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THE OSNABRUECK CURRICULUM: MATHEMATICS AS A TOOL FOR THE REPRESENTATION OF KNOWLEDGE
– AN EVALUATION STUDY ON THE BASIS OF TIMSS-INSTRUMENTS

Abstract:
The first part of this paper gives an overview of a curriculum project in German grammar schools for grades 7 to 10. To improve the culture of mathematics lessons, the cognitive processes of pupils have to be put at the centre of lesson arrangements. In particular, meta-cognitive activities of pupils have to be stimulated. Thus areas such as

- results of research in cognitive science as regards the thinking processes of pupils dealing with mathematics (including their individual differences),
- research (in mathematics education, regarded as a design science) into the provision of a suitable learning environment,
- postulates of constructivistic teaching-learning-research, and
- the establishment of a discursive teaching culture

have to be taken into consideration.
In the second part of this paper, the question is discussed as to whether the considerable shift of emphasis necessary for the achievement of these changes is at the expense of the performance in achieving the learning targets of the regional curriculum. In order to examine this question, a comparative study with TIMSS-instruments has been undertaken. The implementation and evaluation of the study are introduced and the results discussed.

1. Introduction

Systematic explanations of the differences in performance which have been revealed by TIMSS show that it is the quality of mathematics lessons which must become the centre of attention. In particular, the results of the video study support the fact that it is not the general and social forms of interaction during lessons which are responsible for different developments in performance, but rather the problem formations and the cognitive processes produced during the working process (BAUMERT, LEHMANN et al. 1997). Measures taken in order to improve the culture of mathematics lessons therefore have to put the cognitive processes of pupils at the centre of lesson arrangements and the meta-cognitive activities of pupils have to be especially stimulated (COHORS-FRESENBORG 2001b). Thus such areas as

- results of research in cognitive science as regards the thinking processes of pupils dealing with mathematics (including their individual differences),
- research (in mathematics education, regarded as a design science) into the supply of a suitable learning environment,
- postulates of constructivistic teaching-learning-research, and
- the establishment of a discursive teaching culture

have to be taken into consideration. This has to be supported by both conceptual continuity within the curriculum and also by the supply of learning materials (textbooks). In this way pupils see this different approach to mathematics teaching represented in the important media of their mathematics lessons.

In section 2 of this paper, we report on such a concept for teaching mathematics at grammar school level[1] ("Osnabrueck Curriculum", COHORS-FRESENBORG & KAUNE 1993, COHORS-

[1] The curriculum is designed for a Gymnasium, a type of secondary school in Germany which caters for the top one third of students in terms of academic ability.

FRESENBORG 2001a), which has put the above-mentioned central idea into effect. The underlying view of school mathematics causes a shift of emphasis, as a result of which the contents of the classic curriculum have to be taught in a considerably shorter period of time. Several case studies have documented and analysed how the changes made in the culture of problems and lessons led to a deeper understanding by students (COHORS-FRESENBORG et al. 1999, COHORS-FRESENBORG & SJUTS 2001, FRAUENKNECHT 1993, GRIEP 2000, 2001; HOFFMANN 1993, KAUNE 1995, 1999, 2001a-c; SJUTS 1993, 1999, 2001a-c, 2002). The question now remains as to whether the considerable shift of emphasis necessary for the achievement of these changes – the contents of the present curriculum have to be taught in approx. 75% of the teaching time – is at the expense of the performance in achieving the learning targets of the regional curriculum. In order to examine this question, a comparative study with TIMSS-instruments has been undertaken. In section 3 of this paper, the implementation and evaluation of the study are introduced and the results discussed.

2. The Osnabrueck Curriculum

2.1 Cognitive theoretical views on the role of school mathematics

A cognitive theoretical orientation of mathematics lessons means that the cognitive mechanisms, when constructing mathematical knowledge in the pupils' heads, and the process of knowledge organisation and knowledge use form the centre of the teacher's attention.
From the beginning, our consideration for a new orientation of mathematics teaching has been led by the dualism between action and language as two modes of representing knowledge (COHORS-FRESENBORG 1987a). In the maths education community, such a focus is also discussed. From the point of view of mathematical content one may consider the ideas of "procept" by TALL (1991), "APOS-theory" by Dubinsky (COTTRILL et al. 1996) and "operational-structural" by SFARD (1991). For the debate concerning "obstacles provided by the process object duality" see KILPATRICK (1999, p. 53). From a cognitive science point of view in mathematics education, SCHWANK (1986, 1993a) has pointed out that, in relation to those antitheses, two different cognitive structures exist. The existence of individual preferences for so-called functional versus predicative cognitive structures has been proved by experiments in different contexts (SCHWANK 1999, 2001; COHORS-FRESENBORG & SCHWANK 1997, MÖLLE et al. 2000).

Mathematics lessons at grammar schools have to be increasingly aligned with creating workable mental models with regard to the inner mathematics "operation" as well as the processes of abstraction of concepts from colloquial texts and the interpretation of abstract mathematics in concrete situations.

The paradigm of the construction of a cognitive, mathematical operating system in the pupils' heads as the prior goal in the first mathematics lessons at grammar schools gives rise to a new central idea in the discussion of the curriculum. The cognitive science point of view concerning mathematics teaching makes it possible to create a unified concept from the two aspects of (i) mathematics as a discipline in itself and (ii) mathematics as an aid to answering questions from reality. To a great extent, the methodical starting point for precisely stating and formalising intuitively existing knowledge is not tied to whether it is intra-mathematical knowledge or knowledge of so-called real world situations which is dealt with. A specific, irreplaceable way to disclose reality intellectually is inherent in mathematics; it consists in working out the formal aspects of mental actions and thus in creating more transparency and understanding of correlations. In KAUNE (1995), the role played by thinking in functions and their formal representation is explained.

The Osnabrueck Curriculum of grades 7-10[2] is based on experiences gained in two extensive school experiments by the Ministry of Education of Lower Saxony from 1987-1995 with more than 3,000 pupils. These may be summarised as follows:-

- grades 7/8: "Integration of algorithmic and axiomatic ways of thinking in mathematics teaching in grammar schools in grade 7/8 as a contribution to information and communication technological education";
- grades 9/10: "Mathematics as a language for a precise representation of knowledge".

Up to now, it is this curriculum which has been implemented in several grammar schools in Germany.

It has to be pointed out that the changes made in mathematics lessons were constructed in such a way that the complete contents of the regional curriculum were part of the newly designed lessons. Those lessons dealing with the two additional teaching topics cover approximately half a school year so the usual contents of the first two school years at a grammar school are taught in only 1.5 years. The success showed particularly that the number of topics, although often complained about, is not the main problem for the improvement and reorientation of mathematics lessons.

2.2 Construction of a cognitive mathematical operating system

The description of the mathematical knowledge of pupils via the concepts "frame" and "procedure", introduced to cognitively-oriented mathematics education by DAVIS and MCKNIGHT (1979), gives cause for searching fundamental frames and procedures. These have to be developed as parts of a mathematical cognitive operating system in the pupils' heads in the first lessons at grammar school and their combination has to be consciously implemented for the pupils. The individual differences in cognitive structures – predicative versus functional – are particularly taken into consideration.

We have put the construction of a cognitive mathematical operating system in the pupils' heads at the centre of our conceptual work. Its most important elements are the function frame and the contract frame with suitable, attached procedures. Both frames use the frame "formal representation of intuitive knowledge". This requires a fundamentally new judgement of the importance of language and formalisation in mathematics teaching from the teachers, and moreover justifies itself by the increase in value of understanding and communication.

Empirical research (SCHWANK 1993b) has shown ways in which an embedding of the function frames can be effected, taking into consideration the different cognitive structures in the pupils' heads. This is carried out in the microworld of "Einführung in die Computerwelt mit Registermaschinen" ("Introduction into the world of computers with register machines", textbook for pupils, COHORS-FRESENBORG et al. 1995 [3]). Teaching experience has shown that the function frame can be sufficiently established by the end of grade 7.

The subject of the orientation of mathematics teaching in the complete secondary school level 1, as regards contents, can be formulated so that mathematically precise formations of concepts can be made for more and more complicated functions, and algorithms are supplied for the calculation of function values.

Mathematics teaching, especially at grammar school level, is meant to give pleasure in the theory of, and in the capability to use, mathematics. Here a deep understanding of the theoretical mathematical formation of concepts is indispensable. Our starting point offers easy access to this by asking what is actually meant by intuitively existing mathematical concepts.

[2] In Lower Saxony, grammar school starts at grade 7. The curriculum fits this situation.
[3] Unfortunately this textbook only exists in German, but there are two English papers (COHORS-FRESENBORG 1987b, 1993) available in which the underlying mathematical ideas and the didactical transformation are developed.

The cognitive theoretical procedure only has to concentrate on mathematics itself. Thus it is clear that questions of meta-mathematics, the formation of concepts in axiomatic systems and the nature of precise and explicit definitions and proofs become an integral part of the lessons. The axiomatic method provides a means to generally organise a (mathematical) stock of knowledge, to ensure it in itself and to develop it. It has been established to give even pupils of grade 8 a long-term workable framework idea by establishing a suitable microworld "Sätze aus dem Wüstensand"[4] ("Sentences from the desert sand", COHORS-FRESENBORG et al. 1992). Following that the microworld "Vertragswerke für den Umgang mit Zahlen" ("Contracts for dealing with numbers", COHORS-FRESENBORG et al. 1998) offers a frame which provides a unified basis for insight into the formation of concepts in customary school mathematics in fields such as extension of the number field, term rewriting, equations and the method of proving. Probability calculus also fits this frame: it is a contract to talk precisely about the unknown.

2.3 Constructivistic creation of mathematics lessons

The cognitive theoretical view of the role of school mathematics described so far does not only have consequences for a different way of proceeding as regards contents, but it also has to be reflected by a different way of teaching.
The different quality of the developed concept and the intended teaching culture made it necessary to develop completely new textbooks for the pupils and, consequently, extensive handbooks for the teachers as well as didactic material (see list in COHORS-FRESENBORG 2001a, p. 12). Only the topics of geometry are taught according to the normal textbook. The representation of mathematical reconstructions and special tasks shall make it easier for the teachers to teach in a constructivistically-oriented way. By suitable formulations of problems, the pupils shall increasingly be prompted to implement meta-cognitive activities. The effect regarding this concept is analysed in detail in SJUTS (1999). The change of teaching due to a different culture of problems is analysed in detail in KAUNE (2001b). Examples of how this affects lessons can be found in COHORS-FRESENBORG et al. (1999), KAUNE (1999) and SJUTS (2002).

Promotion of meta-cognition by suitable problems

The so-called "Comment-on-that Problems", introduced for the first time by KAUNE in the textbook "Einführung in die Computerwelt mit Registermaschinen" ("Introduction into the world of computers with register machines", (COHORS-FRESENBORG et al. 1995) play a special role in the promotion of pupils' meta-cognitive activities. These problems introduce to the pupils different solutions to a problem, or a summary of a concept, or illustrate how different reasons for the facts may arise, e.g. in the form of a presented dialogue between pupils. They are then asked to comment on the matter (in writing). The teaching arrangements for these problems shall be described briefly (see KAUNE 2001b).
A diagnosis of pupils' mistakes when calculating, or the exposing of a wrong argumentation when giving reasons for facts, always precedes the construction of a "Comment-on-that Problem". The knowledge of what forms the basis of the pupil's idea of a mathematical fact, which might be dependent on different forms of representation or preferences for representations, as well as the knowledge of which ideas invoke certain representations, is presented to the pupils in the form of a discourse:

> As a first step, the (false) idea is, colloquially speaking, "put into the mouth" of a fictional pupil. So the (false) idea is first of all made the object of thinking.

[4] The didactic concept is briefly described in COHORS-FRESENBORG (1987a, pp. 268-271).

In most cases, a discourse is then started by the presentation of a second contribution from a pupil, who comments on the (false) idea.
In the third step, an additional pupil's opinion might follow which describes how a pupil deals with a precise idea in an increasingly successful way.

The tasks can only be used in a promising, i.e. performance increasing, way for the pupils when they have spent sufficient time in their mathematics lessons experiencing that there are different forms of representation for mathematical objects, and that these can again produce different ideas. They have to be aware that there are different mathematical views represented in each statement within these "Comment-on-that Problems" and that they are asked to compare them to their own views and to name possible differences.

Therefore in the argumentation being executed, factual arguments have to be used which refer to the mathematical object, or to their own cognition, or to a pupil's cognition introduced by the problem. The pupil personifies not only a special individual idea, but is often a representative of common ideas.

Although the type of problem "Comment-on-that!" is very open when measured according to the number of possible problem-solving alternatives, the frame wherein the problem is dealt with is set by the teaching culture. The problem is not fruitful if only unfounded agreement or disagreement is expressed, and is only useful if the text is tackled with discursive techniques. Such "Comment-on-that problems" are an important part of a discursive teaching culture in the following sense:

1. they develop the experience gained in a discursive teaching conversation,
2. they make individual work possible in order to perfect discursive techniques (also as homework),
3. the discussion of their solutions often gives cause for further precise discussions while, on the other hand, they also require positively founded pre-experience in leading a discursive conversation.

Our experiences have shown that the type of the "Comment-on-that problems" also makes it possible to get access to an individual's competence in discursive behaviour when testing performance via a written class test. On the other hand, the early occurrence of these types of problems in written performance tests increases the pupils' impression that the teacher is "quite serious" when declaring the necessity and usefulness of discursive behaviour for the understanding of mathematics.
The appeal of such a problem, or its level of difficulty, is not created by the number of possible solutions, but by whether the pupil working on it can represent the facts of the case in different ways mentally.

2.4 Curriculum of grades 7 and 8

The teaching unit "Einführung in die Computerwelt" ("Introduction into the world of computers", COHORS-FRESENBORG et al. 1995) is dealt with in grade 7 and "Einführung in die axiomatische Auffassung von Mathematik" ("Introduction into the axiomatic view of mathematics", COHORS-FRESENBORG et al. 1992) in grade 8. Some 13 teaching weeks (out of 60 weeks of lessons) are reserved for these two topics, the dealing with which is not planned for in the usual curricula. This may seem incredible to the experienced teacher as – at the same time – the complete contents prescribed by the regional curriculum are claimed to have been covered successfully.
The main difference, compared to the usual way of proceeding, lies in the fact that our concept is not oriented towards subjects, but basic structures. Whereas the relevant guidelines

suggest teaching the list of learning goals for the subject one after the other, we proceed in a different way. The central didactic idea is the creation of model ideas mentioned in the two teaching units; i.e. on the one hand the teaching of functions with several variables, on the other hand the teaching of contracts (axiomatic systems).
After successful work with the teaching unit "Einführung in die Computerwelt" ("Introduction into the world of computers") the function language can be used by the pupils as a tool (also see KAUNE 1991; GRIEP 2000). This includes the excellent handling of the mathematical formalism of functions of several variables as well as the concept 'variable'. It allows for work on realistic application problems, whose complexity goes far beyond that of conventional schoolbooks. This is proved by a multitude of problems in the textbook "Modellbildung mit Funktionen" ("Model forming with functions", COHORS-FRESENBORG et al. 1997) and by documented pupils' solutions (KAUNE 1995, GRIEP 2000). The excellent handling of representations of functions of several variables is also useful in order to look at very different orders from a uniform viewpoint in computer algebra systems (see GRIEP 2000).

After finishing the series "Sätze aus dem Wüstensand" ("Sentences from the desert sand"), "Vertragswerke" ("Contracts") offers a mental model of axiomatic systems to the pupils. This world of ideas is then used in an axiomatic consolidation of the expansion to the number field by the inclusion of the rational numbers (COHORS-FRESENBORG et al. 1998). Furthermore, "Sätze aus dem Wüstensand" ("Sentences from the desert sand") provides intensive training when dealing with formalisations. This ability is also available as a tool in all subsequent teaching units.

Pupils' capability in dealing with formalisms free of any emotional points of view allows a simpler explanation of the teaching of equations on the basis of linguistic logic of identity and the concept of function. The three months which we put into the development of model ideas seems to be rather high, but this time has been made up as early as grade 8.

3. Evaluation study on the basis of TIMSS-instruments

3.1 Design and implementation of the empirical inquiry

In order to answer the following questions:

1. whether the reorientation of mathematics lessons of the Osnabrueck Curriculum mentioned earlier and the changes in teaching culture thus achieved occur on account of the performances with regard to the contents prescribed by the regional curriculum, and
2. whether a change in pupils' epistemological ideas can also be ascertained quantitatively,

the following evaluation study was carried out on the basis of TIMSS-instruments (KLIEME et al. 2000).

Two grammar schools (Gymnasium) of Lower Saxony, which had also implemented the Osnabrueck Curriculum as their school curriculum after the two school tests, were chosen for this study. In each school all four grade 8 classes took part in this evaluation, which was undertaken in spring 1999. Complete data records of 179 pupils are available for further analysis.
The following TIMSS-instruments were used:

- the TIMSS-mathematics test as the central performance criterion (i.e. one of the eight rotated test booklets) which reproduces quite well the standard curriculum of grades 7 and 8 as curriculum analyses have shown (BAUMERT, LEHMANN et al. 1997),
- scales as regards the interest in both the teaching subject and also the relevant interest, i.e. interest in mathematical topics,
- a scale for the self-perception of mathematical talent, and

- a set of questions for the recording of epistemological ideas (“mathematical views of life”), which includes four dimensions:
  (a) mathematics as a pure game with numbers, signs and formulae,
  (b) a platonic picture of mathematics,
  (c) mathematics as the application of general laws and procedures on special cases, and
  (d) everyday relevance of mathematics (see KÖLLER, BAUMERT & NEUBRAND 2000).

Moreover we used different components of the TIMS-study as monitoring variables:

- Two sub-tests (verbal or figured analogies) from the cognitive capability test KFT (HELLER & PERLETH 2000). These tests record basic cognitive abilities and serve as a measure for checking the initial selectiveness of the two schools.
- Various scales for the perception of the teaching atmosphere and quality which were developed within the study “Educational course and psycho-social development in adolescence”, were to describe the implementation conditions of the curriculum in the classes involved.

Performance tests and pupils’ questionnaires were scheduled for a working time of two lessons each. External testers, having been trained by an employee of the Max Planck Institute for Human Development in Berlin, ran the tests under conditions which were equivalent to those of the original TIMSS-study. All data were coded, entered, and statistically evaluated at the Max Planck Institute. A sub-sample from the original TIMS-study was used for comparisons. This subset of TIMSS contained those students from grammar schools (Gymnasium) in those Federal States which showed a similar expansion rate to grammar schools as in Lower Saxony. This comparative group consisted of 377 pupils from 16 schools. For 376 pupils of this comparative group we have data concerning performance in mathematics according to the TIMSS-metric.

Regression analysis and analysis of variance procedures were used for the statistical evaluation. If the different cognitive basic abilities of pupils are to be taken into account, a hierarchic linear model, which differentiates between individual and group-level effects, would be suitable. The experimental schools were compared with those chosen from the TIMSS-study as regards epistemological conviction, interests, self-perception and teaching perception of the pupils. In such a situation, a systematic control of intelligence is not useful and hence the data have been analysed at an individual level. At the same time, however, the clustered structure of the data (with an intra-class correlation of approx. 0.20) was taken into consideration by reducing the number in the sample size from 555 (for which valid data exist) to an “effective” number of 102.

The comparison of the experimental schools with schools from the representative TIMSS-sub-sample guaranteed an appropriate establishment in the national reference frame. Through the limitation to comparative groups of grammar schools (Gymnasium) from Federal States with a similar expansion rate, essential contextual conditions could be kept constant. Furthermore, examination of the basic cognitive abilities of the pupils was added. Finally the methodically reflected choice of analysing procedures (multilevel analysis or correction of the effective sample size) also took the limits of the meaningfulness of the data into account. Due to the inclusion of all classes of one grade, the internal variation within schools was considered.

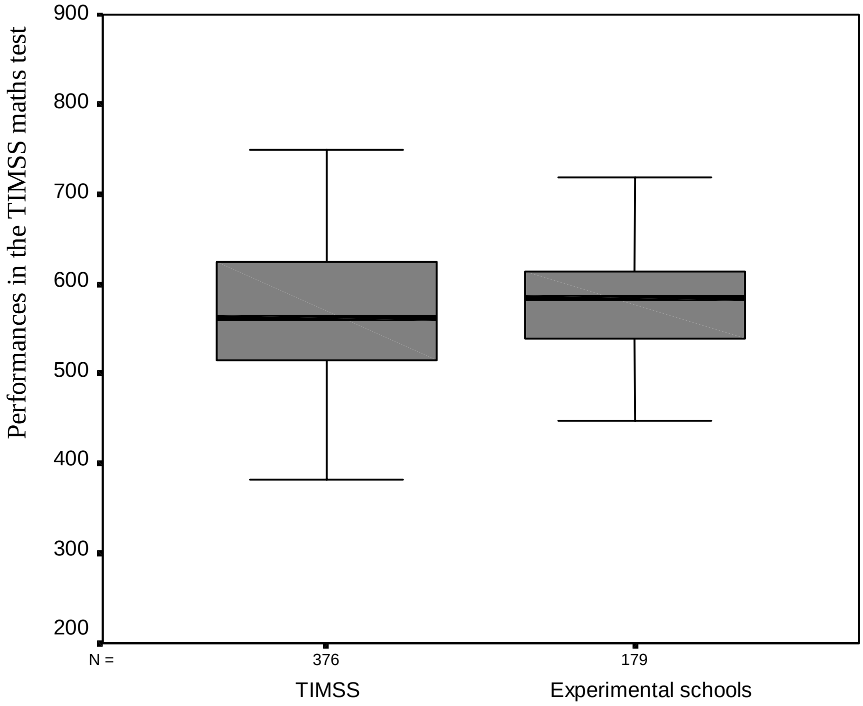
3.2 Results of the Evaluation

3.2.1 The mathematical competence of pupils

The following table contains the central tendency of maths performance (median) and the measures of dispersion for the two samples. Since the TIMSS database is used to estimate measures of the population, the scores spread slightly more than in the experimental classes (see standard deviation). The scores for the mean achievement (median) and the 25%- and 75% percentiles (borders of the middle quartiles) as shown in the table are illustrated in the figure below.

	mathematics score[5]	
	TIMSS	OS
median	562.7	583.6
standard deviation	77.2	57.0
5% percentile	447.7	495.0
25% percentile	514.1	539.3
75% percentile	623.8	612.6
95% percentile	692.0	676.7

The central result of the TIMSS mathematics test is shown in the following illustration. The number of points achieved can be read from the vertical axis. All tested pupils' performances (apart from a very few so-called "break-outs") lie between the bottom and the top horizontal line. The grey-hatched areas mark the two middle quartiles and the thick black line indicates the median. The illustration shows that the experimental classes have certainly reached the level of comparative TIMSS-grammar schools with respect to mathematical competence.



Distribution of math-performance in the experimental schools and the comparative TIMSS-classes. (The grey-hatched areas show the measured values for 50 % of the pupils. The wide line marks the median. Furthermore, the range of the measured values is shown.)

From the illustration, one can conclude that the pupils in the experimental schools achieve higher results in absolute values than those of the comparative sample. If the different basic cognitive abilities are taken into account, this advantage reaches a value of 19 points on the

[5] A direct comparison of the variance of scores between the TIMSS sub-sample and the experimental classes is not valid on an individual basis because in both samples slightly different algorithms for estimation had to be applied when determining the performance scores.

TIMSS-scale. The multilevel analysis, which allows a correct examination in this case, does not show a significant difference between the experimental and the TIMSS-classes, in relation to mathematical competence ($t = 1.09$, $df = 1$, $p = 0.29$). The difference between the TIMSS and the single experimental classes only explains a small part of the performance variance (i.e. 2.1 %). The differences between the single school classes are a lot more significant, explaining 20 % of the variance.

Satisfactorily, the illustration shows that the lowest values of performance and the border of the lower quartile exhibit only a difference of about one half of a standard deviation in comparison to the international average of the TIMS-study (500). In comparison to the TIMSS sub-sample, it is also obvious that the highest level of performance has not been reached. Summarising, one can say that the observed classes achieved a satisfactory level with respect to basic mathematical competence as has been determined by the TIMSS instruments. The slightly higher average in absolute values gives rise to an optimistic assessment of the development in performance among the pupils who are taught according to the Osnabrueck Curriculum.

Apart from these global statements, it is interesting that the experimental pupils achieved the expected good results in those items for which the Osnabrueck Curriculum is particularly useful i.e. successfully working with the cognitive tools which are especially supported by this curriculum. These items are B08 and B12 (TIMSS-expressions) which refer to the understanding of variables, as well as items U02b and V02, which ask for an additional explanation of the solution.
For the two items (B08 and B12) in multiple choice format, a tendency to a higher probability for a solution in the experimental classes can be proved by a simple χ^2 – test (on an individual level without correction of the sample size, $\chi^2=3.34$ / $p=0.07$ or $\chi^2=4.24$ / $p=0.04$). In the following table, the portion of pupils is listed who have chosen the correct answer to the above-mentioned items:[6]

	B08		B12	
	TIMSS	OS	TIMSS	OS
correct	67.7 %	76.3 %	87.6 %	93.9 %
wrong	32.3 %	23.7 %	12.4 %	6.1 %
n	192	177	194	179

For the items V02 and U02b, one can also observe a slightly higher portion of correct answers in absolute value which, however, can not be proved statistically. The type of problem, classification and results of the test problem V02, which proved to be one of the most difficult, is to be found in KAUNE (2000).

3.2.2 Interests, views and teaching perception of the pupils

With reference to their mathematical interests and their self-perceptions of mathematical talent, the pupils of the two experimental schools do not differ from their peers at other grammar schools (t-tests on the individual level with corrected sample size, $p > 0.05$). This means that

[6] The values for correct solutions reported in COHORS-FRESENBORG & KLIEME (2000) are due to a different choice of the TIMSS-comparative sample. In the table shown here, only those Federal states with a similar expansion rate for grammar schools are taken into consideration while in the table presented in COHORS-FRESENBORG & KLIEME (2000) some more schools from other Federal States had been taken into account.

the curriculum has no specific promotional effect on the technical or factual interest or self-perception of the pupils.

It is very interesting to see that the most significant effects arose in reference to epistemological convictions. Mathematics is not a "game with glass beads" to the pupils in the classes being tested ($p<.001$), but they are of the opinion that mathematics occurs in everybody's everyday life. They are not convinced that the solutions of mathematical problems are definite. They know the meaning of derivations and proofs. The typical way in which mathematicians proceed when applying general rules and procedures to special problems is more familiar to them than to the pupils of the comparative group ($p<0.05$).
The fact that the pupils of the experimental classes have significantly lower scores in the scales "Relativistic view of mathematics" and "Mathematics as well-defined knowledge" can be explained by the way pupils apply their mathematical beliefs. This has been deeply influenced by the way of teaching which is inherent in the Osnabrueck Curriculum. Through the textbooks used in class and the lesson script, they have experienced that mathematics is not a finite construction of ideas that one masters by memorising definitions and formulas and by applying algorithms. Furthermore, the pupils in the experimental classes are more often convinced that there is more than one way of solving a mathematical problem.
At first glance, it seems to be astonishing that the pupils of the experimental classes have significantly lower scores on the scale "Relativistic view of mathematics" even though some of the textbooks (COHORS-FRESENBORG, E. et al. 1992, COHORS-FRESENBORG, E. et al. 1998) deal with an axiomatic approach to mathematics in a highly formal manner. This can be explained by the fact that the creation of the formal language and the abstract ideas are taught in a constructivistic way.

4. Summary

In summary, it can be said that the construction of the above-mentioned cognitive mathematical operating system makes it possible to make up the necessary teaching time as early as the end of grade 8 and to achieve – at the individual level - significantly better competitiveness when referenced against the TIMSS-scale. We also conclude that the "amount of teaching material" is not the main hindrance to the improvement of mathematics teaching, but that the quality of teaching has to be improved.

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