Wilfried Herget, Halle (Saale)

"PICTORIAL PROBLEMS"
ONE QUESTION, BUT MANY WAYS, AND MANY DIFFERENT ANSWERS

Abstract:
"Maths means calculating!" – All right, but that is certainly not the whole story: there is far more to mathematics than that!
The aim of maths lessons is to teach pupils how to think mathematically, how to translate into the powerful language of mathematics, and how to find different suitable solutions. In this article, some unusual open-ended problems are presented which may be used in secondary school.
Here, calculating is not the main focus of attention, but rather all the steps before the calculations: "Here is a situation. Think about it!" (Henry Pollak) The true value of such a problem and its solution lies in the pleasure you have derived from courageously taking your own steps, from being creative and bold in search of the right answers, and from experiencing what it is like to find a rough solution by yourself, instead of reverently looking up the answer, or getting somebody else to work it out for you.

The person who fits this giant shoe must have enormous feet!
Antal Annus, a 73-year-old shoemaker from the Hungarian village of Csanádapáca, is depicted here, proudly presenting his hitherto most impressive "creation". To this very day, we still do not know whether he really made the shoe for one of his customers.
Goslarsche Zeitung, 7.1.1995

What size is this giant shoe?

In some of my lectures and seminars, and on teacher training courses, a newspaper article depicting a giant shoe (see figure above, HERGET; STUCK) is used as a starting point: “What size is this giant shoe?” Everyone seems to find a task like this rather unusual, and I am always intrigued to hear the different ways of solving the problem.

Many different ways of solving the problem

The standard approach is to use the man in the picture’s glasses as a yardstick, or his head, or the width of the apron he is wearing: it is quite easy to measure these things, both on the picture and in reality. A few simple calculations suffice to give us the length of the shoe.
Now we have obtained the length of the shoe, but we still do not know its real size! Have you ever thought about the relationship between the length of the shoe, and the various parameters indicating a shoe’s size? That might well turn out to be an interesting research project!
A female colleague, who also teaches art, came up with another idea about how to solve the problem at hand: she said: “A shoe is about the same length as a human face!” Assuming 41 to be the standard size shoe in Europe, we simply have to do our sums, providing the ratio: *shoe length to shoe size* is linear.
A school-girl came up with a third, quite fascinating solution: imagine we turn the shoe at 90 degrees around the man’s belly button, then we’ll discover that the shoe is a little smaller than the man. If the man is about 1.70 m tall then the shoe must be approximately 1.5 m in length.
A fourth possible solution, which is rather similar, was put forward by two pupils who conveyed their idea very clearly using body language: if we imagine the man in real life, then – with his arms stretched out – he spans at least the length of the shoe. In the case of an average-sized human this would be about 1.60 m. Therefore, in reality, the shoe is approximately 1.5 m in length.
But how reliable are the different approaches to the problem? The results lie somewhere between 1 m and 1.5 m. So how accurate are, in fact, the various measurements and estimates? In the end, a critical comparison of each method might well reveal a slight difference, but we still haven’t come up with “the right solution”!

Finally, our task is to look at the relationship between normal shoe sizes, and the length of the foot in centimetres. So, where do we start? One way would be to collect data by measuring various shoes. (Measuring big and small shoes lends itself well to homework – there are bound to be some “giants” and “dwarves” in the family and neighbourhood.) Another possibility, which is rather unusual in maths lessons, would be to make some enquiries in local shoe shops. If we are lucky we might find some tables on the shoe boxes. Finally we will get a type of relationship *normal shoe sizes* ↔ *length of the foot in centimetres* and some respective formula.

“Maths means calculating!”

“Maths means calculating!” – All right, but that’s not the whole story: there is far more to mathematics than that! A typical feature of a set task is that calculating is not the main focus of attention. Instead, we are required to “think mathematically”, and come up with a suitable strategy for solving the problem: “Here is a situation. Think about it!” (Henry Pollack). In the end, we do not produce the perfect answer in terms of: “The shoe is size 127 ½, or the length of the shoe is 197.386 cm.”. Setting such a task and solving a problem of this nature in a maths lesson is very unusual, both for the pupils and for the teachers. Here we aren’t talking about small pieces of food, which are just the right size and can be spoon fed, bit by bit, in the right order. There is no unique, perfect result. However, it should be borne in mind, of course, that it is basic mathematical considerations which enable us to calculate the size of the shoe roughly – without which we wouldn’t stand a chance.

Very precise … and very rough!

In maths lessons in the classroom, accuracy tends to dominate: “If $a = 4$ cm, $b = 7$ cm, and $c = 5$ cm (of course, very precisely!) then how big is V?”
It goes without saying that this precision has its value: using precise data in maths lesson, one can explore and develop ideas, and, above all, practise solving problems in the classroom. This is tiring enough. However, in this way, an imaginary world is conjured up in the minds of the pupils: if, after long calculations involving fractions and square roots, the result is a whole number, then this indicates that the pupil has done the calculation properly. He or she is not at all bothered by the fact that the set task didn’t initially look like it would yield such a beautiful result. (SCHEID 1994, p. 179)
Maths lessons are characterised by precision and “keeping on the safe side of things.” But this obsession with accuracy “flies straight out of the window” the minute mathematics becomes involved with the “rest of the world”. Then, most of the numbers which crop up are only approximately correct. This is inevitable and unavoidable! Likewise, the results are only rough estimates.

We can describe the constellation of the stars quite precisely; we can find out about the waxing and waning of the moon, and the rising and setting of the sun from the newspaper. We refer to a calendar if we want to know when there will be a full or new moon. Eclipses of the sun and moon are predictable. However it is absolutely impossible to be precise if we are forecasting the weather or the exchange rate the next day. In the long run, population and raw material prognoses are only roughly correct due to the difficulties with data and the numerous factors influencing the whole situation. The meteorologists, with their climatic models, who are constantly competing with each other, have to correct their somewhat contradictory predictions every few years. In the case of economic prognoses for the coming year, whole teams of experts frequently make mistakes.
Maths lessons, which aim at meeting the requirements of general education, prepare pupils to approach everyday facts and figures critically, and to deal with a whole host of data and predictions frequently encountered in our everyday lives.

A picture tells a story of well over 1,000 words!

In our lessons, one of our tasks is to bridge the gap between two different worlds: the accuracy so typical of mathematics, and the lack of precision in the rest of the world. This is imperative because both worlds are important, and both worlds are indispensable. How can we possibly learn the true value of the precision and certainty of mathematics if we have not yet learnt that, in the rest of the world, this precision and reliability is something which is very difficult to achieve? On the other hand, one can only learn to cope with this inaccuracy and blatant lack of precision as well as possible, if one has learned to exploit the many possibilities offered by the very precise field of mathematics.

So how is it possible to bridge the gap between mathematics and the “rest of the world?” At this point I propose a very special method: setting tasks which are mostly based on rather unusual newspaper cuttings and which I am apt to call “Pictorial Problems”. Tasks, which are based on a real-life situation, are often far too cluttered up with text to be truly effective. Describing a real-life situation inevitably involves using an abundance of words and information. This is where the picture, supplemented by the pupils’ general knowledge and imagination, comes in handy: “A picture can say far more than a thousand words!”

Although, in itself, a fairly simple situation, our starting point is, indeed, mathematical. Here, our main aim is to deliberately make *creating mathematical models* the focus of attention in our lessons: “The problem situation must be simple if the pupil is to develop models by himself successfully – far simpler than when he has the model provided for him by the teacher.” And “Only mathematics that is very well absorbed seems to be usable in modelling.” (BURKHARD 1981, p. 15).

The Adenauer Memorial
by the artist Hubertus von Pilgrim.

GENERAL-ANZEIGER,
Bonn, 15.6.1999

This memorial features the head of Konrad Adenauer, who was the first president of the F.R.G. from 1949 to 1963.

▶ **How big should a memorial be if it is to represent Adenauer from 'top to toe' on the same scale?**

The Adenauer Memorial "from top to toe" (Task according to HERGET; JAHNKE; KROLL)

What could be used as a yardstick? It's the children in the photo, of course. Therefore, there are at least two different ways of answering this question.

Solution 1

First, we work out the photo's proportions: the girl on the left is approx. 8 cm long. So, how big is the girl in reality? How old is she?
Once again, you are required to demonstrate your general knowledge, and a certain willingness to compromise – let us say she is 1.30 m. Then we have to multiply the measurements, taken from the photo, by approximately $130 : 8 \approx 16$ in order to come up with an answer which is anything like the real measurements.
On the photo of the memorial, Adenauer's head is 9 cm high; therefore, in reality it is about $9\text{ cm} \cdot 16 = 144\text{ cm} \approx 1.4\text{ m}$ high.
Then how big should the memorial be "from top to toe"? Again we have to resort to our general knowledge. In adults, the ratio: *length of body to height of head* is roughly the same. In the case of children, this ratio is somewhat smaller as children have relatively big heads.
If the teacher gives us permission we can take some measurements and ascertain that adults are roughly 7 times as long as their head.
Therefore, the total height of an intact Adenauer memorial is $1.4 \cdot 7 \approx 10$ m.

Solution 2

Instead of working out the photo's proportions, we now determine the ratio of the memorial's head to that of an adult. It is advisable to take the head of the boy at the base of the memorial, or the head of the girl, as a yardstick. Once again we have to resort to our general knowledge. How much bigger is a typical adult head?
In class we then take some measurements. We measure our teachers, and, thus, calculate the factor by which an adult's head is bigger than that of the girl or boy depicted in the picture. Naturally there are bound to be heated discussions about the possible results, and, here, it is imperative to reach a compromise. Therefore we'll say that an adult's head is probably about 1.1 times the size of the girl or boy's head.
On the photo, the boy's head is about 1.2 cm, and, if we measure from the chin to the parting, the head of the memorial is about 9 cm. Thus we get a magnification factor of

$$k \approx \frac{9\text{ cm}}{1.1 \cdot 1.2\text{ cm}} \approx 6.8\,.$$

So how tall is an adult? Let us assume that an adult is 1.80 m. Hence an Adenauer memorial would be $1.80\text{ m} \cdot 6.8 \approx 12\text{ m}$ "from top to toe". This result is different to the first result, calculated using the first method. So, who is right? Which is the correct result?

Now we have a rather unusual situation in our maths lesson. This will certainly take some getting used to – both on the part of the teachers and the pupils.
In fact, *both* answers are correct because in the light of unavoidable inaccuracies, both solutions: "approximately 10 m" and "approximately 12 m" are indeed very close to each other. We can calculate an "interval of tolerance" for any solution by fixing a lower and an upper limit for each step, and then doubling the result – a good mathematical exercise which,

undoubtedly, requires some thought because it involves both multiplication and division! So, according to our final result, an intact Adenauer memorial would be roughly 10 m tall.

Finally, a few more questions may well crop up – some in the field of mathematics and some in other fields (see also HERGET; JAHNKE; KROLL):

- Are there really any great memorials depicting a huge person from "top to toe"?
  Yes of course there are: the New York "Statue of Liberty" is about 46 m from the foot of the statue to the tip of the torch. The figure of Jesus on the sugar-cone in Rio de Janeiro is just under 30 m. The Colossus of Rhodes – one of the "Seven Wonders of the World" in the days of Antiquity – was some 30 m. Built from 292 to 280 B.C., it was destroyed in 227 B.C. by an earthquake.
- How much wool would we need for the right size woolly hat?

Depending on the given situation, individual pupils or groups can further pursue these questions, and later present them in class. The results can then be displayed on a wall frieze or in a school exhibition.

On Saturdays and Sundays this magnificent hot-air balloon, from the twin-town of Oldenzaal in the Netherlands, can be seen soaring into the air at Osterfeld.
If you are interested, the departure times are displayed on a stall on the market-place.

extra Wochenblatt, 18.3.1999

▶ **How many litres of air does this hot-air balloon hold?**

How many litres of air does this hot-air balloon hold? (Task according to HERGET 2000b)

Of course, first of all a model should be made – as accurately as possible – of the hot-air balloon with the help of an object which is easy to describe. The wider the range of mathematical instruments available, the more instruments can be used to solve this task.
In a calculus course, the interpretation of the set task would be based on rotated solids. Modern pocket calculators, with built-in programs for regression analysis, have no limits whatsoever with regard to the type of function. If the resultant integral is difficult to solve, then, of course, we can solve it numerically with the help of a pocket calculator. But do these extremely precise mathematical models and instruments produce precise answers?
In this case – as in many other cases – there is a much simpler solution. We simply take a look at the familiar geometrical shapes in our model building kit, and select something suitable.
So, what would be suitable for our task? What would fit our needs?

Solution 1
A model is made of the upper section of the hot-air balloon using the shape of a hemisphere. A model of the lower part is then made using a cylindrical cone.
Well-known formulae for the volume of a hemisphere and for the volume of a cone are, respectively, $V = \frac{2}{3}\pi \cdot r^3$ and $V = \frac{1}{6}\pi \cdot r^2 \cdot h$.

Now, how big are r and h? Our only source of information is the photo. Using it we try to guess the size of the balloon, and, thus, calculate its real dimensions. The people in the photo may be used as a yardstick. Most suitable are those people standing directly beneath the basket of the hot-air balloon. First we measure the height of an adult person in the photo – about 1.1 cm. then the diameter of the balloon at its widest point – about 9.5 cm. Now we must use our general knowledge i.e. the average height of an adult is 1.80 m.

Thus we already have the first of our two values: $r \approx \frac{1}{2} \cdot \frac{9.7\text{ cm}}{1.1\text{ cm}} \cdot 1.8\text{ m} \approx 8\text{ m}.$

Now we have to measure the height of the cone on the photo. This is not so easy because the photo was not taken from the side, but from below, at an angle. One way of solving the problem is to measure the height, up to an imaginary diameter of the balloon at its widest point. In the process, our view will certainly change. Thus the measurements vary between 7 and 8 cm. so we'll say 7.5 cm. The balloon is also a little bit wider in the middle than in the model with the cone. On the other hand, the balloon does not have the tip, featured in the model. Therefore our results should balance out a little.

Hence the result is roughly: $h \approx \frac{7.5\text{ cm}}{1.1\text{ cm}} \cdot 1.8\text{ m} \approx 12\text{ m}$.

When taking measurements, we are obviously very much aware of how inaccurate these values are. There is little point in making a note of the figures which appear after the decimal point as shown on the pocket calculator. Perhaps, in a second step, we could perform some calculations using upper and lower values, and thus come up with a kind of "interval of tolerance" as an answer. Finally, we obtain the following result for the total volume of the balloon:

$$V = \frac{2}{3}\pi \cdot r^3 + \frac{1}{6}\pi \cdot r^2 \cdot h \approx \frac{2}{3}\pi \cdot 8^3\ \mathrm{m}^3 + \frac{1}{6}\pi \cdot 8^2 \cdot 12\ \mathrm{m}^3 \approx 1100\ \mathrm{m}^3 + 300\ \mathrm{m}^3 \approx 1400\ \mathrm{m}^3.$$

But, in the light of the evidently unavoidable inaccuracies when measuring, estimating and modelling we cannot possibly justify anything more precise than V = 1,500 cubic metres.

Solution 2
It is possible to make an even simpler model of the balloon: let us take a big sphere as a suitable substitute. This is chosen by eye so that the lower part protrudes a little bit over the balloon. In the middle it goes around the inside of the balloon to create, as far as possible, a suitable volume equilibrium. The radius of the balloon may be determined from the picture. When measuring the diameter of the substitute sphere, it becomes quite clear that there is considerable leeway for taking these measurements – i.e. they are between 8 cm and 9 cm.
In the end we may have to opt for 8.5 cm. This is how we obtain the real radius:

$$r \approx \frac{1}{2} \cdot \frac{8.5\ \mathrm{cm}}{1.1\ \mathrm{cm}} \cdot 1.8\ \mathrm{m} \approx 7\ \mathrm{m}$$

and, finally, for the volume of the sphere:

$$V = \frac{4}{3}\pi \cdot r^3 \approx \frac{4}{3}\pi \cdot 7^3\ \mathrm{m}^3 \approx 1400\ \mathrm{m}^3.$$

In this case, too, any result which is more precise than V = 1,500 cubic metres would be very difficult to justify in the light of the inevitable inaccuracies involved.

Solution 3
There is an even simpler way of solving this problem: let us envisage a cube, the corners of which are hanging over the hot-air balloon, and the surfaces of whose sides are partly dipping into the balloon. Here the surplus volume and the volume deficit should balance each other out.
In my own experience, teachers tend to find this simple form of mathematical modelling rather unusual. In our modelling kit, there are far more ‘sensitive’ geometrical substitute models. But behind this simple idea there is an extremely fundamental mathematical idea with far-reaching consequences, e.g. the approximation of curves, surfaces and solids by straight lines, rectangles and cuboids – compare, for instance, with the Archimedes’ approximation for the area of a circle, and the Riemann’s sums for the integral.
So, let us boldly insert our substitute cube into the photo! How big would the lengths of its edges be on the photo? Let us say roughly 7 cm. Therefore, the real lengths of the edges are:

$$a \approx \frac{7\ \mathrm{cm}}{1.1\ \mathrm{cm}} \cdot 1.8\ \mathrm{m} \approx 11.5\ \mathrm{m},$$

and the volume of the die is: $V \approx 11.5^3\ \mathrm{m}^3 \approx 1500\ \mathrm{m}^3$.
Of course anything more precise than V = 1,500 cubic metres would not be such a good idea in the light of the unavoidable inaccuracies.

Sometimes we have to calculate a little more, and sometimes a little less. But, in every case, no matter how we adjust our considerations to fit the given situation, all these models yield a volume of approximately 1,500 cubic metres.

Other questions directly related to this problem might be:
1. Approx. 1,500 cubic metres – How many litres is that?
2. How long would we have to pump it up if we used a bicycle pump?

3. How many m^2 is the area of the hot-air balloon's cover?

… and pupils may use the Internet to find out some technical data pertaining to hot-air balloons.

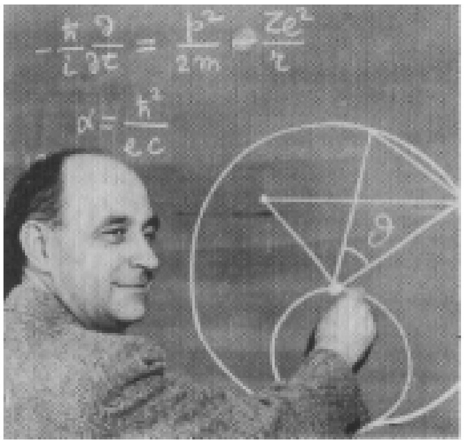
Different ways but many common ideas

When solving such mathematical problems, the following steps are usually taken:

- The real situation remains the focus of attention and, in the process of trying to find a solution, it must be kept in mind. It certainly does not only serve to disguise the calculation model from the last lesson.
- The material, in the form of a picture, is analysed, the points which are mathematically relevant are extracted, and the sometimes interesting but mathematically irrelevant information is "put aside".
- You must look for a suitable *yardstick*, e.g. the child next to the head on the memorial or the people standing next to the hot-air balloon.
- Photos are used to take interesting measurements. Taking your own measurements helps you to become aware that inaccuracies are practically unavoidable.
- The pupils' general knowledge is activated, e.g. "How big is the girl, the boy, a grown-up?" "How wide are the aprons, glasses, etc.?" It might well be necessary to get information from other sources, e.g. "What about the relationship *normal shoe sizes* ↔ *length of the foot in centimetres*?"
- The relationships between the selected dimensions, and the sizes being sought, are described more precisely and the correct mathematical terminology is used.
- Thus a suitable mathematical method is not simply presented on a plate. It is necessary to search for this method.
- If need be, problems are simplified, e.g. "Are the girl and the boy roughly the same size as the children in our class?" "Is Konrad Adenauer about the same size as our teacher?"
- A simple mathematical substitute model is chosen (a cone, a sphere, or even a die may be taken as a substitute for the hot-air balloon).
- In the problem-solving process, other closely related questions often arise – some in the field of mathematics, and some in other fields. These questions should really be further pursued, depending on the classroom situation, e.g. "Is there such a thing as a huge memorial?" "How much wool is needed for a woolly hat which fits properly?" "How big is the area of the cover of the hot-air balloon?" "If we used a bicycle pump, how long would we need to pump it up?"

Today the authority of teachers is certainly being eroded due to computers and the advent of the Internet. Teachers, rather than being responsible for conveying knowledge, are now responsible for presenting it. It is becoming increasingly important for pupils to know how to deal with a whole wealth of information, to know how to do basic research, and be able to differentiate between what is important, and what is unimportant. Pupils need to learn how to design investigations, skilfully compile information for other people, and collect and apply self-acquired knowledge. Of course, there is no such thing as a correct method, nor is there such a thing as a correct answer.

Fermi Problems



Enrico Fermi (born on 29.9.1901 in Rome, died on 28.11.1954 in Chicago) was awarded the Nobel Prize for Physics in 1938. He discovered new radioactive elements, and nuclear reactions, triggered off by slow neutrons. At the end of World War II he was involved in the discovery of the American atom bomb, but afterwards became a fervent opponent of nuclear weapons. Above all, he actively opposed the making of the hydrogen bomb.
Enrico Fermi preferred direct, improvised and bold solutions rather than sophisticated, complex, and time-consuming methods. In order to teach this approach to his students he devised quite unique sorts of tasks, henceforth known as "Fermi problems".

"How many piano tuners are there in Chicago?" This is probably Fermi's most famous task. At first you haven't the slightest clue what the answer might be. In any case, you are pretty sure that you don't have enough information to be able to solve the problem. However, if you start to tackle the problem, and make a few bold guesses, then you very soon come up with a rough answer, step by step:

- How many people live in Chicago? Chicago has a population of some 3 million people. You may get this information from an encyclopaedia or from the Internet.
- There are about three people in an average family. How many of the families will have a piano? Maybe every tenth family?
- ... hence, there are approximately 100,000 pianos in this city.
- How often is it necessary to tune a piano? Maybe every tenth year?
- ... then that makes 10,000 tunings per annum.
- How many can be tuned by a single piano tuner per day? Maybe four pianos a day? Not forgetting that he or she has to get to the respective pianos …
- How many days of the year will a piano tuner work? Maybe 200? It should be not more than 300 days. Therefore maybe 250?
- ... then on 250 working days with 4 pianos he or she manages to tune 1,000 pianos (we are lucky in having chosen these nice numbers) ...
- ... therefore, there must be about 10 piano tuners in Chicago.

Yes, you are quite right, the answer is not particularly accurate. The answer could just as well be only five, or even 100 piano tuners. But, nevertheless, we should appreciate this experience, that, following different ideas, we can make rough guesses which are all "about right".

"How many hairs are there on a human head?" Many schoolboys and schoolgirls have had hours of fun toying with this Fermi problem: usually, the part of the head covered in hair is regarded as a hemisphere, and by placing a ruler on their neighbour's head, the pupils roughly calculate the number of hairs per square centimetre.

"How much paper is used in our school per month?" "How much water is used by a pupil per week?" Problems like these could be tackled both at primary school and also at higher grades (PETER-KOOP 1999a, b).

It was Archimedes of Syrakus who tried to calculate the number of the grains of sand which fit into the universe – to refute those who claimed there was an infinite number. Here you have some more problems (see also HERGET 1999b):

- How many balloons fit into the classroom?
- How many marks per annum will be given at all the schools throughout the country?
- How much money will be spent per annum on coaching, on CDs, on ...?
- How many kilometres will be covered by all the parents of our school in carrying their children by car to school?
- How many square metres are all the classrooms of our country all together?
- How long would the strip be, if you squeeze a tube of toothpaste empty?

Showing the diversity of mathematics

The different processes pupils (and their teacher, too) go through in search of a solution to these problems are very exciting. They make their way boldly towards the solution, one by one, in pairs, or in small groups instead of reverently looking up the suggested answer, or getting somebody else to work it out for them. Bursting with creativity, step by step, they search for a solution, argue about the effectiveness of their method, laugh at blatant mistakes, and are disappointed about unexpected cul-de-sacs. When criticised, they fiercely defend their own methods, discover (when it is perhaps already too late) a way to save time, and can, finally, enjoy the most rewarding experience of all – namely managing, at long last, to find a rough answer by themselves.

Here, mathematics proves to be a very diverse subject. It must not be perceived as merely being confined to finding “the correct solution”. For every single step taken in an effort to find a solution to the problem at hand, there are many different possibilities – not only for translating the initial situation into the language of mathematics but also for finding different ways within the field of mathematics itself.

To accomplish this aim successfully, the pupils must have enough leeway for being creative, and the teacher must adopt a friendly approach to errors to enable pupils to find a solution to problems on their own, and share this exciting experience with others. In this way they develop the ability to make constructive criticism thanks to a healthy dose of self-confidence.

Modelling as a central theme

The discussion of all these points underlines what is really imperative for the creation of mathematical models: the translation of a question from “the rest of the world” into the language of mathematics certainly requires simplification. We constantly have to ask ourselves “What is suitable?” and “What are the consequences of this simplification?”

The focus of attention is not so much on the formulae and the calculations but rather on the different steps leading up to the calculations: “Here is a situation. Think about it!” (Henry Pollak)

Nowadays, tasks requiring pure technical calculation can be solved with the help of a pocket-calculator or computer software. Consequently more demanding activities are gaining importance e.g. the analysis of problems affecting the “rest of the world”. Equally important are clever translations of the task into the language of mathematics, into fitting a mathematical model, internal treatment of this problem in the field of mathematics right up to its solution (or several solutions), and, finally, a deliberate interpretation and critical examination of the results obtained: “Has our original question really been answered? How accurate and how reliable is the result?” Thus, using carefully selected examples, the typical process of mathematical modelling may become a central theme in the classroom, including both modelling methods and the accuracy of the mathematics involved – of course without losing sight of the discrepancy between the mathematical model and reality.

Carry on collecting and compiling

Additional information and many further examples are available under HERGET 2000b and HERGET; JAHNKE; KROLL. But, dear colleagues, you will surely find up-to-date pictures, perhaps even in your local newspaper, featuring events which are of interest to the pupils, and are closely related to *their* world.

References

BURKHARDT, H. 1981: The Real World and Mathematics. Glasgow: Blackie.

HERGET, W.; STUCK C. 1996: Wie groß sind Sieben-Meilen-Stiefel? In: *mathematik lehren* 74, pp. 19-21

HERGET, W. 1999a: Ganz genau – genau das ist Mathe! In: *mathematik lehren* 93, pp. 4-9.

HERGET, W. 1999b: Ungefähr ... richtig! Mathe-Welt. In: *mathematik lehren* 93, pp. 23-46.

HERGET, W. 2000a: Rechnen können reicht ... eben nicht! In: *mathematik lehren* 100, pp. 4-10.

HERGET, W. 2000b: Wie groß? Wie hoch? Wie schwer? Wie viele? Mathe-Welt. In: *mathematik lehren* 101, pp. 23-46.

HERGET, W.; JAHNKE, T.; KROLL, W. 2001: Produktive Aufgaben für den Mathematikunterricht in der Sekundarstufe I. Berlin: Cornelsen.

HERGET, W.; SCHOLZ, D. 1998: Die etwas andere Aufgabe – aus der Zeitung. Mathematik-Aufgaben Sek. I. Seelze: Kallmeyersche Verlagsbuchhandlung.

PETER-KOOP, A. 1999a: Open Real-World Problems in the elementary Mathematics Classroom. In: Meissner, H.; Grassmann, M.; Mueller-Philipp, S. (Eds.): *Creativity and Mathematics Education. Proceedings of the International Conference, July 15-19, 1999.* Münster: Westfälische Wilhelms-Universität, pp. 95-100.

PETER-KOOP, A. 1999b: „Das sind so ungefähr 30 000“. Schätzen und Überschlagsrechnen „aus der Sache heraus“. In: *Die Grundschulzeitschrift* 125, pp. 12-15.

SCHEID, H. 1994: Der Fetisch der Ganzzahligkeit. In: Pickert; Weidig (Eds.): *Mathematik erfahren und lehren*. Stuttgart: Klett, pp. 179-184.

Prof. Dr. Wilfried Herget
Martin-Luther-Universität Halle-Wittenberg
FB Mathematik und Informatik
06099 Halle (Saale)
GERMANY
E-Mail: herget@mathematik.uni-halle.de