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# ARGUMENTATION PROCESSES IN MATHEMATICS CLASSROOMS SOCIAL REGULARITIES IN ARGUMENTATION PROCESSES

**Abstract:**

In the project "argumentation processes in mathematics classrooms" the focus is on special kinds of interaction processes between the teacher and the students in early grades. The characteristics of these processes are that the participants explicitly request and develop reasons for statements. The focus of this paper is on describing social regularities of these types of interaction processes in a qualitative way i.e. how do the participants organise the development of arguments? Three different theoretical approaches to argumentation processes will be described briefly and their incorporation into the presented project will be illustrated by the interpretation of two example transcripts.

In this article, some results of the research project "argumentation processes in mathematics classrooms" (Schwarzkopf 2000) will be presented. The focus is on argumentation processes between teacher and students of early grades. These processes are identified as a special type of interaction by the following characteristics:

- a reason for an expressed statement is explicitly requested
- reasons are given in response.

The project's research goals can be divided into two aspects: firstly to develop theoretical terms to analyse orally-produced arguments in mathematics classroom interaction, with regard to their different content-related functions, and secondly to describe social regularities with regard to the participants' organisation of argumentations. Of course these aspects are closely linked to each other, but it seems useful to distinguish between them in terminology. The first aspect of analysing argumentation processes deals with the content-related, subject-specific structures ("arguments") produced by the participants in mathematics classroom interaction. The second aspect is more socially related and questions how the participants organise the development of arguments in the interaction process ("argumentation").

In the project, mathematics lessons being given to four fourth grade (final year of primary school) and four fifth grade (first year of grammar school) classes were videotaped and analysed. In each class five lessons were observed. These lessons were of "regular type", i.e. the researcher was neither involved in the preparation nor in the delivery of the observed lessons. Those episodes in which argumentation processes were identified, as described above, have been transcribed and analysed in detail. It should be mentioned that the focus of the project is on teacher-student interactions, i.e. group working of the students is not analysed.

The analysis in this research project takes its structure from the interpretative paradigm. This universal theoretical framework uses theories of symbolic interactionism and microethnography, as modified for qualitative research in mathematics education by Bauersfeld, Krummheuer and Voigt (e.g. Bauersfeld / Krummheuer / Voigt 1988, Krummheuer / Voigt 1991, Voigt 1984). For the specific goal of the project, namely analysing argumentation processes, theories of argumentation from the pragmalinguistic

(Klein 1980, Weingarten / Pansegrau 1993) and microethnography (Krummheuer e.g. 1997) viewpoints are consulted and modified. Furthermore, to analyse the arguments produced in mathematics classroom interaction, the "functional analysis of arguments" developed by the science philosopher Toulmin (1969) was incorporated.

While the articles published in previous selected papers focused on a method for analysing orally-produced *arguments*, the paper in hand deals with social regularities of *argumentations* in mathematics lessons. Of course, any description of social regularities in argumentation is closely linked to the theoretical understanding of what should be termed argumentation. The author's definition of argumentation is the result of both citing literature about theories of argumentation and observing and analysing mathematics classroom interactions. In the following section the author gives some of the approaches which are cited in the project concerning social regularities of argumentation processes. Later, two short episodes will be presented and briefly interpreted to illustrate the researcher's method of analysing argumentations in mathematics lessons. This will highlight both different and common aspects between the author's approach and those of some of the already existing theories of argumentation.

## Three approaches to argumentation processes

### 1. A pragmalinguistical approach to "everyday" argumentations

Wolfgang Klein (1980, see also Miller/Klein 1981) develops an important pragmalinguistical approach to theories of argumentation. He is interested in analysing "everyday–interaction" in social groups in a descriptive way. Klein's research goal is not of the kind "how to argue in a correct way", but is to analyse how social groups practice arguing in reality. For his approach it is important to distinguish between the "collectively valid" and the "collectively questionable" of a social group. Klein names everything that would be accepted by a group at a special moment *the collectively valid* of this group at that point in time. It contains e.g. statements and some rules which are necessary within the group to allow conclusions to be drawn from statements in an acceptable way. None of the participants need be conscious of what belongs to the collectively valid of the group. Importantly, neither the statements nor the rules belonging to the collectively valid are "well defined", but depend on the specific interaction situation. Briefly put, the collectively valid contains everything the participants may routinely use in the communication process without calling it into question. Everything that cannot be used routinely in the interaction process is called the *collectively questionable*.

The collectively valid and the collectively questionable are highly dynamic and what is assigned to the one or to the other can change in any group at any time (except where there are underlying institutional rules, e.g. the laws in judicial courts). In Klein's terminology, an argumentation process starts when a social group is confronted with a "quaestio" – i.e. a question dealing with something collectively questionable for which none of the members has got an answer which the group will accept. An argumentation is the interaction process of the group, in which the members try to develop an answer to this question in a rational way. To achieve success, the social group develops an answer that is accepted by everyone due to rational reasoning (and not due to the social power of one of the members). Rationality, particularly, somehow involves a "democratic" balance of power. In other words every member of the group is allowed to bring his doubts to the discussion. In the event of such a success, the collectively valid is extended by the answer of the quaestio due to the argumentation. Hence, an argumentation in the sense of Klein is a special kind of interaction, identified by its function for the group to transport something from the collectively questionable into something collectively valid. This transport has to be organised by the participants i.e. they have to manage the following three tasks, while they are developing an answer to the quaestio.

> The first task of the members is to check *whether produced statements can be accepted*. Statements may be from the collectively valid and thus be accepted without any reasoning. In other cases, the members have to offer reasons and check afterwards whether the statement can be accepted due to these reasons.
>
> Secondly, they have to make sure that *coherence between several statements* is given when reasoning one with another. The participants have to check whether the rules between statements are legitimate ones regarding the group's demands in the special situation. Furthermore, they have to decide about the grade of detail required to accept an argument. Sometimes it is necessary to produce explicit rules between statements, in other cases it may be enough just to mention several statements.
>
> In many argumentations, there are several arguments necessary to solve the problem. Hence the participants thirdly have to *co-ordinate differently produced arguments,* which, combined, shall lead to an answer of the quaestio. They especially have to decide whether the arguments may or may not help with regard to the main interest of the argumentation.

These tasks are essential for every argumentation from the viewpoint of Klein's theoretical approach. However how, and by whom, these tasks are done, e.g. the question of what a rule has to look like to be accepted as a legitimate link between two statements, are empirical questions depending on the special arguing group. It should be mentioned that Klein is interested in argumentations, which often deal with moral problems and are typically very complex (e.g. Klein 1980, 1983). One can find analyses of complex argumentations, in the sense of Klein, which deal with physical problems, in Miller (1986). None of the aforementioned investigations deal with classroom interactions.

## 2. A pragmalinguistical approach to everyday argumentations in classroom interaction

The pragmalinguistical researchers Rüdiger Weingarten and Petra Pansegrau (1993) analyse instruction in regular classrooms in a qualitative way and apply the approach of Klein. According to their observations, there is no balance of power in the classroom between teacher and students but the teaching person has the "power of definition", bestowed by the institution–made rules of school. Hence, the authors conclude that in classroom interactions the categories of "collectively valid" and "collectively questionable" are not relevant, because it is the teacher who decides due to his "power of definition" whether a statement has to be reasoned, accepted, or rejected. Because he does this with regard to the institutional rules of the school, especially the curriculum, the authors name everything the members of classroom interaction may accept as the "institutional valid". From their point of view, there is no argumentation in the sense of Klein in classroom interaction. The authors indeed find language markers such as "why", "because", and others as being typical for argumentation, but they only understand them as aspects of a special kind of "language-style". The participants only *simulate* argumentations *on the surface of language* and Weingarten and Pansegrau name this kind of interaction process "*as–if–argumentation*". The function of this interaction process is to allow the children “to save face”, i.e. to help them not to feel as powerless as they in fact are. In their approach, an “as-if-argumentation” does not start because of a confrontation with a quaestio, but is initiated by the teacher, following institutionally given teaching goals. According to Weingarten and Pansegrau, the teacher does almost all the tasks in organising an (as-if-) argumentation in classroom interaction. It is he who decides whether a statement can be accepted, whether rules produce coherence between statements and how to co-ordinate several arguments. The task of the children is only to produce arguments until the teacher will accept one.

## 3. A microethnographic approach to argumentation in mathematics classroom interactions

In mathematics education Götz Krummheuer (e.g. 1997) investigates classroom interaction in a qualitative way, especially in mathematics. His goal is to develop a theory of teaching and learning from the perspective of microethnography. Thereby, he follows the theoretical approach of Miller (1986) and understands “collective argumentation” as a basic social requirement for children to construct any new knowledge. This theoretical perspective is a special one regarding approaches to mathematics education. While mathematics educators usually see argumentation as one learning goal amongst many others, Krummheuer understands argumentation theoretically to be a basic social requirement for any learning by children. Following Krummheuer´s approach, collective argumentation is a special kind of negotiation of meaning, in which the participants attribute and show rationality (Krummheuer 1997a/b). By negotiating meanings (see Voigt 1998,1994, or 1984), the participants give hints to their underlying situational frameworks i.e. their individual point of view in which they understand the mathematical problem. To make the communication process become stable, and especially to act in a co-ordinated way, the children may thereby “modulate” their frameworks. Thus they may change their point of view when having difficulties in understanding the others within the previously activated framework. In other words, they may learn how to understand the mathematical problem in an adequate way in order to solve forthcoming problems more effectively. Krummheuer (1997) finds, in his empirical analysis of group interaction in primary school levels, that the children rarely ask for reasons explicitly. Nevertheless he reconstructs learning processes in these interactions and understands them as collective argumentations. In these processes, the children show rationality especially by sequencing the story and thus allowing the others to follow their presented thoughts. This organisation of collective argumentations looks more typical of narrative discourses than of argumentation processes in the approach of Klein. Hence, Krummheuer describes collective argumentation as *integrated in narrative discourses*.

### The approach of the presented project

In this investigation, the researcher’s approach is more like the pragmalinguistical ones than the one of Krummheuer. The author understands argumentation as a special kind of interaction process in mathematics classrooms that can be distinguished from other mathematical or everyday activities, like problem solving, calculating, telling stories etc. According to this basic image of the researcher, argumentation is identified when the participants explicitly constitute the need for reasoning for a (mathematical) statement. According to the symbolic interactionism and microethnography, the researcher was looking for some language markers indicating that the members of classroom interaction produced a discourse by showing an emergence of reasoning from their point of view. These markers were like “why”, “can you explain this”, “because”, and others fitting the characteristics of argumentation processes in the sense of the author, i.e. indicating that the participants explicitly ask for and produce reasons for statements.

In the following, the author gives his interpretations of two short episodes as examples for the start of argumentation as they were typically observed in his investigations. Afterwards he discusses the approaches described above regarding aspects in common with, or different from, the approach of the presented project.

**An example from a fourth grade class**

This episode deals with the following word problem:

“The kilometre indicator in Silke’s car showed 1252.8 km before the drive. After the drive the counter pointed to 1271.4 km. How many kilometres has Silke driven?”

The first sentence of this task is written on the blackboard. The question afterwards was orally developed in classroom interaction some minutes before the episode begins. Every student was to solve the task individually. During this working process the teacher walks around the class, observing the students’ work and helping if necessary. At the beginning of the episode, the teacher Mrs. Burmeister goes to the blackboard and talks to the whole class[1]:

| | | |
|---|---|---|
| 1 | T | While walking around a few minutes ago I saw that some children did the task wrongly. I will write it onto the blackboard and you can figure out whether it is right or wrong and, respectively, why. I won’t write the question as we have mentioned it already and it should be right. The calculation, first number (begins to write “1252.8 km” onto the blackboard, pointing thereby at the first number in the task). |
| 2 | S | It’s wrong. |
| 3 | T | Well, I don’t know, everything is copied correctly from the task. |
| 4 | Ss | It’s wrong, wrong (teacher writes “– 1271,4 km” pointing at the second number in the task) |
| 5 | S | Correct. |
| 6 | Ss | (*Louder*:) Wrong. |
| 7 | T | Who was first saying wrong very loudly? So, why is it wrong, Petra please? |

The blackboard shows the following calculation.

$$\begin{array}{rl} & 1252.8 \text{ km} \\ - & 1271.4 \text{ km} \\ \hline \end{array}$$

At the very beginning of this episode, the teacher gives hints that the following calculation will be wrong. One can interpret her first remarks as “I will write the wrong way of calculation that I have seen several times in the class onto the blackboard”. However, she then asks the students to figure out whether the calculation will be right or wrong. After the notation of “1252.8 km” by the teacher, one student already remarks that it is wrong. The teacher reacts in an uncertain way, as if the students would have to convince her that the calculation is wrong. Thus she makes it clear that some aspects of the tasks are copied correctly. In fact, the written number is one of

[1] The author has translated the transcripts. For the original transcripts in the German language see Schwarzkopf 2000. Due to the impossibility of a “1 to 1” translation, aspects like accents etc. were left out.

those appearing in the task. One can say that the teacher shows by this the necessity of a reasoned statement and forces the children to produce arguments – the simple term "wrong" for the calculation is not enough and the need for an argumentation is stressed.

Indeed there is one student asserting that the calculation is correct (line 5), but most of the children show agreement with the judgement of the calculation as "wrong". In line 7 the teacher subscribes to the second judgement and asks the children to give reasons for it. Even if she does not explicitly mark the calculation as wrong in line 7, the teacher's way of asking for reasons makes clear that it is. (According to the teacher's remark there is one of the children who already knows that – and probably also why – it is wrong.) This is how the participants organise the beginning of an argumentation in this episode. Before discussing the above in terms of theoretical approaches, another example of the start of an argumentation will be interpreted.

**An example from a fifth grade class**

In the context of this episode, the class is working on some new basic algebraic laws to legitimise rules for clever calculating. Some minutes before, the distributive law was the topic of classroom interaction. The episode shows some seconds of the discussion of the students´ homework. The title of the homework tasks is "calculate in a clever way". The following solution for the task "$(50-2)\cdot 14$" was accepted by the teacher as correct:

"$(50-2)\cdot 14 = 50\cdot 14 - 2\cdot 14 = 700 - 28 = 672$".

At the beginning of the episode, the student Franz poses a question:

| | | |
|---|---|---|
| 1 | T | Franz has another question. |
| 2 | F | Isn´t it then also correct, 48 times 14 equals 40 times 14 plus 8 times 14? |
| 3 | T | I will write it onto the blackboard as you say it. What did you say? |
| 4 | F | 48 times 14 equals 40 times 14 plus 8 times 14 (*teacher writes onto the blackboard*) |
| 5 | S | (*silently:*) mhm |
| 6 | S´ | (*silently:*) Wrong |
| 7 | S´´ | (*silently:*) Correct |
| 8 | S´´´ | (*silently:*) Clearly |
| 9 | T | Is this correct or is it wrong? |
| 10 | Ss | (*silently:*) Correct |
| 11 | T | Correct. Why? I can tell you that this is correct. Why is this correct? |

The teacher Mr. Zander writes the proposal of Franz onto the blackboard: "$48\cdot 14 = 40\cdot 14 + 8\cdot 14$". Some of the children decide silently about the correctness of this proposal in different ways. Maybe at this time in the classroom there are in fact several opinions about the correctness of the equation, which might lead to an argumentation in the sense of Klein. Maybe the correctness of this equation belongs to the collectively questionable of the students and could be transported into something collectively valid by arguing. But this doesn't happen: in line 9 the teacher asks whether this equation is correct or not. One can't say whether he can hear one or all of the students in the lines 5-8 and 10, because they are talking very silently. However, the teacher answers his question in line 11 himself. He marks the equation as correct and asks the children to reason this. Hence the students already know that the equation is correct, without any argumentation.

**The beginning of an argumentation: discussion of the approaches**

Both episodes are typical of the lessons observed in the presented project regarding the beginning of reasoning processes. The students produce an assertion, which seems to be problematic in the view of the teacher. In the first episode the teacher probably tries to stress a widespread mistake of students working on word problems. The students do not only have to read key words to identify which kind of calculation has to be done and copy the given numbers in the order of appearance, but they have to attend to the context of the word problem[2]. In the second episode it seems to be important for the teacher to make clear that the lesson is about algebraic laws[3]. However, this organisation of the beginning of argumentations is typical of the author's observations: the teacher *firstly* judges about the correctness of the statement and *then* asks for mathematical reasons to accept or reject it.

This social regularity is different from the theory of argumentation developed by Klein. According to his thoughts, argumentation starts when a participant brings something "collectively questionable" to the discussion, i.e. the group does not know whether a produced statement can be accepted or not, otherwise he would not identify this interaction process as an argumentation. Indeed, there are signs in the presented episodes that some children do not agree with the correctness of the assertion. However because of the judgement given by the teacher the students already know whether the assertion is correct or not *before* they have to search for reasons. One can say that a potentially possible controversy *about* the correctness of the assertion is conceded and by this a process of reasoning *for* the correctness may start. In particular the function to transport an assertion from the collectively questionable to the collectively valid, the main identifier for argumentations by Klein, cannot be reconstructed in the data collection of the presented project. The question in most cases is like "is this statement correct?" and the answer ("yes" or "no") is given by the teacher before anyone starts to reason. Hence the question (or potentially existing quaestio in the terminology of Klein) is cleared, before the participants start a process of reasoning. Because of this observation, which is typical regarding the data collection of the presented project, the researcher does not follow Klein's approach in identifying argumentation processes in classroom interaction.

At first sight, this interpretation seems to fit the approach of Weingarten and Pansegrau. In fact the author of this paper also understands the start of an argumentation as an "initiation" but, according to the microethnographic approach of the project, this initiation is not only seen as a consequence of the teacher's power to decide whether a statement has to be reasoned. The initiation is understood to be a consequence of "interactional obligations between routines" in classroom activity (see Voigt 1994)[4].

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[2] Of course, this cannot be shown by only this short episode, but it is a result of one interpretation of the whole argumentation, which can't all be presented here (for this, see Schwarzkopf 2000).

[3] This also is a result of one interpretation of the whole argumentation. The transcript of this argumentation is given completely in the following section with a brief interpretation (for a more detailed interpretation, see Schwarzkopf 2000).

[4] For a brief discussion of this aspect see Schwarzkopf 1999, for a detailed discussion see Schwarzkopf 2000.

Furthermore, searching for reasons for statements which are undoubtedly true seems not to be unusual in mathematics. For example Vollrath (1980) proposes reasoning in mathematics lessons for already well known assertions. In the view of Winter (1983), it is typical for mathematicians to prove nearly every assertion, even if its correctness seems to be clear. In mathematics and mathematics education, proofs or argumentations are not only seen to fulfil the function of figuring out whether an assertion is correct, but whether the proof itself is of interest. Hence proofs are seen in functions like specifying knowledge, bringing knowledge in hierarchical order and building a network of knowledge (e.g. Winter 1983). One can suppose that the start of argumentation as an initiation would be typical for mathematics lessons.

Even if the presented project uses many aspects of Krummheuer´s research results, the understanding of argumentation, as integrated in narrative discourses, is not shared. The author could reconstruct, in his data collection from a pragmalinguistical point of view, that the participants do separate argumentations from other discourses such as narrative ones[5]. Indeed, argumentations could not be identified by the function to transport something collectively questionable into something collectively valid but (only) on the surface of language, as mentioned at the beginning of this article. However the researcher could reconstruct typical tasks to organise the argumentation processes in the approach of Klein, regarding argumentations defined by the author at the beginning of this article. This aspect will be discussed in detail in the following section, where the ongoing argumentation of the second episode is interpreted. Due to the space restrictions on this paper, the interpretation will be given very briefly and will concentrate only on the following question: In which way do the participants organise the tasks, which are typical for argumentation according to the approach of Klein?

**The ongoing argumentation: Organising the argumentation by doing specific tasks.**

In the presented project, the researcher uses a scheme developed by Stephen Toulmin to analyse the content-related structure of arguments with regard to several argumentative functions. Due to space constraints, the author will not use the complete scheme here, but only some terminology regarding two functions of arguments. One is the “conclusion”, i.e. the assertion for which the participants argue. The other is called “data”, i.e. statements which are seen as undoubted facts and given to support the acceptance of the conclusion. For a presentation of the scheme and its usage in the project’s analyses see e.g. Schwarzkopf 1999.

The teacher asked the children to reason the assertion $48 \cdot 14 = 40 \cdot 14 + 8 \cdot 14$. In the terminology of Toulmin, this assertion is called the conclusion if the members do indeed argue for it. The student Peter gives a first proposal for a reason in line 12:

12 P Because 40 plus 8 makes 48. Consequently, the solutions must be the same.

Peter gives a statement (40+8=48) and does this to support the acceptance of the conclusion (“because”, “consequently”). His statement obviously does not have to be reasoned. Implicitly, he shows an understanding of the necessary coherence between statements to develop an argument for the conclusions: “If we can be sure that some parts of the

[5] The author does not doubt that Krummheuer´s approach fits for group work among students. It is clearly important that in the data collection of the presented project the focus is on interaction between teacher and students and especially that the teacher does some important argumentation-specific tasks. Additionally, most of Krummheuer´s data collection is made in earlier grade classes.

calculations on both sides of the equal sign make the same result, than the equation is correct”. Hence, Peter does task 1 and 2. The teacher reacts as following:

| | | |
|---|---|---|
| 13 | T | Yes, well... and 50 plus 2 makes 52 but what does this- |
| 14 | S | Why plus 2. |
| 15 | S´ | I have minus 2. |
| 16 | S´´ | I too. |
| 17 | T | Yes okay, but. |
| 18 | Ss | (*laughing silently for a short time*) |
| 19 | T | This does not satisfy me as a reason. You have forgotten to say something in your reason. This was a little bit different, wasn’t it? Put your hands down please. Would it be enough when I say here 5 plus 5 is 10 as a reason. This is also true, isn´t it? |
| 20 | Ss | (*laughing silently for a short time*) |
| 21 | T | And because 2 times 3 equals 6, this is correct (points to “$48 \cdot 14 = 40 \cdot 14 + 8 \cdot 14$” on the blackboard). It’s rot what I’m telling you here, isn´t it, but nevertheless 2 times 3 equals 6. What has the assertion of Peter, 40 plus 8 equals 48, to do with this task (points to “ $48 \cdot 14 = 40 \cdot 14 + 8 \cdot 14$” on the blackboard)? Michael. |

In lines 13 to 21 the teacher tries to make clear that producing undoubted facts in the data function would not be enough and a more detailed argument has to be developed. Thereby, he does not doubt the correctness of Peter’s statement but he points to other clearly true statements. These statements would surely not be accepted in the function to support the conclusion. Following this interpretation he requests a link between data and conclusion to complete the given fact to an argument. In the terminology of Klein, one can say that the teacher tries to initiate a discussion on the coherence between data and conclusion (second task). Some of the students criticise in a more or less serious way the relevance of the teacher’s remarks (lines 14-16). Doing this, they show that the teacher’s statements would not be coherent to the questioned conclusion. They thereby force the teacher to repeat his critical remarks in more detail. The teacher accedes to this demand and hence the students have the “power” to do the second task.

| | | |
|---|---|---|
| 22 | M | It is easier to calculate. One can figure out 40 times 14 by calculating 4 times fourteen and then hanging a zero after and 8 times 14 is easier than calculating 48 times 14, too. |
| 23 | L | You mean that it’s correct, because it’s easier? |
| 24 | S | It does not become easier at all. |
| 25 | L | It is a more clever way, that’s right, but- Kevin |

Michael points to another aspect of the given task, namely the cleverness of calculation. Probably in his point of view, the title of the homework (“calculate in a clever way”) is of importance for the topic of the argumentation. The task is done right if the calculation is done in a clever way. In these lines, a frame-difference between the students and the teacher can be attributed. While the teacher obviously wants the students to argue for the algebraically correct term “manipulating”, Michael argues for the cleverness of calculation. By his reaction in lines 23 and 25, Mr. Zander is doing two different tasks of argumentation. On the one hand, he agrees to the correctness of Michael’s statement (task 1) – no reason for this will be necessary from his point of view. On the other hand, he shows doubts about the possibility of the co-ordination between arguments for the cleverness of calculations and those for the correctness of equations (“that’s right but” and then calling for another student). This means that he also does the third task of argumentation, i.e. he co-ordinates different arguments regarding the assertion that he wants to be reasoned. At the same time in line 24 a student doubts the correctness of Michael’s statement and thereby does the first task of argumentation – in his opinion, the proposed calculation seems not to be very clever.

The student Kevin then tries again by arguing:

| | | |
|---|---|---|
| 26 | K | It’s correct because at the beginning of the task there is already 50 minus 2 in brackets times 14. |
| 27 | S | Yes but this is not the task that we are doing, we have 48 times... |
| 28 | K | If one does it like Alexandra. Alexandra also calculated first what’s in the brackets and this is what he could have done too. Then he wouldn’t have 50 times 14 minus 2 times 14 but 48 times 14. This is cleverer. |
| 29 | T | Franz. |

Kevin compares the calculation proposed by Franz at the beginning of this episode with a calculation done by Alexandra some time ago. Obviously, Alexandra’s calculation was accepted to be right and was based on firstly calculating the addition in the brackets and subsequently multiplying the result. This can be seen as a reason based on a comparison between an already accepted calculation and the one under discussion. So Kevin does task 1, stressing experiences with accepted calculation methods. He also mentions the aspect of cleverness and shows by this a calculation–bound framing of the argumentation. Following this interpretation, Kevin produces another statement that fits both the cleverness of calculation and the correctness of the equation. The student in line 27 stresses that Kevin is arguing on the “wrong” task, without doubting the correctness of the reasons. Hence he is co-ordinating arguments (task 3) by stressing that Kevin’s remarks would not lead to an argument for the “right” conclusion.
The teacher does not commend these remarks but calls for another student.

| | | |
|---|---|---|
| 30 | F | Because 40 times 14 plus 8 times 14 has exactly the same value like 48 times 14. |
| 31 | T | This is what you assert, but for this I have to calculate it again, I’m too lazy to do this. |
| 32 | S | How shall one explain this in a different way? |

Franz, the student who produced the question leading to the initiation of this argumentation, argues with aspects of values. The teacher’s reaction in line 31 can be classed as two tasks at the same time. On the one hand, he stresses that Franz’ statement has to be reasoned by calculating, thus it can’t be accepted immediately (task 1). On the other hand he marks this kind of reasoning as out of the question. This is the second task of argumentation i.e. the teacher stresses that the only possibility to reason for Franz’ statement (namely “calculating”) would not lead to a coherent argument. By this, Franz’ statement is marked as irrelevant regarding the argumentation. In line 32 a student expresses the difficulties of this argumentation for at least some of the children, namely that they have to produce reasons of a strange new kind.

| | | |
|---|---|---|
| 33 | T | Bernd |
| 34 | B | Because we have used the distributive law of multiplication and addition. And there we've got it standing in the brackets. Left bracket, 40 plus 8, right bracket, times 14 is what we could also write there. (*teacher writes on the blackboard*) This would be all the same; we have the distributive law of multiplication and addition in there. |
| 35 | T | (20 seconds, cleaning parts of the blackboard) |
| 36 | S | Couldn't one leave out the second line? |
| 37 | T | One can, but as you have seen, we left it out in the middle step and there were several (problems), I don't know why this was so difficult. Did you just think, "I don't have to mention the distributive law" because it was used all the time or Michael, didn't you even think about the distributive law? |
| 38 | M | Hmm, I don't know. Actually I did not. |

After the teacher's writing, the blackboard shows the following:

"$48 \cdot 14 = (40 + 8) \cdot 14 = 40 \cdot 14 + 8 \cdot 14$".

In line 34 Bernd produces an argument which is immediately accepted by the teacher. The remark of a student in line 36 again indicates that a calculation-bound framing is obviously nearer to the students than an algebraic one. At this point, the argumentation ends and the participants change the subject to another task.

According to this interpretation, the participants play different roles regarding the tasks, with which they organise the argumentation. As one can see, it is not only the teacher demanding the children to produce the reason that he wants to hear. Even if the children do have to produce a special kind of argument (using the distributive law), they are actively included in organising the argumentation and may do every necessary task. In the presented episode, the students do every one of the three tasks. The author selected this episode to illustrate that the students can do this in contrast to the approach of Weingarten and Pansegrau. They are especially "powerful" enough to oblige the teacher to do the tasks of argumentation in an adequate way.

Typically for the observations of this project, the students often do task 1 and 2 at the beginning of argumentations. The pupils produce statements (and reason them if necessary) to suppose the questioned conclusion. The teacher then asks the children for links between these statements and the assertion which has to be reasoned. The children often do not produce these links in any kind of formal rules, but they produce further statements and by doing this, they give hints as to their underlying frames. In fifth grade classes, the researcher could reconstruct two typical argumentation frames which can be classified as "calculation bound" and "more algebraic bound" (see Schwarzkopf 1999). Many students understand the content of argumentation in the form of calculation bound frames. They then often argue for the cleverness of a calculation. Their arguments are not wrong, rather that they only do not lead to the questioned mathematical conclusion (from the viewpoint of the teacher). In these cases, the teacher typically does task 3 of argumentation and tries thereby to make clear that arguments based on this frame would not lead to the questioned conclusion. Hence, especially task 3, i.e. to make sure whether different arguments can lead to the questioned conclusion, may fulfil two important functions for the learning process of the children. They firstly have to "modulate their frames" (see Krummheuer / Voigt 1991), i.e. they have to understand the content of the argumentation in a more adequate (in the sense of school mathematics) way. Secondly, the students may generally learn and practice arguing. The task to be done in argumentations is not only to produce correct arguments, but it is also necessary to co-ordinate arguments that lead to the questioned conclusion. This co-ordination depends on standards which are bound to the subject of the argumentation and to the demands of the arguing group.

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