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SPRICH DAZU! LINGUISTIC EVENTS IN THE MATHEMATICS CLASSROOM

Abstract:
In this paper I will discuss some aspects of the linguistic events which happen during a mathematics lesson and which help determine the teaching, learning and understanding of mathematics. First, problems which may arise from the use of the special language of mathematics in the classroom will be described. The second section focuses on the use of this language under the aspect of cognition and in the third section I will show the embedding of empirical didactic research into linguistics. The final remarks will give an overview of items and methods of modern linguistics and conclude with some thoughts about interdisciplinary research work.

Introduction

During my time at school one of my maths teachers was not content with the abilities of his students until they commented on their calculation or formula which they had written on the blackboard: 'Sprich dazu! Speak while you are writing!' The students did not like it for they thought it more difficult than the mathematics itself. Visiting lessons nowadays one can hear teachers asking their students 'Tell us what you are doing!' or 'Could you describe or explain what you have done?' In the recent didactical discourse one can read about speechless children and children as authors of mathematical texts like 'Rechengeschichten', 'Reisetagebücher' and written 'Eigenproduktionen' (GALLIN/RUF 1999, SCHÜTTE 1997, SELTER 1997). There are discussions about technical items of language, the communicative function of students' remarks or frames of communication in educational situations (KRUMMHEUER 1984). The language in the mathematics classroom has become a subject of didactical research.
In this paper I will first discuss some of these aspects, then take a view both from the didactical perspective on the language of mathematics education and also from the linguistic perspective on mathematics education. Thus thoughts and results of research on didactics of mathematics will be combined with those of linguistics. Two reasons suggest this double view: (i) Linguistics have been referred to by other scientific disciplines for several years (see the so-called linguistic turn in social sciences). A number of linguistic methods are already used in the didactics of mathematics. However, many observations are lost in some of the research work, because of the lack of a proper embedding into a linguistic theory. (ii) Secondly, as one clearly cannot teach without using language, so one clearly cannot describe and evaluate mathematical instruction (in or out of the classroom) without looking into the forms and functions of language.
At first, of course, we have to investigate in this paper what kind of language is spoken and what problems may arise while children learn, and teachers use, the special language of mathematics. The second section focuses on the use of this language under a special aspect: the cognitive function of language. While the first two sections look at tasks for the children, the third section deals with the linguistic task for researchers and teachers. Thus in the third section of my paper I will describe empirical didactic research as a linguistic science, since evaluations are mostly based on texts. At the end of each section I will summarize the main linguistic aspects into a list, all of which will be collected into a table in the final section.

1. The Special Language in the Mathematics Classroom

Maybe the most significant feature of communication in the mathematics classroom is the use of an unusual kind of language: unknown words, new meanings for known words, strange kinds of sentences or the variety of symbols, i.e. the use of the *special language*[1] of mathematics. In general one cannot compare the acquisition and use of a special language with that of a foreign language. A German special language is part of the German language system. The special language of mathematics, for example, is a so-called *variety* (kind of speech) of the German language. Varieties have features at all linguistic levels: sounds and letters, words, sentences and texts, as well as the area of validity and social prestige. Each language encloses a large number of varieties, for each person converses differently depending on his racial, regional, social, cultural or occupational background. Special language can be called a *functional variety*, for its use is bound to a function or a special aim (e.g. description of a mathematical fact).[2] The function of a special language is to describe a fact or object as exactly, distinctly, concisely and objectively as possible, while remaining intelligible for a wider range of scientists.
Special languages can be specified under two criteria; horizontally with respect to the subject (mathematics, chemistry, medicine, language,...) and vertically with respect to the degree of specialization. It is impossible to use a high level of special language in an open public discourse and it would be nonsensical to ask students to learn and use it in their courses. The context and, above all, the partners of a communicative event determine the degree of specialization. On the other hand the subject itself determines the variety spoken by the specialists. Mathematics surely needs more symbols than philosophy.
The special language of mathematics is not the first variety that children have to deal with when entering school. In the southwestern part of Germany, children generally talk at home in a dialectal variety like Südrheinfränkisch, Alemannisch or Schwäbisch. These differ from the standard variety on the levels of sounds (‘isch’ instead of ‘ich’), words (‘babbeln’ instead of ‘sprechen’) and even inflexion of words (‘mir laufet’ instead of ‘wir laufen’). Hence they have to learn the standard variety ‘Hochdeutsch’ as a ‘new’ first language in school. In addition, special languages for other subjects like reading, writing etc. also have to be learned.

Students want (or at least they should want) to learn something. They get used to measuring the quality of their learning by the reaction of the teacher and so students aim at meeting the expectation of the teacher. With regard to language this seems to be the use of those strange new words in special situations. Let me give two examples.
When learning to add two numbers, first grade students have to use the words ‘ist gleich viel wie’ (‘equals’). They learn to use it, but their use of the word is not necessarily an indicator of a proper understanding of the principle of equality. Maybe this understanding was not an aim of the lesson. However the teacher possesses this understanding; he will be implicitly aware of it while using the words, and somehow require it from the students when he talks about addition.
Another typical situation arises from instructions on how to do a calculation, where students have to use these words: ‘7 plus 5 gleich 12, schreibe 2, übertrage 1’ (prescribed in the

[1] I will use this abbreviation for the term ‘language for special purposes’.

[2] Other *functional* varieties are for example the language of politics or the language of economics. Their aims can be to convince someone of a special opinion or to analyze the market situation. A good overview is provided by ROELCKE 1999.

Bildungsplan 1994, 230). Third grade pupils, whom I observed, focused all their attention on the right sequence and the right accentuation of words, while the process of calculating remained disregarded. The result is an artificial, external language, which will never be good for understanding or talking about mathematics. Mathematical language therefore becomes an additional topic in the mathematics classroom, not a help as it should be.
Maybe one could avoid this dilemma by allowing the use of a wider range of words and verbal phrases. The task of students should not be to use the prescribed words automatically, but to use proper words in a well considered way. These can be their own, individual words for the description of a mathematical process or fact. The duty of the teacher is to mediate the remarks of students. He can offer them technical words as possible, desired and – in his opinion – best terms for a process or fact. Preconditions are social forms of teaching that allow or rather call for discussion and description of mathematical topics between students. Some possible forms are the aforementioned Reisetagebücher or Rechengeschichten. Another possibility is to allow the students to search for, and find, their own way to solve a problem ('eigene Rechenwege'). It is important however that the student comments on the way he tackles a problem. He should find words for his actions i.e. he should relate something.

- symbols
- abbreviations
- technical vocabulary: new words, new meanings of known words
- typical texts (e.g. instructions, calculations)
- individual symbols, expressions
- communication between teacher and students, between students
- imitation of the language of the teacher
- call for talking about mathematics
- verbalization of actions and thoughts
- special language as a functional variety of German
- vertical division respecting the subjects, horizontal division respecting the degree of specialization
- understanding processes and facts

List 1: Special Language of Mathematics

2. Talking About and Understanding Mathematics

Why is it so important for a student to find words for mathematical processes and facts? While a person is searching for proper words he reflects on his actions and thinks about the given items. Thus language used in the foreground for documentation also becomes an instrument for thinking processes. This function of language is called the cognitive function of language (Erkenntnisfunktion)[3], for here the language is used by a person to recognise objects and facts in the world.
The result of a process of understanding is a representation of an object or fact in the mind of a person: he will build up a concept. He integrates this concept into the system of his other concepts and will nearly always change it when-ever he has to somehow refer to the object or

[3] Other functions are the communicative, symptomatic and representation function of language (Kommunikations-, Darstellungs- and Symptomfunktion, see REICHMANN 1976).

fact. Hence building up a concept, that means understanding something, is always a dynamical process for an individual person.[4]

One can refer to a concept in a person's mind in different ways, for example by drawing a picture or executing a special movement, but it is more common to refer to the concept in a (spoken or written) word. In structural linguistics a word is described as a linguistic unit with the following three properties:

(1) it is bilateral, i.e. a word is an union of two sides: signifiant (the thing that signifies, sound/letters) and signifié (the thing signified, concept);[5]

(2) the relationship between the two sides is arbitrary;

(3) the relationship is a product of convention within a human community.[6]

The last two qualities are important for the successful use of language in the mathematics classroom. Teachers impart mathematical knowledge to students mostly by using words.[7] The student hears or reads the sound or letters of the words and tries to pick out the right concept in his/her mind. At this moment several problems may arise:

(i) The concept in the student's mind does not correspond with that of the teacher (see the example 'equals' above. This is almost always the case since if they corresponded precisely, the student would not have to go to school any more). If the teacher takes his own concept as a basis for further instructions, the student will not be able to follow them for they do not fit with his concept. So teachers should try to take the concepts of their students – whatever they may be like – as a basis.

(ii) There is a concept in the mind of the student. It may even almost correspond with that of the teacher, but the word used does not evoke the concept because the student has bound the concept to another sound image (e.g. 'abziehen' and 'subtrahieren' for 'substract'). In this case students might be able to recognize the concept by analyzing the context of the teacher's speech.

(iii) In the worst case, there is no concept at all. Unfortunately this occurs more often than is generally assumed. Teachers cannot readily realize the lack of an appropriate concept, for students learn to use the right word at the right time without knowing exactly what it means. They are even trained to learn whole sentences by heart (see the example 'instructions' above).

In all three cases the transmission of knowledge is impaired. The student at least has difficulties in building up a useful concept of a mathematical object or fact based on verbal information from teachers. It is possible however, to address these problems by encouraging students to talk to each other about mathematics:

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[4] For details and a closer view on the process of understanding see STEINBRING 1994 (epistemologisches Dreieck), GÄRTNER 2001/1.

[5] If there are two concepts signified by the same sound image, one speaks of polysemy. Some words seems to be polysemic, but the two meanings belong to different varieties: 'Körper' means 'body' in colloquial language, 'field' in the special language of mathematics. The influence of the colloquial meaning on the use in special contexts deserves closer investigation.

[6] When a child asks 'Why is a tree called tree?', the proper answer would be 'because of a convention in the English-speaking community'. Germans say 'Baum', French people 'arbre'.

[7] In the first grade there certainly are other forms for imparting knowledge but language is one of them. Maybe it is not sufficient, but it is undoubtedly necessary.

(i) If students learn to describe their concepts, the teacher can decode and try to reformulate what they said before, according to this concept. Should there be any significant faults in the student's concepts, he can address them;

(ii) If it is only the term that differs, a real dialogue between teacher and student will show this and allow both partners to find a common term. Thus the imparting and understanding process can proceed on a proper basis;

(iii) If there was no concept at all, this will become evident while talking about it. One needs very good rhetorical abilities and eloquence to talk about something one does not really know. Only few students will be able to do this, and no teacher should shrink from asking once again 'what did you mean?'

Therefore language not only supports the thinking processes of the learning person, but is also a diagnostic instrument for the teacher. He can measure the results of transmission and the degree of concept development by his students while listening cautiously to them. The basis for this review should never be the repeating of mathematical expressions, but rather the ability to *speak* about mathematics. It is clear that students have to learn to use the special language of mathematics but this is pointless as long as their use of it is externally motivated. Only internal motivation for the use of special terms i.e. when the student is convinced that they are the best terms for the fact, will lead to efficient usage. Only the language of the individual student can be an instrument for cognition.[8]

- representation function of language
- word: a bilateral, arbitrary and conventional linguistic unit
- communicative function of language
- linguistic actions produce mathematical reality
- symptomatic function of language
- individual special language of mathematics
- cognitive function of language
- individual language: instrument of cognition
- understanding: building up a connection between sound and concept

List 2: Talking About and Understanding Mathematics

3. Linguistics in the Didactics of Mathematics

Quite a few didactical publications are based on empirical research. Researchers visit and observe mathematics lessons. Sometimes they impose some restrictions on the teacher, sometimes they observe it 'as it happens'. Recording lessons on videotape offers the possibility of reviewing the actions, maybe several times, under new aspects. Researchers will mostly try to transform the video recording into a written text, i.e. they will make transcripts. Additionally, they take notes during the lesson and afterwards, or interview other people who also observed the lesson. The result is a file of texts which will be the basis of his investigation. The next steps (like sequential and specific interpretations) are to analyze, interpret and evaluate the texts. For this purpose the researcher will use methods of text analysis which are generic methods of linguistic research work. Let us therefore take a look at linguistic methods and their theoretical basis.

[8] Besides, using their own words will make it easier for students to accept mathematics as a part of their own thinking and their own world.

The linguistic concept of ‘text’ includes all linguistic utterances, written or spoken. A letter is a text of course, but essays, calculating papers, telephone calls or oral examinations are also texts. Each of these units can be interpreted on at least three levels: the level of grammar, of semantics and of pragmatics. Grammar includes, for example, the number and form of words, length of sentences (especially in German special languages) or distribution of tempus or modus verbi. Semantic aspects are the topics of texts or the types of sequences of propositions. The last, pragmatic, takes the text as a special kind of human action, namely a linguistic action. A question of higher priority is “why did a person say something?”.
Analyzing a text means to describe its features at these three levels. This process leads to characteristics which are typical of a special type of text (‘Textsorte’) and differ from the characteristics of other kinds. As an example I will show the text type ‘word problem’:

> There are three vases on the table. In each vase there are six flowers.
> How many flowers are there altogether?

Word problems are texts in which a mathematical problem is wrapped up in an everyday situation. Therefore one can find many terms for everyday objects like ‘table’, ‘flowers’, ‘vase’ and other items of colloquial language (names of persons, 3rd person active). The syntax should be easy and difficulties should not arise in understanding the text. Although the text describes a situation, the task of the student is not to understand and comment on this situation, but to do a calculation. This becomes clear by the differences between word problems and colloquial narration, e.g. the number words or the question at the end of the problem. So ‘word problems’ are a typical text type of the mathematics classroom.[9] Other typical text types are number calculations, definitions, proofs and examinations, but also included are dialogues between teachers and students (introducing a new topic) or between students (discussing a problem).[10]

The results of the analysis are typical differences which allow a closer view on the linguistic aspects of the education process. It becomes evident why some forms of texts are more appropriate for the special aims of education. The original form of the flower word problem did not include the question. Rana, a good second grade pupil, solved the problem by drawing three vases with wonderful flowers. She could not be blamed for her solution, only the situational context – a mathematics lesson – made the real intention of the problem, i.e. doing a calculation, obvious.

The next example shows that one often requires a wider concept of text to explain in which aspect there is the decisive difference between two linguistics actions. Dealing with more difficult additions, second grade pupils could not follow my explanations but had no problem when their schoolmate Laura explained the calculation, using – at least in my opinion – the same words as myself. In this case one has to add the para- and non-verbal events to the analysis of the verbal utterances. Not only the words alone decide about a successful or unsuccessful communication process. The sound of the voice – emphasis, quivering, speed – facial expression and gesticulation can be decisive for the message reception by the partner. (In written texts the form of the handwriting bears these signs.) Laura did indeed use the same words and, probably, it was just the social position of the partners which made the one process successful and the other not.

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[9] You will never encounter a text of the type ‘word problem’ in your everyday life.

[10] Dialogues between students about mathematical topics have, as yet, been paid little attention.

It is a difficult problem to highlight these signs in a transcript. At the grammar level there are objective methods to describe a text but the description and interpretation of semantic and pragmatic aspects remain dependent on the personality of the researcher. All attempts to set objective methods can only give us results with a higher degree of interpersonality. Yet this degree often suffices for setting up useful deliberations on education or for understanding processes.

• types of texts • grammar	• text: linguistic action within a communicative action • mathematical communication as cooperation of linguistic, para- and non-verbal actions	• oral and written texts

List 3: Linguistics in the Didactics of Mathematics

4. Didactics of Mathematics and Linguistics

We took a closer look at three features of mathematics education. Many linguistic topics did indeed arise from this small part, which I listed at the end of each section. I will compile these topics and classify them in this section under linguistic aspects. The following table is therefore based on a linguistic view of mathematics education.

(i) The first column includes units of the structural system of language. Usually the field of linguistics that deals with linguistic units and the rules of connections between them, is called grammar. One distinguishes the structural levels of sounds/graphs, words (inflexion and building), syntax and texts, where, at each level, form and meaning (semantics) of the linguistic units are of interest.

(ii) In the second column the word ‘action’ is striking. The listed items are subjects of the linguistic field of pragmatics, which takes language as a special kind of human action i.e. as an interaction between two (or more) partners. There are different forms of this linguistic interaction/communication in each community. The aim of the science is to describe and fix the extralinguistic parameters of this interaction (context, partners, subject) and their influence on the use and choice of linguistic items. The investigation of all aspects may enable one to catch the intention of the active, i.e. the speaking, person.

(iii) The entries in the third column make it obvious that the choice of speech depends on the particular social environment or the purpose of the communicative act. The theory of language varieties describes the differences between varieties and standards of usage which are valid in a society or culture. It is the subject of sociolinguistics.

(iv) All entries in the last column belong to the field of language and cognition. It questions the nature of the grammar and ‘dictionary’ in the mind of each individual person and asks why and how a person is able to transform these entities of his mental language (langue) into grammatical and sensible verbal utterances (parole). This field is called psycholinguistics.

Only three aspects of mathematical instruction resulted in many questions, these being spread over four important fields of modern linguistic research. There is no question that it would be easy to fill up the fields by including further linguistic events in the mathematics classroom.

topics of language in the mathematics classroom	• symbols, digits • technical terms • texts, types of texts • word as bilateral, arbitrary, conventional verbal unit • representative function of language	• forms of communication • linguistic, para-, non-verbal actions as reality • verbal action makes mathematical reality • communicative function of language	• special language of mathematics as functional variety • horizontal division of the special language • individual language of the student • symptomatic function of language	• cognitive function of language • role of language by building up concepts • individual language as instrument of cognition
	→ units and rules of language	→ language and action	→ language and social reality	→ language and cognition
field of linguistics	**grammar**	**pragmatics**	**social linguistics**	**psycholinguistics**
topics of linguistic research	• phone, phonemes; graphemes, words, sentences, texts • semantics	• communication • linguistic actions	• theory of language variety • norms of language used in a community	• knowledge of language • process of verbalization (producing, perceiving, understanding language)
related discipline	philology	theory of (inter)actions	sociology	neurolinguistics cognitive psychology

Conclusion

Language is an important factor in mediating mathematical knowledge therefore aspects of linguistic research should be included in investigations on mathematics education. However it is necessary to also apply the methods of linguistic research. As linguistics is an old academic discipline, one can base research on elaborate and well-proven theories. To do real interdisciplinary work, one has to make oneself acquainted with these theories and methods to such a degree that one is able to adapt them to the new subjects. Certainly this is supplementary work but it opens up new possibilities of interpretation and points of view. I will conclude my paper with a last example. Many children count 'one, two, three, ...' when entering school. The sequence of the words however is no proof that a concept of numbers is really developed. Probably they learned the sequence by heart but that is not a special problem of mathematics education. Children also say 'a, be ce, ..' before beginning school without having a clear idea of sounds or letters and the connection between them. Both observations seem similar in many aspects, so it would be worthwhile to examine both situations comparatively.

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