Dynamic Geometry Software (DGS) in Teacher Education

Peter Bender
University of Paderborn

At Paderborn University the course in elementary Euclidean geometry for prospective primary and lower secondary school teachers has been changed from classical chalk-and-blackboard work to a multi-media environment which is based mainly on the use of the DGS Cinderella. Encouraged and supported by a research programme initiated by the administration of the German state of Nordrhein-Westfalen we started in the year 2000 to investigate the effect of this environment on how students learn mathematics, in particular geometry. In this article I give an outline of the research design and report on some preliminary results.

The paper is based on lectures given by the author in 2001 on several occasions, in particular at the annual meeting of the GDM 2001 at Ludwigsburg.

1. Introduction

Although geometry plays a rather modest role in mathematics education in German schools, it is at the same time an unquestionable part of the university curriculum for prospective primary and lower secondary school teachers. In Paderborn we offer a course in elementary Euclidean geometry of the plane to our first semester student teachers. Our course scarcely goes beyond the contents of the secondary school curriculum, but requires a higher level of reflection, in particular geometrical reasoning. For a long time, we gave the course in the classical pencil-and-paper and chalk-and-blackboard manner.

When in the mid 1990s new media became more readily available, my colleague Prof. Hans-Dieter Rinkens changed the character of our geometry course from manual to electronic media. He presented the geometric situations through Cabri geomètre, the world's first dynamic geometry software (DGS; released in 1988; see Laborde 1985), embedded in a Power Point presentation. He could do so because the students were not meeting geometry for the first time, but had already some experience of it at school, at least with graphite drawings on paper. Of course, there was still much chalk-and-blackboard as well as pencil-and-paper work and a great amount of verbalization. This ongoing importance of old media is part of the concept, not least bearing in mind our students' future profession as school teachers.

In the winter of 1999, Prof. Rinkens switched over to another DGS, Cinderella (developed by Richter-Gebert & Kortenkamp 1999), because this one could already be used on the internet at that time. Since then the students have access to the written material via the internet, and can make use of the main strength of DGS when working with the material, i.e. they themselves can continually change the given geometric situations.

One year later, in Prof. Rinkens' second run of the course with Cinderella, we started a research project on the impact of this kind of multi-media environment on our students' learning of geometry. From August 2001, this research has been supported by the competency network "Universitätsverbund MultiMedia NRW" (UVM) founded by the administration of the German state of Nordrhein-Westfalen (NRW). This is part of their global research programme about the effects of university teaching with multi-media. The main work since then has been done by Dorothee Maczey who is writing her doctoral thesis about this subject. This report deals mainly with the pilot phase in the winter of 2000/01, the original ideas and methods and a few preliminary results.

We were not only interested in the media aspect, but we also concentrated on the issue of didactics of geometry (in a wider sense), and we adjusted our work to the following leading questions:

— How do the students perceive the use of new media — cognitively, emotionally and socially?

— How does a multi-media environment change the learning of geometry?

— How will the curricular selection of contents as well as the epistemological character of the geometrical concepts change? (This latter question concerns the teachers much more than the students and it requires long-term consideration.)

33 persons attended the course, 31 took part in the test at the end and 24 passed it (but one can learn more about the students' performance from the interviews than from the test). We interviewed 18 students, most of them in December 2000 when the course was still running, and a few in February 2001 after the test.

Of course, our students are not 13 to16 year old school pupils, the main target group of didactics of geometry, but 19 to 22 year old university students, and they were not dealing with Euclidean plane geometry for the first time. Still, conclusions can be extrapolated from our results to secondary school level as there are rather similar traits in the whole area of teaching and learning geometry (notwithstanding the very different social structures). This is especially true since, in general, only little substance from the geometry lessons taken at school seem to be retained. We also expected to benefit from the higher age of our students through their experience, reasonableness, and professional inclination. They should be better able to undertake strenuous interviews, show a wider horizon when answering our questions, and produce more serious statements about their own thinking processes. In the end our expectations were confirmed.

The course itself is a normal university course: 16 weeks long, three 45 minute lectures per week given by the professor in a "normal" lecture room and based on Cinderella drawings. These are downloaded from the internet, via a laptop, and projected at the wall with the help of a beamer. There is a lot of chalk-and-blackboard work and, of course, verbal instruction including questions and answers from both lecturer and students. Each week the students had to do some homework in groups of two or three: they usually had to find and work out geometrical constructions and theorems with proofs, either as applications or as supplements to the lectures. Very often they had to create Cinderella drawings and then manipulate them in order to formulate and work out ideas. They were stored on a diskette and handed over together with the written part of the homework to two older students who gave written comments and corrections.

There was another 90 minute "training" lesson per week for discussion of the students' solutions and general questions. It took place in a special room with 11 computers (one for the tutor, ten for the students), ordered in a circle with the screens facing out, so that the students and the tutor could communicate easily. As at most two students shared one computer, the whole group was divided into two subgroups, and the "training" lesson was given to each subgroup separately (by Prof. Rinkens' collaborator). All the computers were linked in a so-called pedagogical network which could be controlled by the tutor who could transfer the contents of one screen to the other screens thus allowing the students to show their solutions to the others as the basis for the discussions. This computer room is open all day and, when there are no lessons in it, students can work there which is an important option for those who have no access to a computer elsewhere.

The expectation that students can and will work independently and self-reliantly is cited as one of the main benefits by the advocates of the use of computers in education. We share this belief, but with an accent on didactically-related subject matter and only indirectly on the following pedagogical categories:

The students shall develop the ability to solve and acquire strategies for solving geometrical problems independently. DGS seems to be created exactly to support this goal;

When continuously moving and deforming geometrical figures, the students can observe greater or lesser striking changes and invariants. By using given procedures ("macros") or creating their own, the students realize one of the most important problem-solving strategies, namely to divide a problem into smaller ones, to solve these individually and to use them as modules for the solution of the global problem (which, unfortunately, is not yet implemented in Cinderella, but is, for instance, in Cabri);

The students can make exact (in a certain sense), neat and coloured drawings in a relatively short period of time. Thus they are freed from technical details and can concentrate on higher level activities, namely problem solving.

2. Method of research

Obviously, quantitative methods (including the final test of the course) are not suitable for pursuing our research questions. (This is true of a lot of educational research, because in most of the cases where such methods are applied the samples are not representative, the participants are not independent and the questions are not valid.) Instead, we worked in a qualitative manner: we interviewed the students (always two at a time for economic reasons, but also with the hope that they would stimulate each other), recorded the interviews on video, made transcripts of several scenes which at first glance seemed to be interesting, and interpreted these video scenes with the help of the transcript...

The interviews were based on a detailed questionnaire which included two geometrical problems. As I myself was the interviewer, I took the liberty of slightly modifying the questions spontaneously, leaving out some questions because of lack of time, or speeding up the discourse by intervening now and then. (In the winter of 2002/03 I was the one who gave the lectures, and then, of course, I was not involved in the interviews, nor did I take part in the interpretation sessions.)

This kind of (partly) unrestrained interview, as well as the central role of the interpretations, are both founded on the "existence of generally shared rules of social actions, which have a claim of validity on their own, and a hidden objective structure underlying the interaction" between the interviewer and the interviewee, in the sense of Oevermann (1986, 22ff). The objective character is intensified in that the subject of the discourse is part of a strongly rationalized domain, namely elementary geometry, as well as the thinking and learning processes related to it. However the structure of the meaning of the discourse is very different from some fictitiously objective system of mathematical concepts, and at first it really exists only in a latent manner. Yet the interviewer claims to be an expert in geometry and in the teaching of geometry as well as being experienced in talking with students. He therefore should be able to note the latent structure of meaning apparent during the interviews, at least rudimentarily. Whether he is right must be judged from the interpretations.

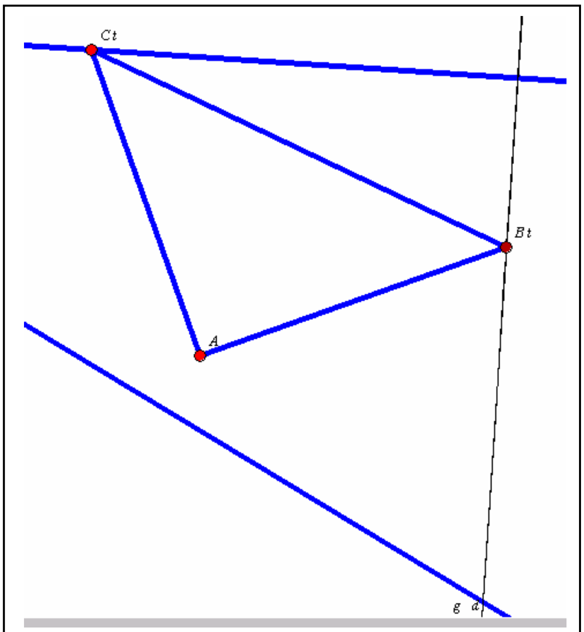

Fig. 1

Here is, as an example, one of the geometrical problems from the interviews, as well as the way how J, one of the students, dealt with it, and our interpretation of the scene.

Between the two sides g and h of a suitable (I will not discuss here what "suitable" means) fixed angle there is a fixed point A. The students shall construct a right-angled isosceles triangle with the right angle at A and the two other vertices on g (say, B) and on h (say, C).

This problem had been discussed in the "official" course as a prototype for a special DGS strategy, namely the locus strategy. One puts one vertex, e.g. C, on its side (h) (I will call it C_t, as it still can be moved on h), and constructs a right-angled isosceles triangle with A as the right angle vertex and C_t as the second vertex, i.e. one has to construct the third vertex B_t (choose one of the two possibilities, e.g. that one where AB_tC_t is oriented counter-clockwise). Then all constraints of the problem are fulfilled except one, because B_t does not lie on g (see fig. 1).

This is the well known (n-1) strategy, a notion which comprises the locus strategy but stresses a different feature, namely: at first only $n-1$ constraints out of n have to be fulfilled, and then the situation has to be changed so that the n^{th} constraint will also be fulfilled without dropping one of the others (cf. Weth 2002). In DGS one of the main variants is to find and to analyse the locus of some point.

It is now clear that if C_t is moved along h and the $n-1$ constraints are kept fulfilled, B_t changes its position, depending on C_t, and it is hoped that there will be at least one point C_t whose corresponding point B_t lies on g. In fact, one can see (not logically, but visually) that there is such a point, and where it is situated, by moving C_t along h. In Cinderella, as in all other DGS, an option is implemented which produces the whole locus of B_t at once. There is a special button with which one can activate this locus option. In general: If the motion of C_t can be considered as a "sufficiently tame" function $C: (0; 1) \to \mathbf{E}: t \mapsto C_t$ from the real interval $(0; 1)$ into the Euclidean plane $\mathbf{E}$ with a line h as its image, and if B is a "sufficiently tame" function from h into $\mathbf{E}$, then the composition function $B \bullet C: (0; 1) \to \mathbf{E} \to \mathbf{E}: t \mapsto C_t \mapsto B_t$ delivers the induced motion of B_t on its domain d, the locus of B which is again a line. The locus option of Cinderella requires identification of the point C, its locus h, and the point B; it then produces the locus d of B.

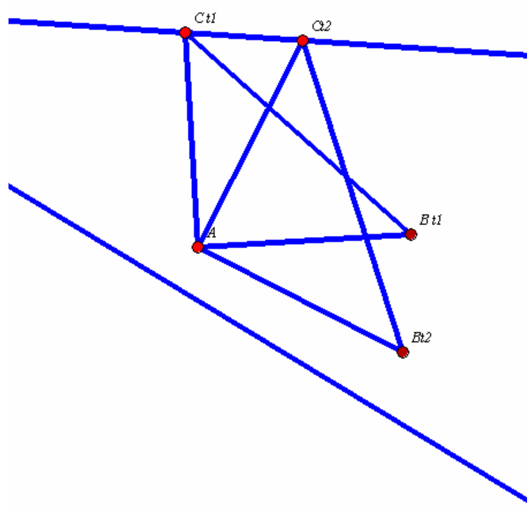

Fig. 2

In our case d is a straight line, perpendicular to the straight line h. One has only to find the section $\{B_s\}=d\cap h$ in order to produce the exact position of the locus d and its section $\{B\}=d\cap g$ (which exists because of the supposed suitability of the angle), thus completing the whole construction. This is because the right-angled isosceles triangle with the vertex A (with the right angle) and this vertex B ($\in g$) fulfils all constraints, in particular $C\in h$. B_s can be constructed easily: Draw a line from A perpendicular to h, paste a 45° angle to it with vertex A (in the right direction), and the section of the second side with h yields B_s.

One important question is still unsolved: How does one know that the locus d of B_t is a straight line? If one knows this, one can do without the fact that it is perpendicular to h. One just takes two different points C_{t1} and C_{t2} on the line h, constructs their corresponding (different) points B_{t1} and B_{t2}, and draws the straight line d through them (see fig. 2). This geometrical problem yields the following central didactical problem: The students must, in short, understand that there is still a need for a proof that the locus d, which looks like a straight line, really **is** a straight line. In my opinion, the curricular solution of this didactical question determines whether the attempt to keep geometry at school alive, with the help of DGS, will be a success or a failure.

In transformation geometry the construction of the locus d of B is not very difficult. Consider the clockwise rotation about A with angle $90°$, assigning to each point C_t its corresponding point B_t and to the straight line h its image k, which is also a straight line, inclined by $90°$ to h. As all points C_t lie on h, all points B_t lie on k. It is also clear that for each point B on k there exists a point C on h, whose image under the rotation it is (some more considerations are actually needed, because, in fact h and k are both only half lines and one actually has to prove that they really meet. However this is not the crucial question here). So $d=k$, and $k \cap h$ is the section in question.

In our course this problem was given rather early, before transformation geometry had been developed, and the straightness of the locus d had to be concluded, with no little effort, on the basis of the theorems of congruency. However the arguments of transformation geometry are not really easier, as they make use of the fact that the images of straight lines under congruent transformations are again straight lines. In the didactics of classical transformation geometry (up to the 1970s) the epistemological role of this fact was one of the main issues: Either it was given as an axiom, or, if the theory was built on axioms of congruency, it had to be proved or at least be identified as having to be proved. A lot of supporters of transformation geometry in schools did not recognize the didactical question (whether the pupils should develop insight into the essence of this fact or indeed even prove it), let alone tackle it (cf. Bender 1982, Schwartze 1990).

In the actual discussion about DGS in schools there seems to be much more awareness of didactical questions of that kind, at least in published papers. There it is clear that every important fact, which can be observed visually when dragging drawings or producing loci, has to be proved mathematically or at least be made plausible on some, maybe very low, level of formalization and exactness.

One essential didactical task is to support students in developing the conviction that important facts which are given visually have to be proved as a matter of principle, even if there seems to be no doubt about them. According to this fundamental goal of geometry teaching our students differ considerably from 15-year old pupils. During the lecture course the necessity of proof was articulated on many occasions. A lot of proofs were given by the professor or had to be done by the students, and in the interviews they all expressed their insight into the importance of proofs in geometry. We tried to find out whether this insight really was part of their cognition or whether it was a mere repetition of the professor's words.

Now I come to J’s way of tackling the problem of the right-angled isosceles triangle and our interpretation of her actions. J started with the construction on the base of two points C_{t1} and C_{t2} and their corresponding points B_{t1} and B_{t2}, but I influenced her to work with the locus button because this was what we wanted to see. After the locus d of B had been produced in the correct way and the point C had been dragged to the position C_{t0} so that the corresponding point B_{t0} lay on g, I asked J whether this was a construction in the sense of what she had learned in the course. J said no to this question with the following arguments (translation by me): " If I did this by hand, that is, with only pencil and ruler and compass, it would not work because I produced the locus even though I had only one point at my disposal, and one cannot do this, if one ... uses the ruler and the compass. That is, I would need two points. For that reason I had the idea that one needs two triangles. Then one has two points, and one can construct the locus through these points. If one works with a ruler, for example, one can connect these two points." Even though J was one of the better students, we refused to take it for granted immediately that for her, on the one hand, the drawing of the locus by the computer based on one point, and, on the other hand, the classical construction of the locus with pencil and paper based on two points, really meant two different principles of reasoning in the logical, mathematical and epistemological mode. Instead we first discussed the option that for her the difference could lie mainly in the mode of media used. However when J continued her work on the problem using the two-point construction she had started with and emphasized that the locus, which had been drawn with the help of the locus button and was still shown on the screen, could be wiped out without interfering with her construction, we finally conceded that she had had insight into the different status of the two ways of producing the locus in every mode (although she managed only to produce the construction, not the proof of its correctness).

As already mentioned, our method of interpretation follows the system of "objective hermeneutics" in the sense of Oevermann. The analysis of the interviews requires a lot of time and for economic reasons only a few scenes (out of twenty hours of video material) can be interpreted in a sufficiently painstaking manner. We believe that the objects of the communication, with their subject matter structure (geometry) as well as with their didactical structure (learning of geometry), have an essential influence on the structure of meaning in the social situation (interview). It plays no role, if the meanings of the objects are constituted by, and agreed between, the participants (at some time, in a special context, in a learning process under more, or less, control etc.). Possibly Oevermann himself, and some advocates of the pure theory which only admits sociological categories for the interpretations, would not join in our didactical definition. However why should one be restricted to sociological categories when one finds obvious, relevant non-sociological structures. Hölzl (e.g. 1994), vom Hofe (e.g. 1998) and Friedrich (2001) in his doctoral thesis overcame these restrictions advantageously.

3. Some students' reactions

I will first give a summary of the answers to the questionnaire, and then I will outline the second geometrical problem discussed in the interviews and how the students dealt with it in more detail (having already introduced the first problem in the second chapter).

Today, nearly all students have already worked with computers when they come to university, usually through word processing programs or computer games. For a few computer science had been an (optional) subject at school and one person had even met the computer in geometry teaching. They all had had geometry lessons at school and had met some of the same content as in our course, but, in contrast to it, largely without formal proofs or informal argumentations (as far as they could remember).

The students express considerable satisfaction with how the computer is integrated into the course. Their relations, and communication, with the professor, the tutor and each other are not affected by working with the computer. On the other hand, nobody wants the use of the computer to be intensified or even to dispense with the professor's presence in the lessons. Their views can be summarized as: "We want to be addressed by a human being." I am convinced that this would still have been the case if they had been a group of 500 (instead of 33) as then all individuality is lost.

There are people who predict that within the next ten years the majority of university courses for the majority of the students will take place without the presence of the professor and his collaborators. I have severe doubts that the principle of teaching at universities will develop in this direction, and, what is more, I think that this would be detrimental to the students' interests and welfare. My idea of good teaching includes the students being addressed personally. This holds more for students who aspire to a profession where they will have to work with (young) people. (Indeed, even this latter argument is disputed by some people claiming that in schools teachers can, should or will be replaced by machines. All well-meaning members of the educational system should stand up against such a development.)

The students approve of working with computers at school, but only in an appropriate manner. In particular, they think that the ability to work with ruler and compass is still important and they envisage the computers being installed in a separate room instead of being available to the pupils in their classroom all the time (in Germany it is the teacher who changes room to work with the next class, whereas the pupils in general stay in the same room during the whole morning). This vision of a separate computer room probably indicates that the students' statements are not always based on sound didactical reasoning (which no-one had claimed), but possibly on reminiscences of their own time at school.

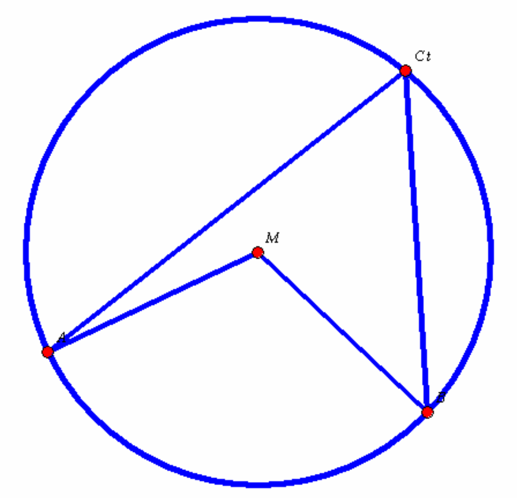

Fig. 3

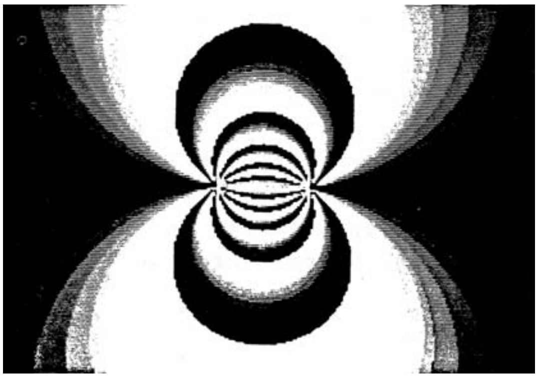

Fig. 4

I also asked about the benefits and shortcomings of DGS, the students' views of geometry with and without DGS, the ontological status of points, figures, loci and figures in motion and corresponding heuristic strategies with and without DGS. I will now concentrate on the question: "Why do you drag?" (What is the drag mode good for?)

The students mentioned, in a rather hesitant manner, "to see what happens", "to get hints for the solution", "to create special cases", "to test conjectures", "to observe the invariants and the changes". They cited a few examples from the course, like the Pythagorean Theorem, the Euler line, similar triangles and proportion and the problem of the biggest visual angle, but in the end almost nobody was able to demonstrate the essence of the named didactical functions with a geometrical example in a passably convincing manner. We know from many oral examinations of the widespread inability among older students to substantiate such catchwords, never mind first year students. Perhaps (from a university didactical point of view) the professor did not concede enough autonomy to the students through his teaching style. Maybe (and in fact, this is my opinion) he was too cautious and over-estimated the students' competence to develop such insights by working autonomously or by following his actions during the lessons without always making his ideas and strategies explicit.

Consequently the students knew little about angles inscribed in a circle and the corresponding proofs, although this had been a subject of the course. Given: a fixed circle with centre M, two different fixed points on the circle A and B, one of the two open arcs of the circle determined by A and B named d, and a (semi-) variable point C_t on d. Then for all points C_t on d the angles AC_tB have the same size. This follows from the fact that all such angles AC_tB have half the size of the central angle AMB (see fig. 3).

When I gave the course two years later, I embedded this theorem in a more general situation (see fig. 4). At first, I did not draw the circle at all, only the two different fixed points A and B, and let the domain of the third point C_t be the whole plane (without A and B). We then conjectured that the lines, on which measurement of the angles AC_tB does not change, are circular arcs from A to B (a concept similar to contour lines in geography, isobars and isotherms in meteorology or lines of constant cost in economics). Here we have a prototype of functional thinking at its best; maybe the students develop the feeling (and even a sense of urgency) that a proof has to be given. The theorem is now much more general, and one immediately obtains the converse of the theorem about inscribed angles as it is often used in geometrical applications.

In fact, because of lack of time, one generally starts with the situation where a circle is already drawn. When I gave this geometry course in the 1980s, I also did it this way. I remember that at some point I always had to check my notes to see how the proof continued. I can therefore sympathise with the students who were unable to reproduce even a basic version of this proof unless given a lot of help. During several phases of the proof I urged them to drag the point C_t along its arc so that they would meet special cases. They could then determine whether they had found all cases, or perhaps be inspired to transfer the treatment of one case to another. The issue here is the role of continuous movements and deformations as a means of structuring the body of a proof (cf. Bender 1989).

In the course, the starting point consisted of a circle with M not on the segment AB, and the arc d for C_t being the longer one of both arcs between A and B (as in fig. 3). At first the special case of A, M and C_t being collinear was taken into consideration (fig. 5). There the (central) angle BMA is the exterior angle of the triangle C_tMB in the vertex M and has the same size as the two interior angles in the vertices C_t and B added together. As the sides of the triangle MB and MC_t are radii of the original circle, they have the same length and consequently the angles MBC_t and BC_tM have the same size, that is, in particular the angle BC_tM on the arc d has half the size of the central angle BMA.

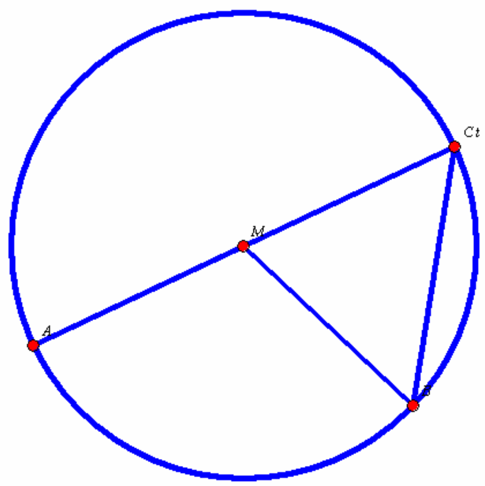

Fig. 5

Several cognitive obstacles exist however:

The students must know the properties of exterior angles and be able to apply them flexibly;

They have to consider a triangle (C_tMB) that is different from the one they started with (ABM), as well as a different angle (BC_tM instead of BC_tA);

They then must see the correspondence between the exterior and the interior angles which is impeded by the occurrence of the chord AB (which in fact could be omitted) as well by the free side MA cutting the exterior angle BMA (because one is used to such a side being infinitely long). By the way, if one could actually draw this line longer, namely as an infinitely long half line from C_t through M, one would be in a stronger position to appreciate the next step i.e. leaving the special, collinear case and entering the general case when A no longer lies on this half straight line. However in the special case there is no reason to draw this line.

In visual mode the structure of the general case is completely different from that of the special case: there is no longer a triangle and an exterior angle. On the basis of the students' actions in the interviews, we had intensive discussions about the question as to whether one could expect the students to come up with the idea of drawing this very helpful half line from

C_t through M. This is because of the following two strong arguments: (i) So far, no use had been made of the fact that C_t lies on a circle through A and B and therefore the segment C_tM is as long as the segments AM and BM . Just draw C_tM (as part of the strategy: "draw any special line which may be relevant and see what happens") and thus get two triangles of the type as seen in the special case. (ii) Then try to transfer the application of the properties of the exterior angles from the special to the general case (as part of the special case strategy). In the end however we decided that one has to know the proof already before one is really in a position to apply such strategies. This is the crucial point with the whole field of problem solving: All those nice strategies are appropriate for structuring problem-solving processes subsequently, but even if the most experienced experts apply them systematically, there is no guarantee that they can find the solution of a really new, non-trivial problem in a straightforward way. Even the DGS does not help in our situation, although one could drag the point C_t a little bit to the left and to the right, thus leaving the special case, and one could observe what happens. Mentally, the special case does not easily transform into the general case, and the additional half line does not readily appear.

There is another special case which possibly is more suggestive: If one draws C_t exactly in the middle between A and B , the whole diagram is symmetric with respect to the axis through C_t and M , and it is hoped that the students "see" and then draw the axis of symmetry, thus producing two triangles with their interior and exterior angles (fig. 6). These two parts can now be treated separately, each like the special case. Of course, once again the properties of exterior angles must be at one's disposal. By subsequently dragging the point C_t it can be seen that the argumentation remains valid when symmetry is eliminated. In fact, the symmetry is completely irrelevant; it only helps to "see" the axis of symmetry, which, however, is not used as such but for some quite different purpose. One can regard the role of this axis as an inadequate crib (as well as the drawing of the radius C_tM , on principle), but using cribs is an important mega-strategy in the field of heuristics, in particular geometry.

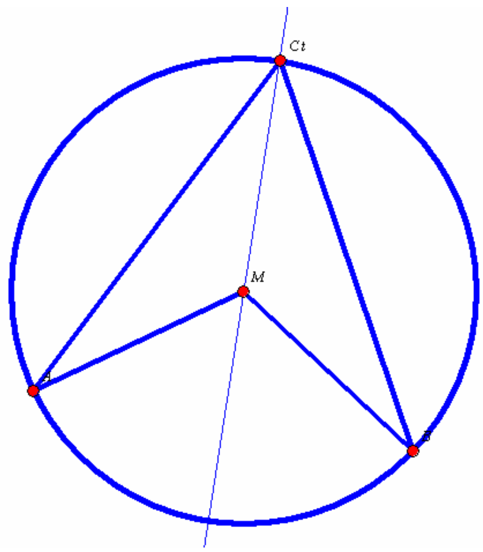

Fig. 6

In fact, it turned out that the best approach for all concerned was for the proof of the theorem to be given by the interviewer and then summarized by the participants with the help of the drag mode. This had been the starting point for this part of the interviews. It was not important that the students should work out the proof, and so I completed the special and the "normal" general case (with acute inscribed angle and addition of the two parts) myself. After the dissection into two triangles the students should only inspect all cases by moving C_t around the whole circle (including the second arcs). They took into account the case with the obtuse angles only after prompting from me, but they did not recognise the existence of the case where the two angles have to be subtracted. They moved the point C_t much too fast and only slowed down when I urged them to do so; the subtraction case then appeared right before their eyes yet still they did not become aware of it.

Usually I am sceptical about teachers' reports on what "the" students are able or not able to do, because in the ordinary classroom situation they can only know about the performance of a few of their students. Even then they have to rely on some transitory and vague remarks, and written tests also only disclose a rather narrow section of the students' conceptions, abilities etc. Compared with this, our observations are based on much more exhaustive research (although throughout we had two students per interview and I did not manage to talk with all of them about angles inscribed in a circle. However it can be assumed that in general the mathematically stronger ones were those who took the lead roles in the interviews).

4. Some preliminary findings

4.1 In general, students do not show enough patience and perseverance to change geometrical situations deliberately and purposefully and to observe the resulting striking phenomena. Of course this is due to the interview situation but I can also identify the so-called bully effect (i.e. the human user feels provoked by the computer to perform some action regardless of its goal or sense). In my opinion this effect is reinforced by some traits of modern pedagogy which overestimate students' autonomous behaviour. Instead students must be educated and trained to cope with learning subjects sensibly, systematically and persistently, together with some flair for contemplation. They require extensive assistance as well as careful guidance at least at the beginning of the learning process. Obviously our students are at the beginning of their work with DGS. Dragging and observing has not yet become second nature, even if the professor has demonstrated it and had made them practise it intensively. We, often incorrectly, assume this to be trivial.

4.2 When looking for suitable problems for the interviews, we noticed that a classification system related to DGS and a corresponding overview is still missing. There are attempts to rectify this (e.g. Schumann 2001), but they need to be extended and refined.

As Hölzl (2000) remarked, arguments from transformation geometry are often suitable for proving conjectures which were discovered when applying the drag mode to some geometric situation. This induces the danger that a special kind of gap in the proofs is just shifted from the drag mode to the corpus of transformation geometry. A lot of people are aware of the problem of whether the locus of a specific point, produced with the help of the locus button, is a straight line, but at the same time they tend to assume that the image of a straight line under congruent transformations is also straight. However this gap can be closed in transformation geometry once and for all, and then this fact can be used when dealing with DGS, as for example in the problem with the right-angled isosceles triangle where there is no need of an extensive argumentation based directly on axioms of congruency.

4.3 In general, our students knew that a conjecture, found visually with the help of DGS (as well as the correctness of a construction), has to be substantiated in some more or less formal way if it is to be turned into an assertion. They seemingly had no conceptual problems with the existence of two such antagonistic cultures within the domain of geometry learning and with the requirement to harmonize them. Compared to the students, this harmonization is much more fractious for 15-year old pupils, and in general they withstand it, as Hölzl (1994) and others observed. It seems that the problem of motivating pupils for proving and reasoning, one didactical core of geometry teaching, will be aggravated by working with DGS, at least in school geometry.

4.4 Because of the different ontological status of the objects, the mentioned antagonism is even deeper than many members of the DGS community suppose it to be. Whereas transformation geometry and the point set interpretation of Euclidean geometry have always been inseparably connected, and congruence geometry can also be treated as being consistent with the universal mathematical idea of static sets, the objects of DGS now are movable. Their mathematical formalization requires a higher conceptual effort because now, in the language of sets and functions, one has to deal with continuous functions from a real interval $\mathbf{I}$ into the Euclidean plane $\mathbf{E}$, and, more generally, with respect to figures $\mathbf{F}$ with continuous functions $\mathbf{I}\times\mathbf{F} \to \mathbf{E}$ which, restricted to $\mathbf{F}$, preserve lengths, angles or areas or ... By the notion of a movable point C' (or similar) is meant the function C and its values C_t ($=C(t)$) with $C_0=C$ (!) respectively etc.

This problem cannot be reduced to mathematical formalism, instead it is an epistemological one essentially involving "basic imagery and understanding" for geometrical concepts (cf. Bender 1998). Maybe one can avoid it in geometry teaching for a long time or even forever, but I think that it has to be a constituent part of the geometry curriculum, and should be discussed with the students, regardless of whether it emerges by itself or not.

4.5 Schumann (1991, 119f) and others always stressed the great influence of mental tools on the formation of concepts. They claimed that even small differences, as between two DGSs, could yield different geometrical concepts. I tended to play down such effects in the interplay of all influencing variables but in our investigations we may have found a remarkable effect of such a small difference: A lot of our students exercised a strange restraint towards drag mode even in those cases where its uses were obvious. We identified the following possible cause:- The DGS Cinderella is not equipped the trace mode (whereas Cabri, for example, has it). In the trace mode the locus of a point B is produced (according to the motion of another point A on which B depends by a geometrical construction) in real time. While A moves and visits one point after the other in its domain, B moves in its corresponding domain (its locus) creating a trace. A change in A's velocity yields a corresponding change of B's velocity, etc. In Cabri the trace is even shown as a discrete (which here means finite) sequence of points. This gradual coming into being of the trace, point by point, seems to be helpful for students to better understand the conceptual basis of the locus strategy. However that may be, Cinderella only has the option of producing the locus of B globally all at once (in Cabri this option is also available), and the local connection between the motion of the point A and the emergence of the locus of the corresponding point B does not grab one’s attention as in Cabri. On the other hand, it must be admitted that several students brought about this local effect by applying the drag mode to A after they had produced the locus of B with the locus button, thus visualizing the two corresponding motions in their respective domains simultaneously.

References:

BENDER, PETER (1982): Abbildungsgeometrie in der didaktischen Diskussion. In: Zentralblatt für Didaktik der Mathematik 14, 9–24

BENDER, PETER (1989): Anschauliches Beweisen im Geometrieunterricht — unter besonderer Berücksichtigung von (stetigen) Bewegungen und Verformungen. Main lecture on the 7th workshop on visualization in Klagenfurt 1987. In: HERMANN KAUTSCHITSCH & WOLFGANG METZLER (eds.): Anschauliches Beweisen. Wien: Hölder-Pichler-Tempsky & Stuttgart: Teubner, 95–145

BENDER, PETER (1998): Basic Imagery and Understandings for Mathematical Concepts. Main lecture on the ICME8 1996 in Sevilla. In: CLAUDI ALSINA et al. (eds.): 8th International Congress on Mathematics Education. Sevilla, 14–21 July 1996. Selected Lectures. Sevilla: S.A.E.M. 'Thales', 57–74

FRIEDRICH, HAUKE (2001): Schülerinnen- und Schülervorstellungen vom Grenzwertbegriff beim Ableiten. Paderborn: Dissertation; see also: Eine Kategorie zur Beschreibung möglicher Ursachen für Probleme mit dem Grenzwertbegriff. In: Journal für Mathematik-Didaktik 22, 207–230

HÖLZL, REINHARD (1994): Im Zugmodus der Cabri-Geometrie. Weinheim: Deutscher Studien Verlag

HÖLZL, REINHARD (2000): Dynamische Geometrie-Software als integraler Bestandteil des Lern- und Lehr-Arrangements. In: Journal für Mathematik-Didaktik 21, 79–100

HOFE, RUDOLF VOM (1998): Probleme mit dem Grenzwert — genetische Begriffsbildung und geistige Hindernisse. In: Journal für Mathematik-Didaktik 19, 257–291

LABORDE, JEAN-MARIE (1985): Projet de cahier de brouillon informatique pour la géométrie. Grenoble: Archives LSD2-IMAG

OEVERMANN, ULRICH (1986): Kontroversen über sinnverstehende Soziologie. Einige wiederkehrende Probleme und Mißverständnisse in der Rezeption der "objektiven Hermeneutik". In: AUFENANGER, STEFAN & MARGRIT LENSSEN (eds.): Handlung und Sinnstruktur. Bedeutung und Anwendung der objektiven Hermeneutik. München: Kindt, 19–83

RICHTER-GEBERT, JÜRGEN & ULRICH KORTENKAMP (1999): The Interactive Geometry Software Cinderella. Heidelberg: Springer, `http://www.cinderella.de`

SCHUMANN, HEINZ (1991): Schulgeometrisches Konstruieren mit dem Computer. Stuttgart: Metzler & Stuttgart: Teubner

SCHUMANN, HEINZ (2001): Methoden der dynamischen Geometrie — eine Zusammenfassung. In: BUS, Zeitschrift für Computernutzung an Schulen 43 (1), 54–59

SCHWARTZE, HEINZ (1990): Zur Stellung der Kongruenzabbildungen im Lehrgang der Kongruenzgeometrie. In: Der mathematische und naturwissenschaftliche Unterricht 43, 387–394

WETH, THOMAS (2002): Der Computer als heuristisches Werkzeug im Geometrieunterricht. In: Beiträge zum Mathematikunterricht 2002. Hildesheim: Franzbecker, 511–514