Hans-Wolfgang Henn, Dortmund

"PROMOTING CLASSROOM CULTURE" THE BLK-SCHOOLPROJECT IN BADEN-WUERTTEMBERG

Abstract:
The project "Promoting classroom culture in mathematics" is Baden-Wuerttemberg's contribution to the programme of the Bund-Länder-Kommission, BLK, (commissioned by the German government and the federal states) "Furthering the efficiency of mathematics and science teaching". This four-year programme for the lower secondary level was developed as a meaningful reaction to the less than satisfactory German TIMSS results. The approach of this project focuses on changing the style of teaching. The main aim is to develop a holistic design of mathematics teaching integrating comprehension, active participation and long-term productive learning. A report on first experiences of the project in Baden-Wuerttemberg is given.

TIMSS and the BLK-Project

The not very flattering German results in TIMSS, the Third International Mathematics and Science Study, have acted as a catalyst for a Germany-wide discussion on educational goals and the content of mathematics teaching. The most perceptible signal is the programme "Furthering the efficiency of mathematics and science teaching" initiated by the Bund-Länder-Kommission (BLK) for educational planning and research promotion. This four year programme for the lower secondary level started with the 1998/99 school year. In preparation for the BLK programme (BLK 1997) a report was drawn up stating the assumptions of educational policy and describing the various problems and principles of learning and teaching of mathematics and science in schools. Consequently the report proposed a Germany-wide school experiment and accompanying measures. Schools and teachers were, however, not confronted with ready-made teaching concepts, but given so-called modules with promising starting points for promoting mathematics and science teaching. In detail, the following modules were indicated:

1. Developing a problem-solving culture in mathematics and science teaching.
2. Scientific work.
3. Learning from mistakes.
4. Consolidating basic knowledge – intelligent understanding learning on different levels.
5. Experiencing the growth of competence: cumulative learning.
6. Experiencing the boundaries of subjects: cross-curricular and subject-related learning.
7. Furthering boys and girls.
8. Developing concepts for the co-operation of students.
9. Strengthening the responsibility for individual learning.
10. Assessment: Comprehending and feedback of competence growth.
11. Securing quality within schools and the development of comprehensive standards.

The German federal states are participating in this BLK-programme with a total of 30 experiments (so-called "Modellversuch"). Six schools, a so-called school set, participate in each experiment. Each federal state has chosen its own modules from the above list, which are put into practice in the schools. The single federal states have organised their respective experiments differently. During joint annual conferences experiences are shared. The overall BLK-project will be evaluated by the IPN (Institute of Pedagogy of Science) in Kiel.

The school project in Baden-Wuerttemberg

The project "Promoting Classroom Culture in Mathematics" is Baden-Wuerttemberg's contribution to the BLK-programme (cf. BLUM/NEUBRAND 1998, HENN 1999). There are three school sets, one with six "Hauptschulen" (lower level education), one with six "Realschulen" (intermediate level education), and one with six "Gymnasien" (higher level education). Obviously schools are working closely together and there is a lively exchange of experiences between the three school sets.

For our chosen schools, we have focused on the following four out of the 11 modules stated in the report:

Module 1: Emphasis is on open problems, appealing to all students. Individual problem solving abilities are challenged. Problems are given in varied contexts, to open up various qualitatively different solutions and to provide systematic and productive exercises.

Module 3: Psychological and pedagogical theories creating the conditions of how to foster learning from mistakes are put into practice in the classroom. This should be used in teaching practice. "Mistake-friendly" lessons can improve students' mental activities.

Module 5: Possibilities for making a vertical net and securing cumulative learning are sought. Long-term orientation and activating their learning history should help students to accumulate increased "learning possession".

Module 10: The development of challenging examination problems, and examination types suitable for measuring comprehension and the flexible application of knowledge, is highly important in improving teaching.

Instruction takes place in all years of the lower secondary level. The approach underlying our project aims less at changing the mathematical content, but rather focuses on a change of teaching style. The aim is the holistic design of teaching which leads to understanding, active involvement and long-term fruitful learning. Central deficiencies of current teaching are

- short-term learning for the next test,
- restrictions resulting from a narrowly guided "questioning-developing" teaching style, and
- a strong calculation orientation of mathematics teaching, whereby calculations are used without insight and understanding.

What has to change?

The central question is not "what is to be learned?" but, "how should learning take place?", "how can mathematical literacy be promoted?" and also "how can learning processes be measured?". A willingness to question and to rethink current teaching, to change one's own acceptance and to realize opportunities brought about by new practice and teaching techniques is important.

The question is, what should students actually "know" by the end of the lower secondary level, when "knowing" is not primarily related to knowledge but rather to basic concepts, ability, competence, and attitudes. The emphasis should be on discussing the teaching content necessary to promote an adequate concept of mathematics and the abilities, competence and attitudes indispensable for reacting to an increasingly complex world.

Our aims

We do not intend to reject everything from the past as bad and to follow a new fashion such as "tasks as open as possible" or "application at all cost". Rather we aim at a reasonable transfer of emphasis, balancing between instruction (by the teacher) and construction (by the students themselves), between teaching and discovery, between convergent routine problems and divergent open problems and between different modes of testing and measuring achievement.

This reorientation must naturally take place throughout the years spent in school from elementary to upper secondary level. Our approach to the BLK-project was highly influenced by the concepts of the Dortmund project *mathe 2000* of the group based around E. Ch. Wittmann and G.N. Müller (cf. Mathe2000). Inherent is the concentration on fundamental ideas of mathematics and long-term development, following the spiral principle, during the years spent at school. Of main importance is pupil-centred mathematics teaching, i.e. to take seriously the answers, ideas and products of the pupils. This means a conscious change in teachers', as well as students', attitudes especially in taking learning processes more seriously than results. A "good teacher" is not characterised by giving "good explanations" but by promoting thinking processes and providing active discovery learning in a productive learning environment.

Therefore, we propose a new model for learning:

- Learning not perceived as building a wall where missing "stones" prevent further development.
- Instead, learning as a continuous meshing and re-structuring of a net whereby the interrelations of the net are important for the development of basic concepts.

We want to move away from the usual procedure

motivating example → rule → practising the rule

and move towards

> researching a new field, many enlightening, eye-opening examples, then examples giving insight into vertical and horizontal relations and distinctions to other fields, independent student activities in "error-friendly" introductory phases, formulation of their own rules right at the end and not before, and finally a rigorous division between working and assessment phases.

Naturally, computational skills have to be developed. Computation, however, should not only be learned off by heart, as is often the case, and used without thought. For example, the rote learning of three basic rules for calculations with percentages rather prevents understanding and perceptive application of competence with percentages. Changing gear correctly when driving a car cannot be learned from a book but by driving. By the same token, students have to gain experience by handling formulae and terms themselves and applying calculations meaningfully in context.

Two main aspects which are not self-evident, at least in teaching at upper secondary level, have emerged: to take children and their products seriously on the one hand and to construct productive learning environments on the other.

Taking children seriously

In “How children compute”, a book worth reading, (Selter/Spiegel 1997) the authors plead that teachers should think and argue with children, listen to them, take their products seriously and not ignore their mistakes, but rather discuss them productively. We usually expect that children think in the same way as we, as mathematicians, do (whereby we often are remarkably blinkered …): However, children calculate, as Selter and Spiegel clearly point out

- in a different way from the way we do
- in a different way from the way we assume they do
- in different ways from other children and
- the “same” problem in different ways.

Consequently it is not enough to question children, but rather to take their questions seriously, discuss and try to understand them. Thus teachers are able to recognize and understand their students’ cognitive structures. *One* way is to gather all solutions without any comment on the blackboard in a first step and then to discuss them. Often learners then point out the mistakes they themselves made. The following examples illustrate this approach.

Example 1: Geometry (grade 6):

The problem is (relating to Fig. 1):

Complete the drawing in such a way that there are two adjacent supplementary angles.

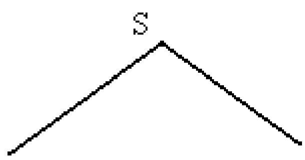

Figure 1

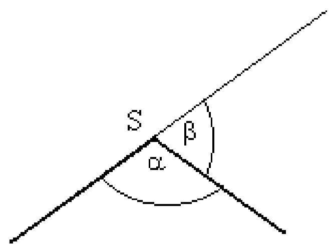

Figure 2

In a normal lesson, pupils would be guided to a solution whereby one line at the side of the angle would be made longer (cf. Fig. 2). Here the teacher had pupils accomplishing the task themselves independently without any hints or guidance. Single solutions were then presented at the blackboard. Questions were only allowed after the drawing was finished. The pupil then had to explain his or her solution and mistakes had to be realized. Solutions turned out to be much more general than the narrow schoolbook solution. Often there were several pairs of adjacent supplementary angles. Fig. 3 shows some of the solutions:

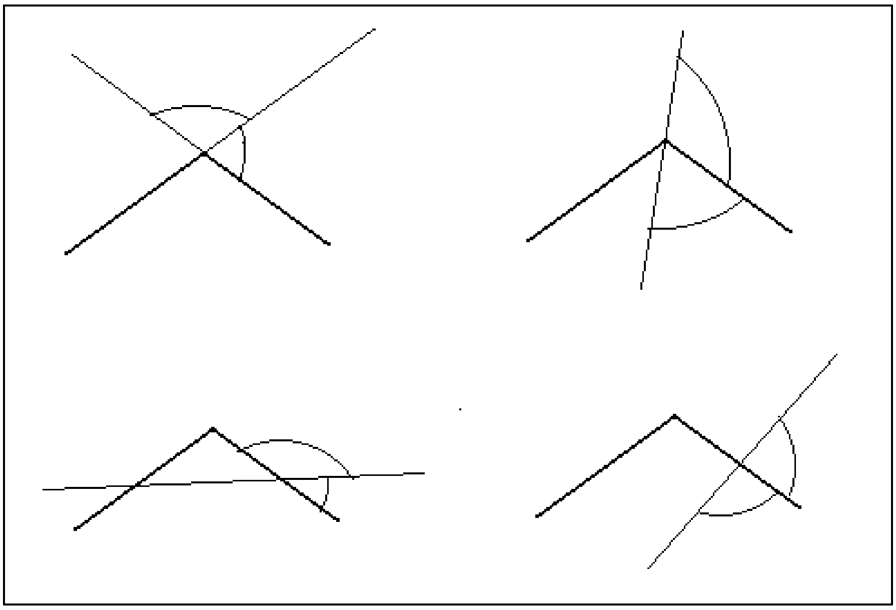

Figure 3

Interestingly enough, the schoolbook solution was not mentioned (and not forced upon the students by the teacher). Taking the children seriously created a productive working atmosphere. "I could easily see the disappointment of students whose solution was already presented by others. A positive disappointment", reports the teacher.

Example 2: Arithmetic (grade 7):
Often teachers think that students have understood 'the subject' when they can regurgitate a variety of problems of the same type, unrelatedly strung together, following a given pattern. However in this way no adequate and sound basic concepts of mathematical constructs are created or even tested. Just as the ability to differentiate a function does not convey any understanding of the concept of derivative, the ability to compute (more or less correctly) with rational or real numbers does not convey anything about understanding these numbers. The manipulation of expressions is no indicator of a meaningful idea of the variable concept. The following question was given to children at the beginning of grade 7:

Is $2.\overline{9} < 3$ *or* $2.\overline{9} = 3$*?*

Examples of their answers reveal much about the existing basic concepts.

Jan: "*For me* $2.\overline{9}$ *is equal to 3 because, when you write down* $2.\overline{9}$ *as a composite fraction, it is* $2\frac{9}{9}$, *and when you simplify* $\frac{9}{9}$, *you get* $\frac{1}{1} = 1$. *Then 2 + 1 results in 3.*"

Christian: "*The periodic number* $2.\overline{9}$ *is equal to 3, because the period is going on infinitely, and therefore, it has to arrive at the number 3 eventually.*"

Katerina: "$2.\overline{9} < 3$. *There is always 0.1 missing. There is no proof. I assume that it is only accepted to be able to compute more easily.*

i.e. 2.99, there is 0.01 missing; 2.9999, there is 0.0001 missing; etc.

The number $2.\overline{9}$ *cannot be equal to 3, because there is ALWAYS something missing. When there is no space between* $2.\overline{9}$ *and 3 then* $2.\overline{9}$ *jumps to 3.0, the next number.*"

It is not at all clear that both boys have a better idea of the complex construction of infinite periods than does the girl.

Example 3: Expressions (grade 7):

In a grade 7 project class the three solids given in figure 4 were presented.

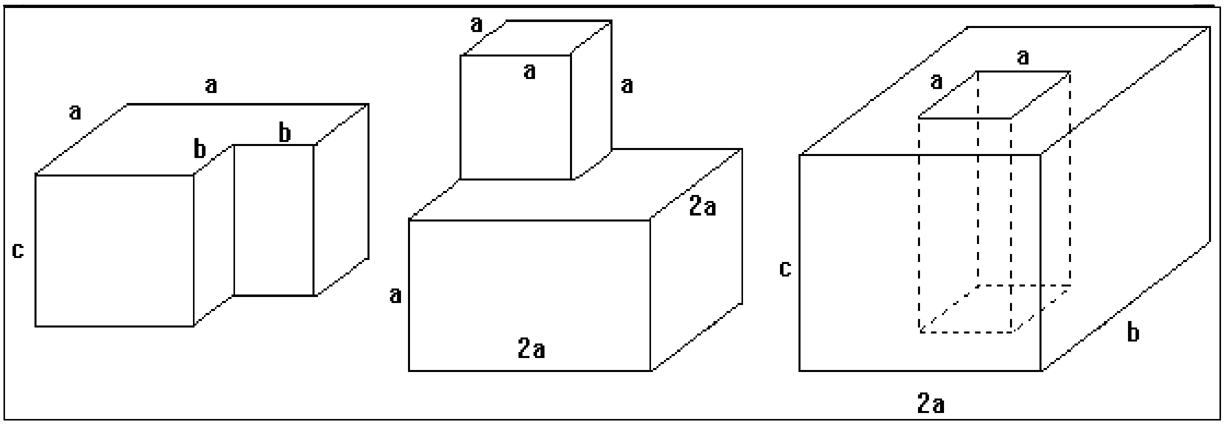

Figure 4

The class agreed to tackle the task of determining surface areas and volumes of the solids individually or in groups. Students started to work and results were written (without comment) onto the blackboard. After some time there were up to eight different expressions for the sought quantities. Now a discussion started about who had found the correct solution and obviously the teacher did not intervene. Some expressions, for example including a term a^6, could be sorted out and found incorrect with respect to the unit. For the remaining terms it was not obvious whether they were correct or incorrect. The students tried intensively to compare the expressions, which provided excellent motivation for finding strategies for the manipulation of expressions. To calculate with the children was of great value also for the teacher: "I was very content how the lesson developed. I gained insight into the cognitive structure of students".

Productive practice

E. Ch. Wittmann describes the didactics of mathematics as a *design science* which develops and researches "productive learning environments". Problems with rich content are worked on holistically. This is presented exemplarily in both *Handbooks of Productive Arithmetic Practise* (Wittmann/Müller 1990/1992) for the four elementary school years. The single learning sections create meaningful relations and propose problems of different degrees of difficulty, leading to a natural differentiation. In contrast to the usual step-by-step teaching, not all obstacles are removed. Pupils gain experience in using 'common sense' and are challenged to think about problems on their own, to judge their own considerations and to test whether they make sense.

One example for a productive learning environment is the following sequence of problems on number walls, developed by a colleague for his grade 5 class:

Example 4: For the empty number wall given in figure 5 the following questions were asked:

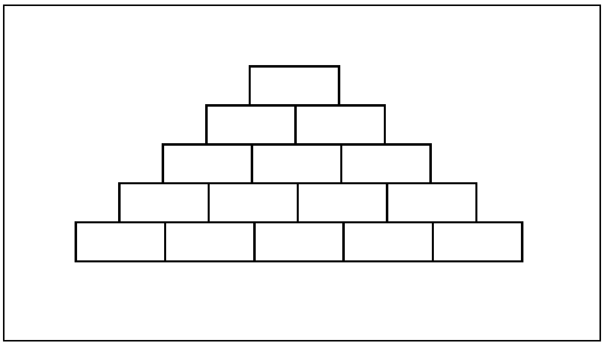

Figure 5

Can you build number walls in each case?

- *In the first line write down any five numbers you like.*
- *Write down only four numbers in line one.*
- *There are only odd numbers in line two.*
- *At the top is a number close to 500.*
- *At the top is exactly 500.*
- *Can you find several walls with 500 at the top?*
- *Is it possible that there are only numbers divisible by three in line three?*
- *Can you write down five numbers in any space and still complete the number wall?*

Number walls are a tried and tested exercise format which is introduced in elementary school. The sum of two adjacent spaces will appear in the space above. This exercise format has also been successfully used for other number spaces and operations, as well as for variables. More complex questions can be easily posed. For example, write down the unit fractions $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ ... from top to bottom on the left side of the wall. Then all other places can be filled unambiguously. What do you observe?

Standard problems, too, can often be improved ‘productively’, by taking away a restriction, by opening up a too rigorous problem statement or by posing the problem rather vaguely. Some examples from our project:

Example 5: In the standard problem

Arrange the following numbers according to their size

$$-1;\ \frac{2}{3};\ 3.\overline{9};\ -\frac{143}{6};\ -184.76;\ \frac{14}{7};\ 8.23;\ -5\frac{2}{9}\ ,$$

the words “according to their size” were omitted in a grade 7 project class. This resulted at first in a creative restlessness among the children - what could be meant by “arrange”? They were used to convergent, unambiguous questions. Only after the teacher pointed out that there could be more than one solution (without, of course, mentioning them) the children started to work and arranged the numbers according to positive/negative, number space, or size.

Example 6: *Represent the numbers from 0 to 10 with exactly four fours, the four operators +,*

-, ·, ÷, and brackets.

For this problem, pupils have to be creative and use the number operations; it is not enough to know them by heart. And they practice too!

Example 7: *Choose four fractions. Use them to make expressions as large as possible (as near as possible to 1, ….).*

Example 8: *Calculate some powers that you like.*

It is typical in such problems that pupils work intensively and offer many ideas, but naturally make mistakes too. However they realize and correct their mistakes. It is time well spent because children are involved quite differently. Emotional AND cognitive aspects are addressed.

Often only standard knowledge and skills are applied to standarized question types. If practice takes place in such a way, thinking and computing are separated. A problem's setting should be looked at from different perspectives. Inverse problems are often only superficially easier, but illustrate missing basic concepts. Here are some tried and tested examples from our project:

Example 9: *State two different fractions between 6/17 and 7/17 or explain why there are none.*

Example 10: *Find all solutions or explain why there are none:*

$$6 \quad (20 - ?) = 144; \; 7 \quad (12 + ?) = 100$$

Example 11: Better than the convergent question "*7 + 5 = ?*" which asks for the synthesis of twelve, is the divergent question "*Which is the most beautiful twelve?*" asking for the analysis of twelve and resulting in many correct and important answers, i.e. 12 = 11 + 1, 12 = 6 + 6 or 12 = 1 + 2 + 3 + 3 + 2 + 1.

Example 12: *Supplement the following equation in such a way that you can solve it:*

$$x^3 + ... = 0.$$

The answer, which is obvious to us, is to insert 0 and arrive at the cubic equation $x^3 = 0$ but this was rarely chosen!

Example 13: *By how many degrees can you rotate the normal parabola, which is the graph of $y = x^2$, around the origin, so that the resulting parabola still is the graph of a function?*

These illustrative examples show that only by choosing another formulation in standard problems can new aims be addressed to further creative ideas, to differentiate the possibilities, to order, and to classify.

Assessment: Measuring and feedback of gain in competence

It is important to differentiate between learning and assessment situations. Understandably pupils try to avoid failure in assessment situations. Nevertheless, problems that are open and ask for personal decisions have to be included in tests. In our experience pupils did not see these problems as something new because firstly they were used to such problems from their lessons and secondly these questions were included sensitively in test situations. In particular, one can not force creativity in relatively little time and under stress. However, some test problems from our project classes (grades 5) show what is possible.

Example 14: *Armin buys a rubber and two pencils from Beate. The rubber costs DM 2, each pencil DM 1. Beate asks for DM 6, and Armin protests. Then Beate writes down an expression and explains her calculation to Armin. Then Armin understands. He tells Beate that the term is correct but that she has broken a rule in her calculation. Armin pays the correct price.*

Which expression did Beate write down and what rule has she used incorrectly?

Example 15: *In the following problem use brackets in order to get a result as big as possible and then calculate: 6 + 4 7 – 5 2*

Example 16: *Form expressions with the numbers 24, 9, 8 and 5 and calculate them. For at least three of the expressions the results should be between 0 and 10. For at least three of them the results should be between 100 and 110.*

Naturally, with this type of question, basic arithmetic knowledge is checked, but in addition, algebraic competence with expressions is necessary. The important computation rules are used independently rather then merely checked out of context.

Example 17: *Write down a problem which includes 2 kg 500 g and 10 days. Then solve your problem.*

Contrary to usual expectations children do not always solve their self-posed problems correctly. However, self-posed problems are worked at with more interest and commitment. Examples show how highly imaginative the children were:

- *Laura wants to lose 2kg 500g in 10 days. How much weight will she lose in one day?*
- *A certain monkey in Frankfurt Zoo eats 2 kg 500g of bananas each day. What weight of bananas does it eat in 10 days?*
- *Tim, a 10 month old baby, weighed 2 kg 500 g 10 days ago. Now he weighs 4kg. How much weight did he gain?*
- *A baby weighs 2kg 500g. What will his weight be in 10 days?* (here the result given was 25 kg!)
- *Katrin's pony weighs 2kg 500g. The pony eats 1kg500g in one day. How much will it weigh when it eats 1 kg 500 g for 10 days?*

Discussion of the answers is also one important preliminary stage for mathematical modelling.

Example 18: *Supplement:*

a) 7   (50 + [ ]) = 350 + 28;   b) 7   14 + [ ] = [ ]   (14 + 6).

Whereas for a) the solution 7 (50+4) = 350+28 is unique, for b) naturally a solution 7 14 + 7 6 = 7 (14 + 6) was given, triggered by the distribution law. In reality, the equation 7 14 + a = b (14 + 6) has the natural number solution a = 2 (10b-49) for every natural b 5. In a few cases further solutions where found by trying out different numbers, which, of course, is a creative, original achievement.

Experiences from the project

In posing more open problems one has to take starting difficulties into account. Pupils tend to ask for a recipe, and are at first unsure. They often are afraid of doing something wrong and do not start at all but, after some time, their attitude changes. “During the school year insecurity grew less, pupils sought more and more problem solutions on their own and accepted that there is more than one way to reach the solution”, reported two of the colleagues involved. One could observe a growing familiarity with more complex, more open problem statements. “The wide range of solutions was only possible when I myself as a teacher retreated at the decisive moment – left the problem completely to the class and did not break the problem down into bite-sized pieces by questioning-answering techniques until they were convinced that the problem could be solved only with a linear equation. Is that not what often goes completely wrong in mathematics teaching?”

The increase in creative and heuristic abilities is hard to measure. Commonly it was reported that problems were increasingly dealt with as a matter of routine, with perseverance instead of resignation. All involved were convinced that pupils gained metaknowledge rather than a collection of easily accessible but quickly forgotten information. The deliberate change in the teacher rôle (to stay back in the working and solution phases, to challenge and to accept solutions, to encourage alternatives) was not restricted to project classes only!

Some problems of realisation

Some problems which occurred should not be concealed:

- The principle expressed in Matthew’s gospel, “to him that has, shall be given” leads to the unavoidable problem of the “scissors” which now emerges at a earlier stage than previously. However, both shanks of the scissors can move. Naturally some parents blamed their children’s failure on the ‘new’ style of teaching.
- The problem of a lack of orientation towards achievement in Germany (which relates to pupils as well as to teachers).
- Practice and teaching methods used in school have to be experienced by teachers in their own university education and this happens rarely or not at all.
- Only teachers who are pursuing mathematics themselves can succeed in sparking off pupils’ interest, have them enjoy puzzles and encourage them to achieve something.
- The “points-orientation” of the centrally-posed Abitur (final examination at the end of upper secondary level) results in too much rote learning, too little deep thinking. The problem sets of the central exams become a hidden curriculum.

The WUM-inservice teacher education

The experiences gained up to now in the 18 project schools have resulted in a newly developed regional inservice teacher education "***W***eiterentwicklung der ***U***nterrichtskultur im Fach ***M***athematik" (abbreviated to WUM, that is to say, development of the teaching culture in mathematics) for schools of all types. Schools themselves request that an inservice teacher education team comes to their school. In one whole and three half days firstly problem awareness is generated followed by a number of short presentations introducing the new methods (productive practice, opening and variation of problems, open-ended approach, non-routine examination problems). The main work is then the independent preparation of teaching material for their own lessons. The tremendous demand shows the great interest of our teachers.

Conclusion

Obviously our experiences from the BLK project after less than two years are not so reliable that we could deduce definite conclusions. It is certain that the teaching climate and the active participation of most learners have improved decidedly. We have reason to believe that through open, more challenging work and innovative problem styles, improved basic concepts will be developed.

References

BLK 1997 Gutachten zur Vorbereitung des Programmems "Steigerung der Effizienz des mathematisch-naturwissenschaftlichen Unterrichts". – Heft 60 der Materialien der Bund-Länder-Kommission für Bildungsplanung und Forschungsförderung Bonn

BLUM, W. U. M. NEUBRAND (Eds.) 1998 TIMSS und der Mathematikunterricht. – Hannover: Schroedel

Henn, H.-W. (Ed.) 1999 Mathematikunterricht im Aufbruch. – Hannover: Schroedel

SELTER, CH. U. H. SPIEGEL (1997) Wie Kinder rechnen. – Klett, Leipzig 1997

WITTMANN, E. Ch. u. G. N. MÜLLER 1990/1992 Handbuch produktiver Rechenübungen.. – Klett, Stuttgart, Band 1 und 2

Prof. Dr. Hans-Wolfgang Henn
University of Dortmund
Department of Mathematics
Institute for Development and Research of Mathematics Education
Vogelpothsweg 87
D-44221 Dortmund, Germany
E-Mail: Wolfgang.Henn@math.uni-dortmund.de