples for the primary school and for the education of primary school teachers respectively:

- To obtain a complete overview of all Archimedean solids and to prove that there can be no other solids of this type. This is one of the few 'theorems' which is accessible to pupils in that they can engage fully with it and for which there exists a simple, clear proof that can be produced by themselves (possibly with some support by the teacher).
- To join together all possible outcomes of a chance experiment in one mathematical space, to impose a structure on this space, and to close possible gaps.

— To engage with addition, subtraction, long multiplication and long division, not merely to acquire skills which can be applied automatically, but as a preliminary completion of arithmetic at the end of primary school.

3.2 Goal: To experience and to appreciate mathematics as a specific and indispensable part of human civilization

With this goal I do not aim at the utilitarian trait of mathematics (as it can be found, to mention just one example, in astronomy and meteorology for more than five thousand years), but rather I aim at the specific ways of reasoning and working which make mathematics prototypical and fundamental for the science-oriented 'rationalist' culture in our society.

3.3 To demand effort and to allow reflection

Many educated people in western societies dissociate themselves from the 'rationalist' culture. This is not only because of the negative image of the natural sciences and technology (including mathematics and informatics) and bad instruction in school, but also because of the effort which needs to be expended on the demanding manner of reasoning needed in these subjects. In particular, mathematical activities require permanent discipline, because normally the slightest inaccuracy leads to failures which often become immediately manifest. As a matter of course, working in the domain of the humanities also requires effort and discipline of thinking, but not every piece of sloppiness is immediately punished. As I pointed out in 3.1, the simple notions of mathematics allow young children to develop some discipline in their thinking, and the primary school is the right place to initiate and gently consolidate it.

This seems to be another cause of the **dsm** pedagogy. The **dsm** school possibly assumes primary school children to be mentally overstrained and, what is more, to be closely controlled because of the early confrontation with the authoritarian structures of mathematics. Maybe this anti-authoritarian motive is supported by a lack of willingness, or ability, to leave the culture of humanities and to enter the 'rationalist' culture. I think that children have the right to be challenged intellectually and to be encouraged to expend effort on thinking consistently. For that one must again and again offer them time and a silent classroom with a stimulating atmosphere for reflection.

I see this close control elsewhere, e.g., when the key problems of the world are forced on the children in a one-sided, 'politically correct' way, or, the reverse i.e. when students are exposed to some excessive pedagogy keeping them away from rational or critical thinking (cf. my example 4.3.2).

4. A view on (primary) mathematics teacher education

4.1 Mathematics for all prospective primary school teachers

I wonder how anyone will undergo the difficult task of teaching mathematics without having adequate basic ways of imagining and understanding, an adequate discipline of thinking and adequate ideas about mathematics teaching ('adequate' being related to the child and at the same time to subject matter). Certainly every adult person once acquired mathematical knowledge in school, but obviously this is not enough to meet the qualifications which are needed for teaching mathematics in primary schools.

Yet, in Germany, nearly all primary school teachers teach most of the relevant subjects, in particular mathematics, although in most of the 16 German states prospective primary school teachers study only two subjects, and mathematics need not be one of these. This disparity between the reality in primary schools and the education of the teachers originates from the same over-estimation of subject matter in the 1960's which entailed the implementation of New Math in schools. As has been the case with secondary schools, (prospective) primary school teachers were to study at university and later to teach only two subjects at school. However this reform was only realized with respect to teacher education, whereas the schools adhered to the old principle, i.e. each class has one main teacher who teaches a large portion of all lessons. The many good reasons for this principle need not be discussed here. Most of the states addressed their teacher education only slightly to the reality of school. For instance, in some states prospective primary school teachers have to take a little German and/or mathematics as part of their pedagogical studies (if they do not study them as particular subjects). NW is one of the few states where all prospective primary school teachers must study three subjects (two of them with half the normal volume), and two of these subjects must be mathematics and German. Unfortunately the NW ministry of education plans to abolish these model regulations (because of extraneous utilitarian reasons).

Of course, didacticians from other subjects could argue in the same way, and I adjudge the subject 'German' in the primary school to be at least as important as mathematics. However mathematics is fundamental to the 'rationalist' culture, and a general high esteem for this culture would require that everybody be acquainted with maths, and that from the primary school on. There is another point - mathematics refuses more stoutly than other subjects to be subsumed inconspicuously by the **dsm** pedagogy. This is because parents and society as a whole notice the local and global success and failure of mathematics teaching more acutely, and they have done so long before TIMSS came into being. In school, mathematics is, in a way, the backbone of the 'rationalist' culture.

John Searl stresses that mathematics is an extended literacy which, again, is the basis of rationalist culture and that in Western culture the humanities are rationalist. This is basically true, of course, but in large parts of the school system, in particular in primary school, a lot of subjects are not treated like that. Consequently many people do not perceive the humanities as rationalist, but attach this mode of thinking to the mathematical-scientific-technical-economic complex and keep some distance from it. If they should try to approach it, they would inevitably get confronted with mathematics, which can turn out to be a severe obstacle when a solid foundation was not laid at school.

4.2 The potential of geometry teaching

Arriving at this point we have the opportunity (and the duty) to take mathematics as a starting point for building a bridge to the **dsm** pedagogy and to supply it with subject matter. In my opinion, geometry is particularly suitable for this bridging, with the following goal at the top:

To structure the real space, and to explore the utilization of this structure.

This is a rough implementation of the intelligence factor 'spatial ability' which is particularly important today in the age of two-dimensional media. However it contains more (Bender & Schreiber 1981, 1985):

— reasoning about the purpose and the functioning of geometric forms, e.g., why are bricks ("Quader": three pairs of parallel rectangles, each pair being perpendicular to the other two pairs) with special measures particularly suitable for building walls?
— activities concerning the production and the use of geometric forms, e.g., making a closed tetrahedron (for the keeping of fluids) by suitably welding the edges of a rectangle, or the Archimedean conveyor for the lifting of fluids, sand etc.
— concept formation by idealization, e.g., to 'force' ideal geometric forms upon real space either by really making them (approximately) or at least by recognizing them in the structures of real space.

Here we have the epistemological pendant to the so-called operative principle (Wittmann 1974) as a didactical principle based on Piaget's theory of cognitive psychology. At the same time, our ideas are obligated to the holistic character of Gestalt psychology but they cover more than psychology as their core is the integration of the concerns of everyday life and of society, thus making geometry really meaningful.

4.3 Examples from mathematics didactics for primary teachers

4.3.1 The primacy of didactics over methodology

Our lectures on didactics in Paderborn are complemented by so-called exercises, where the student teachers translate the contents of the lectures into action and discuss homework. In one of these exercises (for geometry teaching) we wanted to make the students familiar with the old, and at the same time very modern, methodological concept of "Werkstatt-Arbeit" (working in workshops), using the mathematical concept of symmetry in order to supply them with ideas for their future teaching.

We arranged twelve stations in the lecture room for different activities ("learning carousel") promoting experiences with (reflection) symmetry, e.g.,

— to fold paper and to cut out symmetric shapes,
— two persons to move synchronously and symmetrically creating a solid with two components which is always symmetric,
— to look for axes of reflection in plane figures,
— to study the effects of double reflection, etc.

During their own activities, the student teachers could easily duplicate a primary school class working at the stations for longer than one hour and having fun. However I also 'tormented' them with the problem of how to make such scattered experiences persistent and consistent, and how to fit them together and hence constitute a sound concept of symmetry.

The situation, which I just described, is a paradigm for the critical relationship between the **dsm** and the **osm** cultures in our educational system.

For a typical follower of the **dsm** school, working with stations would be the true teaching subject as it provides possibilities for cooperative, self-organized, responsible learning, team-work, communication, withdrawal of the teacher, etc., whereas the mathematics involved might not be so important, and maybe there would be no attempt to distil a sound mathematical concept from the experiences. In contrast to this I think working with stations to be a method, a means for teaching and learning mathematics. It is not, however, a bait which the students have to take to allow me to do my mathematics with them. The material and the activities, well organized in stations, are essential supporters of the process of concept acquisition, and, what is more, according to our concept of mathematical concept, they are an essential part of the concept of symmetry itself.

This is a way, offered by mathematics education, to overcome the disparity between the **dsm** and **osm** cultures. That, for example, this kind of work is favourable to certain deliberate key qualifications goes without saying.

I personally adhere to the core of Klafki's article from 1958. First comes the didactical analysis of some subject (basic ways of imagining and understanding, fundamental ideas, structure of the subject matter, place in the curriculum, stage of the concept formation, etc.), and then comes the fixing of the methodology according to the didactical analysis (and to intentions of other categories, if they do not contradict that analysis).

When preparing a lesson there is no full equality of status for **osm** and **dsm** goals, because concentrating on subject matter, didactics and methodology automatically still effects some **dsm** activities, whereas stressing **dsm** goals can occur without any outcomes on the **osm** side. So even if one thinks **dsm** goals to be more important, it is advisable to start one's concrete reflections with subject matter.

In fact, literally in every minute of classroom work, some key qualifications are addressed although, of course, these are not always the modern, pleasant-sounding ones. Older ones too, even if they are out of fashion, can be useful for example: listening to other people, sitting still for a while, working on one's own or concentrating on something.

Over-emphasizing one or the other type of working arrangement in the classroom can affect one or the other key qualification, and one must concede that there has always been an over-emphasis of teacher-dominated teaching, so that the propagation of a student-centred style was necessary. But the baby must not be thrown out with the bathwater.

4.3.2 To take the students seriously

In my geometry teaching lectures we discussed the following situation where first-graders had to do some measuring in the classroom. It had been observed and written down by a colleague of mine whom I usually regard highly:

"... as they want to exchange their results only with each other and feel no urge to communicate them to anyone outside the classroom, it is not necessary for them to use standardized measures like 1 metre. ... [There were two boys who] measured 'foot by foot' the classroom and got different results (64 vs. 59). As, according to all experience, in mathematics lessons such a difference cannot exist, because it must not exist, they asked the teacher for an arbitration. Her measurement with her shoes, 41, was accepted as the final result."

If this situation is typical for the whole lesson, then, from my point of view, this lesson suffers from a lack of significance. Why do the students perform these measurements in the classroom? It is absolutely unimportant to know that the shoe of one boy fits 64 times, that of the other 59 times and that of the teacher 41 times along one side of the classroom. One could draw at least a little bit of sense from these facts if one would ask the question “whose feet are bigger?”. The answer to this question could have been found more easily!

However the meaning of these measurement activities could just consist of the understanding of the advantage of having a unified measure. It is not at all clear that this should be the size of the teacher's shoes. This situation cries out for further development in the direction of introducing a standard measure in general use, the more so as nearly all first-graders know that there is one. They know rulers with scales, yardsticks, distances, the body length, even though they do not completely grasp the principle of measurement (this is exactly one reason for teaching it).

“However that may be, by exerting a little bit of influence teachers would take their students more seriously, than when they leave them in the very state of knowledge they are just taking up (because they do not express the need for a change). Possibly the students have not enough knowledge about the subject matter or pedagogical experience, possibly they are not able to develop and articulate needs for effort costing promotion, because the teacher permanently conforms too closely to the needs they However that may be, by exerting a little bit of influence teachers would take their students more seriously, than when they leave them in put into words and the non-needs they do not put into words."

4.3.3 Teachers must completely understand what they want to teach

If you ask adults, including (primary school) mathematics teachers,

— to do subtraction and to explain where they put the small 'one's and why they put them there,
— to do long division and to explain what actually happens and why the algorithm works,

nearly all of them are able to carry out the operations and to train children to carry them out. Many of them, however, cannot give the explanations, and if they happen to be teachers, their students are thus prevented from gaining a real understanding. Nowadays, however, the main reason for teaching these algorithms (and they are taught!) is to make the students understand them (in order to get better insight into arithmetic), as the former motive for carrying them out automatically and without reflection has become obsolete in view of the wide availability of electronic calculators.

4.4 Examples from university mathematics lectures

There seems to be a worldwide (rather vague) agreement on how mathematics should be taught at universities, although there are many differences according to the lecturer's personality, to the students' level etc. In any case, there is a widespread lack of intelligibility (not in what the lecturers mean to do, but in what they really do) in favour of (often superfluous) absence of meaning, (often unnecessary) generality and (often exaggerated) avoidance of gaps. Of course, these features are essential for the science of mathematics, and they must also be experienced by the students, but not at the cost of understanding. I have at my disposal several passages of mathematics lectures (from linear algebra, number theory, axiomatic geometry), where one can identify exactly the point where intelligibility is sacrificed in favour of those other features. Because of lack of space I cannot display them here.

5. Final Remark

In mathematics teacher education we should demand and promote much more common sense than we have been doing up to now, and not strangle it, as we often have done. In mathematics lectures as well as in mathematics education lectures, plausible reasoning as a method and as a subject should be a universal principle, in order that the new teachers will arrange their teaching on the same principle, demanding effort from both sides. By this approach we would open up a doorway to the 'rationalist' culture in our society for all students and at the same time make our contribution to helping overcome the disparity between the **osm** and the **dsm** cultures in our educational system.

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