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MATHEMATICAL KNOWLEDGE OF CHILDREN WITH SPECIAL NEEDS IN THEIR FIRST YEAR OF SCHOOL

Abstract

This paper is a summary of an empirical study which evaluated the mathematical knowledge and progress of children with special needs in the first year of school. Emphasis was placed on the question "does the teaching method influence progress in mathematical skills?". Following the results of the study, current approaches to initial mathematical teaching for special education are considered questionable and conclusions for using the "Mathe 2000" approach in special classes are given.

Introduction

A research project conducted by the Institute for Special Education at the University of Freiburg, Switzerland, has evaluated the mathematical knowledge of children with special needs at the beginning of their school education (cf. Moser Opitz 2001a; Moser Opitz 2001b). One definition of "special needs" is children with learning disabilities. In Switzerland, these children are assigned to special classes where, as a criterion, an intelligence quotient between 75 and 90 is employed. Alternatively, so-called "introductory classes" are common in Switzerland. This kind of class is intended for children with a partial delay in their development and, in some cases, when the maturity level required for entering the first class is in doubt. Here the subject matter of the first class is extended over two years. The children then enter either the second year of the mainstream class, or a special class for children with learning disabilities. Unfortunately, children who show unusual behaviour and don't have learning disabilities are often assigned to these classes.

Current approaches to initial mathematical teaching (in special classes) in Switzerland are strongly marked by an understanding of the concept of numbers developed by Piaget. It recommends introducing numbers step-by-step after lengthy pre-numerical practice (several months, up to one year). Classification, seriation and number-conservation are considered as necessary prerequisites for understanding numbers. A textbook for children with special needs (Mathematik Kleinklasse A des Kantons Bern 1989) recommends, for example, not to practise counting skills at the beginning of school education rather to start slowly and to count forwards to six only.

However the "Mathe 2000" approach (Wittmann; Müller 1997, Wittmann; Müller 2000) with the Swiss version of the textbook "Das Zahlenbuch" (Hengartner; Wieland 1995) offers the range of numbers between one and twenty simultaneously. Exercises, which precede concrete calculation, are completely absent. In addition, the emphasis lies on working with images of quantities with a given structure. The reasons for this are given in the following paragraph.

There are many studies showing that the numerical knowledge of first-grade children in mainstream classes is much higher than traditional textbooks assume (Hengartner; Röthlisberger 1995; Hengartner 1999). Lengthy pre-numerical practice, based on the concept of number by Piaget, is ignored, because such an approach to mathematical learning is being questioned. For example, many studies show that number-conservation is not a necessary prerequisite for the development of mathematical abilities (Wember 1989; 1998; Moser Opitz 2001a, p. 48f). Nowadays, it is known that children acquire mathematical knowledge

by solving meaningful mathematical problems and not by solving tasks such as number-conservation and class-inclusion. Furthermore, it is important to present the range of numbers from one to ten or one to twenty immediately. A separate number can only be understood as a part of a whole, in the context of a larger group of numbers. A step-by-step introduction of numbers hinders such an overview and, consequently, understanding. To help children to represent numerical quantities, it is important to use sets with a given structure of five or ten, the so called "power in five" or "power in ten" within the "field of twenty" (figure 1). This structure should help them to internalise the concept of numerical quantities.

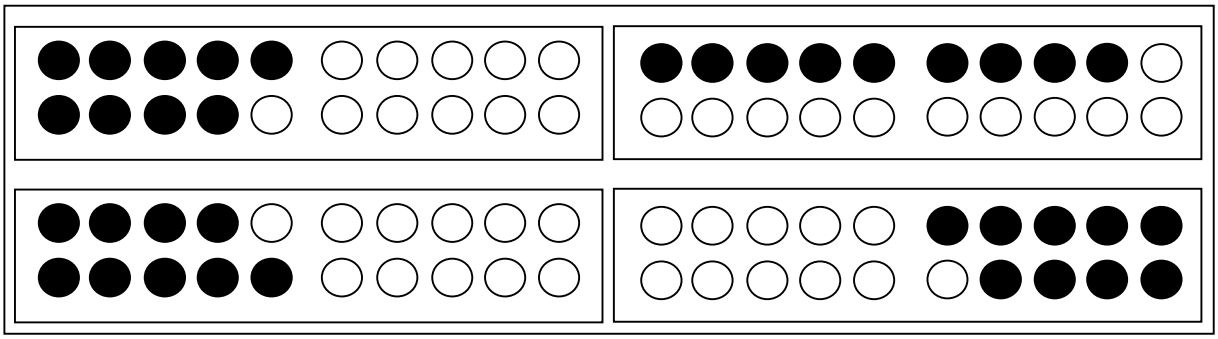


Figure 1: Sets with a given structure within the "field of twenty"

In the practice of special education, some critical questions about this new approach have arisen. It is doubtful if children in special classes have the prerequisites necessary to work in the range of numbers from one to ten or one to twenty immediately. In addition, it is difficult for teachers to accept that pre-numerical practice, which used to and still does characterise the teaching of mathematics in special education, is questionable. Furthermore, there are doubts about whether children in special classes are actually able to acquire the concept of numerical quantities from one to twenty. These doubts have led to the questions the intended research project sought to address:

- What kind of numerical knowledge do children in special classes bring into school?
- Do children in the first year of a special class, who are taught according to the "Mathe 2000" approach, make more, less or similar progress in mathematical knowledge than children taught according to current (special education) methods?
- What conclusions can be drawn from this information for initial mathematics teaching?

Method

The subjects of the study were 162 children (59 female, 103 male) from 28 special classes in the Canton of Berne at the beginning of their school education. The average age was 6 years 9 months. Taking into account the special needs of the sample the test had to include everyday experience and the opportunity to be active. The test was given in the form of a gold coin game (figure 2) and comprised two parts, "prerequisites" and "calculation" (table 1). The children were tested individually in a session of 35 minutes (cf. Moser Opitz 2001a, 126f).

Material: Playing board, dice, counters, gold coins (i.e. coins of 5 Rp., 10 Centimes, 2 p)

Game instruction: The game is played as a common dice game. Each player moves the counter according to the dice. If someone reaches a field with gold coins, the person can take the same number of coins out of a box. The person with the most gold coins at the end of the game is the winner.

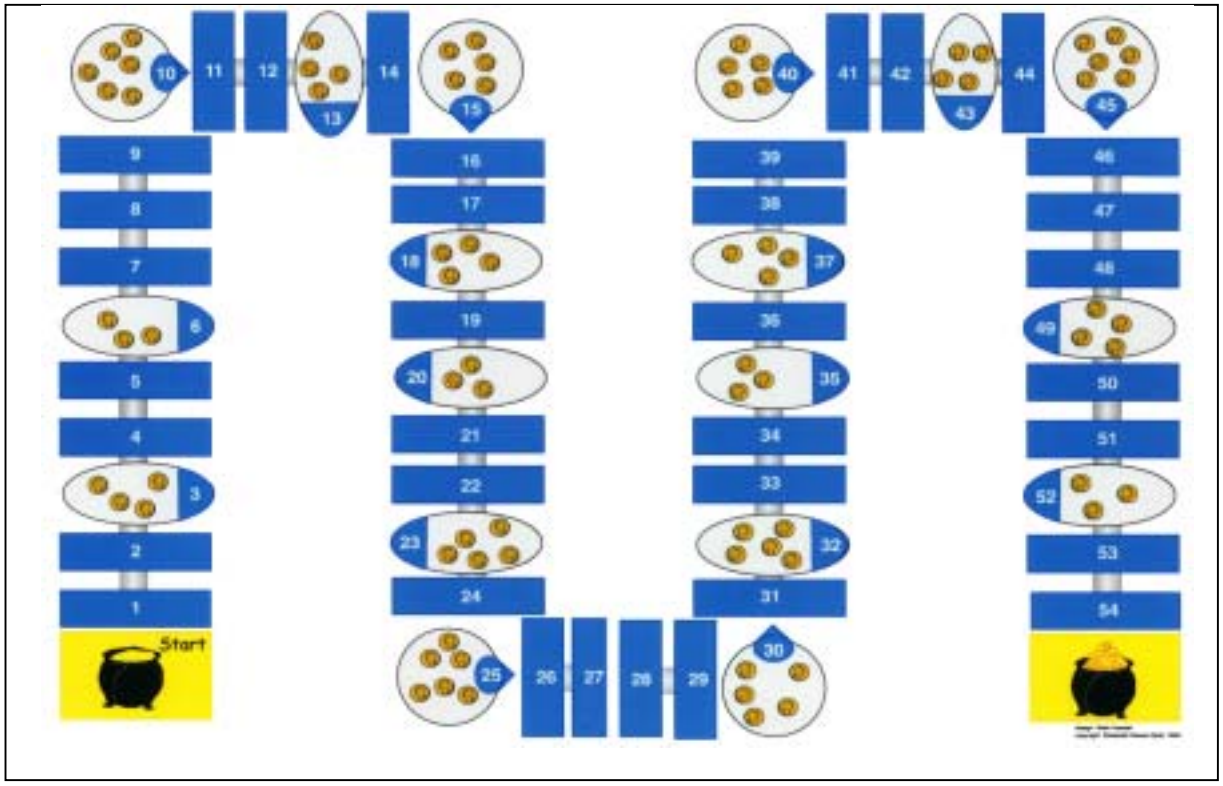
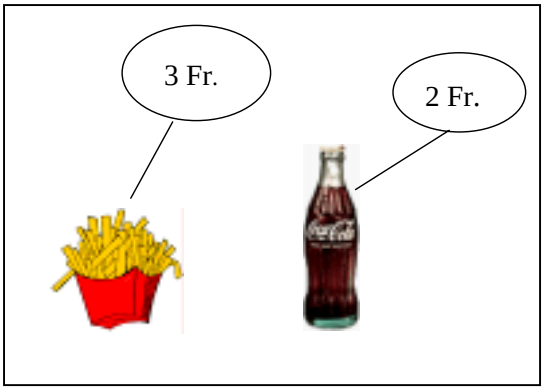


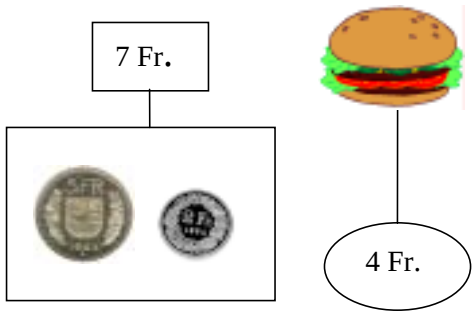
Figure 2: Gold Coin Game

	Task	Realisation
Prerequisites	*Pre-numerical task* Classification Seriation One-to-one correspondence	 Ordering dice and counters Ordering coins of different sizes in the right order Make one-to-one correspondence with two rows of gold coins; moving the counter correctly on the playing board
	Comprehension of numerical quantities	Taking out n coins from a box Comparison of two numbers after having counted the coins (who is the winner?)
	Grasp of numerical quantities	Recognising the dice Recognising unordered sets on the playing board Recognising sets with a given structure on the "field of 20"
	Counting	Counting forwards and backwards (number sequence) Counting cold coins
	Number	Reading the numbers on the playing board Writing numbers on a playing board without numbers
Calculation	Addition/Subtraction from 1-20 with counting aid	Doing addition and subtraction on the playing board
	Addition/Subtraction from 1-20 without counting aid	Shopping situations (figure 3)

Table 1: Tasks of the test



You buy Pommes Frites for 3 Fr. and a Coke for 2 Fr. How much do they cost?
It's an addition task: 3+2.



You have 7 Fr. in your wallet. You buy a hamburger for 4 Fr. How much money have you left after paying?
It's a subtraction task: 7-4.

Figure 3: Shopping tasks

Results

First the results of the numerical competencies at the beginning of school education are given as percentages (table 2). Where a percentage range is given, different scores were given for different tasks.

The results show that the children's numerical knowledge at the start of their school education is higher than current approaches to special education presuppose. Most of the children managed the pre-numerical tasks and had a comprehension of quantities from 1 to 6. More than half of them knew the number words from 1 to 10 and 46% were able to write numbers from 1 to 5. More than 50% were able to count at least to 20 (cf. Moser Opitz 2001; 1999a; 1999b). The addition tasks, with the possibility of counting within the first ten, were completed by 43-66% of the children, the subtraction tasks by 32-40%. It shows that the calculation ability of children with learning disabilities is lower than in mainstream classes where the score for addition is 80% and for subtraction 40% (Hengartner; Röthlisberger 1995, p. 71f). Only a few children were able to solve addition and subtraction tasks without counting aids. Overall, one third were not able to solve any calculation task. Therefore, the calculation competence of children with learning disabilities should not be overestimated. Additionally, the performance of numerical competence of the children in special classes was very heterogeneous (cf. Hasemann 1998; Schipper 1998). The difference included the subject matter of one year or more. Interestingly, boys with a high numerical competence (ability to calculate from one to twenty at the beginning of school education) were over represented in the sample. This leads to the assumption that the reasons for assigning boys to introductory classes might not be learning disabilities or cognitive deficits. It may be presumed that these boys were sent to these classes because of their unusual behaviour, i.e. such as showing aggressive tendencies. It is doubtful if schooling in a special class, where the intellectual level is low, is an appropriate measure for these children and their special needs.

Task	**%** (N = 162)
Prerequisites	
Pre-numerical Practice	
Classification (multiple)	66.7
Seriation	72.0
One-to-one-correspondence	89.5
Comprehension of quantities	
Take n objects	88.3-98.8
Number words from 1-10	69.8-92.6
Writing numbers	
Numbers from 1-5	46.3
Numbers from 1-10	29.6
Counting	
Counting forward to 20 (and further)	55.6
Counting backwards from 6	49.4
Calculation	
Addition with counting aid from 1-10	43.2-66
Subtraction with counting aid from 1-10	32.1-40.8
Addition without counting aid from 1-10	20.3-27.1
Subtraction without counting aid from 1-10	5.5-6.2

Table 2: Numerical knowledge of children with special needs at the start of their school education

The second part of the research project examined the improvement in mathematical knowledge in the first year of school. Based on written reports by the teachers about their teaching methods, the teaching material used and the subjects taught, three different groups were matched with an equal number of boys and girls (table 3). One group was taught according to the current mathematical teaching approaches with lengthy pre-numerical practice and step-by-step introduction of number - called “Group ClassCur”. The second group was taught by the “Mathe 2000” approach from the beginning of their school education. This includes working with the range of numbers from one to twenty and placing emphasis on the recognition of sets with a given structure within the “field of twenty”. This group is called “Group Mathe 2000+”. The third group, called “Group Mathe 2000” started working with the “Mathe 2000” approach during the first year of school.

Group ClassCur (N = 34)	Taught according to the current mathematical teaching approaches i.e. lengthy pre-numerical practice and step-by-step introduction of numbers.
Group ClassMathe 2000+ (N = 34)	Taught by the “Mathe 2000” approach from the beginning of school education. Working within the range of numbers from 1-20 and placing emphasis on the recognition of sets with a given structure.
Group ClassMathe 2000 *(N = 33)*	Started work with the “Mathe 2000” approach during the first year of school (after Christmas).

Table 3: Groups matched by written reports of the teachers

There were no statistically significant differences in the mathematical performance in the pre-test. Eight months after the start of their school education, the children were tested again with the same test. Analysis of variance with repeated measures and a Scheffé-Test were used to determine if there were differences in the mathematical performance between the groups in the post-test. The dependent variables were the different sections of mathematical knowledge (number words, recognizing sets with a given structure, writing numbers, counting forward, recognizing the dice and unordered sets, calculation with counting aids, calculation without counting aids).

Mean		F	Sign.	Scheffé-Test
Number words (N = 101)				
ClassMathe2000+	6.912→14.853	4.72	**0.011**	ClassMathe2000+/ClassCur **p = 0.021**
ClassCur	6.647→12.029			ClassMathe2000/ClassCur p = 0.100
ClassMathe2000	7.091→14.212			
Recognising sets with a given structure (N = 101)				
ClassMathe2000+	5.882→22.029	17.23	**0.000**	ClassMathe2000+/ClassCur p = **0.000**
ClassCur	4.294→6.647			ClassMathe2000/ClassCur p = **0.000**
ClassMathe2000	4.758→18.758			
Writing numbers (N = 101)				
ClassMathe2000+	9.441→34.676	4.93	**0.009**	ClassMathe2000+/ClassCur p = 0.081
ClassCur	8.559→25.176			ClassMathe2000/ ClassCur p = 0.087
ClassMathe2000	6.727→34.606			
Counting forward (N = 101)				
ClassMathe2000+	26.147→39.824	2.01	0.139	
ClassCur	24.606→34.333			
ClassMathe2000	24.909→40.576			
Recognising the dice and unordered sets (N = 101)				
ClassMathe2000+	12.029→13.539	1.07	0.346	
ClassCur	11.441→13.500			
ClassMathe2000	11.121→13.424			

Table 4: Statistical summaries of Analysis of Variance for the section “Prerequisites”

The results show (table 4) that the performance of the groups taught by the “Mathe 2000” approach was, in some areas but not overall, significantly higher than those of the group which was taught using current approaches. The two groups “ClassMathe 2000+” and “ClassMathe 2000” made significantly more progress in the area of “number words”, “recognising sets with a given structure” and “writing numbers”. However the Scheffé-Test, with its conservative measuring, is not always significant. No differences were found in the areas of “counting forward” and “recognising the dice and unordered sets”.

Interesting results were found in the calculation section. Because there was a special interest in whether there were ability group differences in the frequency of using counting aids such as saying the number sequence or finger counting, several analyses of variance were run (table 5). The first one compared the mean of the tasks judged not only as right or wrong, but also whether the tasks were done with or without counting aids. The Mathe 2000 groups performed significantly higher. A second analysis of variance was run with the mean of the tasks scored only with “right” or “wrong”. No significant effect was found. These different values led to the assumption that there must be differences in the use of counting aids. The third analysis of variance was performed only with the mean of the tasks done without counting aids. The main effect was significant for the “Mathe 2000” groups but the Scheffé

Test shows significance only for the group “Mathe 2000+”. Interestingly, only the group which worked with “the power in five” from the beginning of their school education used significantly less (finger) counting strategies in addition and subtraction than the other groups. This is an important finding because it is known that one symptom of children with learning disabilities in mathematics is the frequent use of (finger) counting strategies (Geary; Brown; Samaranayake 1991). From the written reports by the teachers of the group “Mathe 2000+”, it is known that they emphasised working within the “field of 20” and the “calculightning” i.e. the intensive exercise of basic skills (Wittmann; Müller 1997, p. 75ff.), more than the teachers of the group “Mathe 2000”. There seems to be evidence that it is important to work with sets of quantities with a given structure in a very intensive way and over a long period of time. Given the data available, the question of whether it was the influence of time or teaching method which caused the difference cannot be answered.

Mean		**F**	**Sign.**	**Scheffé-Test**
Calculation (Tasks scored differently, with and without counting aids)				
ClassMathe2000+	11.118→41.059	5.20	**0.007**	
ClassCur	9.941→24.471			
ClassMathe2000	10.848→33.636			
Calculation *(Tasks scored right/ wrong)*				
ClassMathe2000+	4.088→13.676	2.07	0.131	
ClassCur	3.912→10.265			
ClassMathe2000	4.061→13.061			
Calculation tasks without counting aids				
ClassMathe2000+	1.500→6.853	6.7	**0.002**	ClassMathe2000+/ClassCur = **0.004** ClassMathe2000+/ClassMathe 2000 p = 0.100
ClassCur	0.941→1.971			
ClassMathe2000	1.333→ 3.758			

Table 5: Statistical Summaries of Analysis of Variance for the section “Calculation”

Conclusions

The results of this study show that the numerical knowledge of children with special needs at the beginning of their school education is much higher than current approaches in special education presuppose. Those common teaching materials, which prescribe pre-numerical practice for several months or a whole year without using numbers, can be considered questionable. If for example more than 50% of the children are able to count forward to 20 or higher, it does not make sense to count to six only, as is recommended in the textbooks. Moreover, there are valid reasons to conclude that working with the “Mathe 2000” approach helps children to develop their concept of number. Early mathematical teaching for children with special needs should take these results into account and adapt its methods.
Classroom experience and theoretical concepts show that the following points seem to be important:

Individual teaching

The results of this study show that the numerical knowledge of children in the first year of a special class is very heterogeneous. Children with very limited ability and children who are able to calculate from one to twenty may attend the same class. The teaching has to take this into account and to set appropriate tasks for low and high achieving pupils. It cannot con-

tinue that children who are able to calculate from one to twenty at the beginning of their school education are taught this for two years in an introductory class.

Emphasise the counting ability

According to Geary; Bow-Thomas; Yao (1992) children with learning disabilities in mathematics are known to be poorer counters than pupils without mathematical learning disabilities. Counting competence is a necessary prerequisite for working within the “field of 20”. Counting is the first and most reliable way to determine the quantity of a set. Whilst working with the “field of 20”, children with poor counting competence are never sure if their counted quantity is really right. Therefore, initial mathematical teaching has to look carefully at the counting abilities of its pupils and to emphasise its practice. Exercises which include number sequence, counting of objects and understanding of cardinality (Fuson 1988) should be a daily part of the initial mathematical teaching at the beginning of school education.

Emphasise reflecting on and practising “basic skills”

Classroom experience shows that many teachers in special classes introduce numbers and then immediately start introducing addition and subtraction. They are not aware of the importance of working out the range of numbers from one to twenty in a reasoned and structured way by the “power in five”, “splitting numbers”, “complements in ten” and so on.
The results of the study showed that only the group ClassMathe2000+ made significantly more progress in addition and subtraction without counting aids. Their teachers emphasised working with the “power in five”, the “field of 20” and the “calculightning”. Many teachers are not aware of the importance of this point. A special education teacher expressed this in the following way:

> “Working with the 'Zahlenbuch' for the third time, I was at long last brave enough to take a lot of time on working with the 'power in five', the 'field of twenty', 'splitting numbers' and 'complements in ten'.
> It was striking that introducing addition was like cutting through butter – even for the children with big learning disabilities. I did not lose time, as I was afraid I would.”

Give more guidance for special education teachers

Interviews carried out with special education teachers (Moser Opitz 1999c), together with classroom experience show that teachers often lack knowledge as to how they should adapt the “Mathe 2000” approach to children with special needs. For example teachers don't know how to cope if children – mostly children with spatial orientation problems or with poor counting competence – do not manage to work with the “field of 20”. Often they don't even know how to manage the rich learning environments they have in the textbook “Das Zahlenbuch”. Neither are they aware which topics should be emphasised and which can be worked out later. It is important to give more guidance to these teachers: guidance which is orientated towards substantial learning environments and special education knowledge. In Switzerland, a so-called “Heilpädagogischer Kommentar zum Zahlenbuch” (Commentary for teaching mathematics with the Zahlenbuch in special classes) has been written (Moser Opitz; Schmassmann 2002). It is a commentary on selected pages of the textbook which

give guidelines for teaching special classes and children with learning disabilities. It includes information about important topics concerning children with learning disabilities, references to necessary prerequisites and previous knowledge for the appropriate mathematical subjects and potential difficulties of children with learning disabilities. It makes suggestions, such as to how to adapt the “Mathe 2000” approach for these children.

To convince special education teachers of the importance of the new approach for children with special needs and to give them the support they need, much more work, more vocational training and more research has to be done.

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