How Do Upper Secondary School Students Respond to Contextualised Tasks? First Results of an Empirical Study

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In a qualitative-oriented empirical study, the effect of context on students' approaches to real world problems is investigated. First results show that the contexts offered in a task can be interpreted in very different ways by individuals. It is also shown that both a student's affective proximity to the context and their substance knowledge can have fostering but also hindering effects on his or her performance.

The theme of the paper is context, a rather blurred concept, used by many authors in different meanings and ways. Sometimes different concepts of context can even be found in the same study. Several names are also given for this concept such as *cover story*, *situation*, *situational context*, *setting*, *social context* etcetera.

On the other hand it is the consensus that context - however it is meant - is a central topic in the discussion about modelling and applications in mathematics education. Since its very beginning, the debate on the teaching of mathematical modelling and applications claims usage of various contexts or contextualised problems. We associate many goals with the use of real world problems, among others the fostering of motivation for the learning of mathematics or as a way to visualise mathematical concepts and methods (Kaiser, 1995). Until now we do not have comprehensive empirical evidence of how far our hopes concerning the use of contextualised problems are fulfilled.

The empirical study we are going to present tries to give a few answers to this problem, although we are far from having final and generalisable answers. Firstly, some results from previous empirical research studies are presented, and then the aim of our own study and its methodical design will be described. In the third part the first descriptive results are detailed and finally the first results of the influence of the so-called *contextual ideas* will be described.

Research Results from Previous Studies

The effects of context have not yet been comprehensively investigated. The studies carried out so far do not provide a uniform picture, and they often refer to primary school children.

Clarke & Helme (1996) proposed distinguishing between *figurative* and *interactive context*. *Figurative contex*t comprises the real scenario in which the task is embedded. *Interactive context* describes the conditions in which the task is encountered by a student. Applying a constructivist perspective, they point out that both kinds of context are individually constructed. In this paper we use the term *figurative context* in the sense of Clarke & Helme (1996) to emphasise the focus of our study, which refers to the real background described in the task.

Papastravidis et al (1999) found that, in a quantitative-oriented study, most students performed better in pure maths than in contextualised tasks with the same mathematical structure but exceptions were also found. A task about the costs of shirt production and another about the time students need to walk home from school provide better results than the uncontextualised tasks based on the same mathematical structure. This led Papastravididis et al (1999) to the conjecture that a familiar figurative context enhances performance. Since the

study followed a quantitative approach, it remains an open question as to what *familiarity* means to the *individual* student.

The culture of the mathematics classroom seems to be a factor as well. Boaler (1993) found in her comprehensive study that in an open, discussion-centred mathematics classroom the figurative context has a smaller effect than in traditional classrooms. However no pattern could be found as to which figurative contexts have a fostering effect and which have a hindering effect. One of the results of the study was interesting and concerned a possible gender bias within certain figurative contexts. Boaler found that boys performed better in the task shown in figure 1 and she suggested that the figurative context of the task could be responsible for this.

FASHION WORKSHOP

There are four people in the fashion workshop, Jane, Darren, Susan and Ramesh. They have got two days to do all of the following jobs (8 hours work each day). They can start a job one day and finish it off the next but they cannot share a job. Work out who can do which jobs. Show all of your working and write down all of the decisions you make.

SKIRTS	
Cutting	5 hrs
Tacking	5 hrs
Machining	7 hrs

SHIRTS	
Cutting bodies	3 hrs
Cutting collars and sleeves	4 hrs
Tacking bodies	3 hrs
Tacking collars and sleeves	3 hrs
Machining	7 hrs

OTHER JOBS	
Deliveries to London	10 hrs
Deliveries to Birmingham	9 hrs
Answering letters and filing	8 hrs

Figure 1. Boaler's Fashion Workshop Task.

She referred to research from science education where girls were found to engage more in the figurative context of assessment tasks and have difficulties abstracting from context. She interpreted the figurative context of the task as *fashion*, which girls are probably more interested in than boys, so she assumed that the fashion context of the task was an advantage for girls. That is one way to interpret this example and we will come back to this idea of an interesting figurative context as an obstacle when we later present the results of our study. This example can also be interpreted in another way: the figurative context of the task can be seen as the context of organising work instead of being a fashion context. This leads us to the conjecture that the figurative context may be a matter of interpretation. Again, we will come back to this later.

Kaiser-Messmer (1993) found differences between boys and girls concerning their preferences for certain figurative contexts. Boys preferred technology and physics, whereas girls preferred biology, medicine and ecology. Since this study focussed on attitudes, no conclusions about the effects on the solving process itself can be drawn.

According to Stillman (1998) the degree of involvement with the figurative context is influenced by the task type. So-called "wrapper problems" provoke less involvement whereas modelling tasks generate more involvement with the figurative context. Generally she found

that medium to high involvement with the figurative context is connected to better performance.

When investigating the problems minority students in the Netherlands have with contextualised tasks, Carvalho de Figueirdo (1999) found that language and cultural background have an effect on the way figurative contexts are understood. It was also found that illustrations can mislead students in their understanding of the figurative context of a task. Furthermore it was pointed out that an unfamiliar figurative context tempts students to rapidly de-contextualise the task.

To sum up: in all the studies mentioned it remains an open question as to how the figurative context offered in the tasks (by words or illustrations) is represented internally and which effects it shows in the course of the solution process. These questions were the central questions of our own qualitative-oriented study.

Aim of the Study and Methodical Design

The aim of the study is the generation of hypotheses about the effects the figurative context can have on upper secondary school students. We are aiming to develop a categorisation of the effects observed. This study uses the case study approach, therefore generalisations cannot be made easily as they have to reflect the special characteristics of the sample and the observation design.

The methodical design has to provide data that allow the reconstruction of the internal representation of the figurative context offered in the task. In addition to that, the data are to give insight into the student's emotional attitude towards the figurative context. The problem is that no direct measurement is possible. To solve this problem we carried out some pre-studies, aspects of which are outlined subsequently. For example, groups of students were asked to write free stories about the task immediately after they had worked on it. This was done with 12- and 19-year-old students. Let us give an example of the story a twelve year girl wrote after working on the following task:

> A dog's first year is equivalent to 15 human years. The second year of a dog is equivalent to 6 human years. Each of the following years of a dog is equivalent to 5 human years.

Following this introductory text some calculation tasks were given. After solving the task correctly the girl wrote:

> I have a dog named Nadja. She was born in 1986 and her age is 13 dog years, which is approximately 76 human years. Anyway, when I was approximately 2 years old I was sitting on the baby's changing unit. I could not feel my legs from the first day on. But somehow, nobody knows how, I tried to crawl. Nadja heard that and came running into the living room. I fell down. Fortunately Nadja was there and I fell on Nadja's back and then onto the floor. Up to the age of 3 I could not feel my legs! All that is true!!!

This is an impressive and touching example of what can be on a student's mind when confronted with a task. This observation, among others, led us to the idea that figurative contexts offered in a task can evoke very individual and unpredictable interpretations.

In another trial, interviews were held after working on the tasks. Stories and interviews done after working on tasks showed interesting results but it was not obvious which ideas came *after* solving the task and which ideas were already present *while working* on the task. In a different approach the students were asked to speak or write about the figurative context simultaneously while working on the task. However this appeared to be a disturbing factor. It was a problem, because statements on the figurative context required a change in the level of reflection i.e. jumping from the solution process to the figurative context level and then back again.

As a result of the pre-studies the following *three step design* appeared to be appropriate: four pairs of students (16 to 17 years old, in their 1st year of upper secondary school, two pairs of boys, two pairs of girls) from four different schools were given realistic tasks. The students were videotaped while working on the tasks. After working on the task the students watched the video individually. They were requested to stop the tape when, during the solution process, something concerning the figurative context crossed their mind. They were asked to describe these thoughts and the statements were audiotaped. The interviewer too could stop the recording, if the student did not do so, should they consider certain parts of the tape as potentially useful for statements. This method is known as the method of stimulated recall (e.g., Weidle & Wagner, 1994). In an immediate follow-up interview the student was asked questions about the statements made during the stimulated recall phase. The purpose of the design was to catch thoughts, associations etcetera concerning the figurative context *at that moment* in which they crossed the student's mind ("in statu nascendi") without disrupting the solution process. To avoid disturbances the statements made during the stimulated recall phase were usually not interrupted by the interviewer. The place and time for questions was during the interview.

We decided to let the students work in pairs and not individually. The main reason was that an essential part of the methodical design, the stimulated recall phase, would not work as well if the problem solving was done individually. It is much easier for a student to recall an association or a thought when it can be connected to a phrase used during the discussion. It could be argued that working in pairs makes it difficult (if not impossible) to match certain thoughts to one of the students. This is true, but given the background of this study this is not a problem. The main focus is *not* the solution process itself, but the role the figurative context plays. Consequently the methods to get access to the *individual* role of the figurative context are applied individually: namely stimulated recall and interview.

The tasks were given on three different days and were matched to different figurative contexts. The tasks differed also in the degree of mathematical modelling needed. The mathematical theory required for all the tasks was taken from earlier years of the students' maths education. There are two reasons for that:

- It was not clear which topics had already been covered at the different schools
- The mathematics should remain in the background for the benefit of the figurative context. In other words the maths was supposed to be well known.

The first task which was given to the students is shown in figure 2. The text is translated from German. Before we refer to the students' results let us comment briefly on the task:

- The data are authentic as they were taken from publications of an oil company. The students had not been informed about the authenticity of the data before working on the task.
- One might be tempted to solve the task by applying a mere extrapolation but in order to determine the consumption in a certain number of years it is necessary to cumulate the consumption figures. This is the main mathematical difficulty in this task.
- The global consumption seems to be almost linear. Indeed if the results of a linear approach are compared with those of an exponential one, little difference is found. Using a linear model one gets a range of 32 years while the exponential approach leads to a range of 31 years. (It might be interesting to know that these figures match well with predictions made by geo-scientists.)

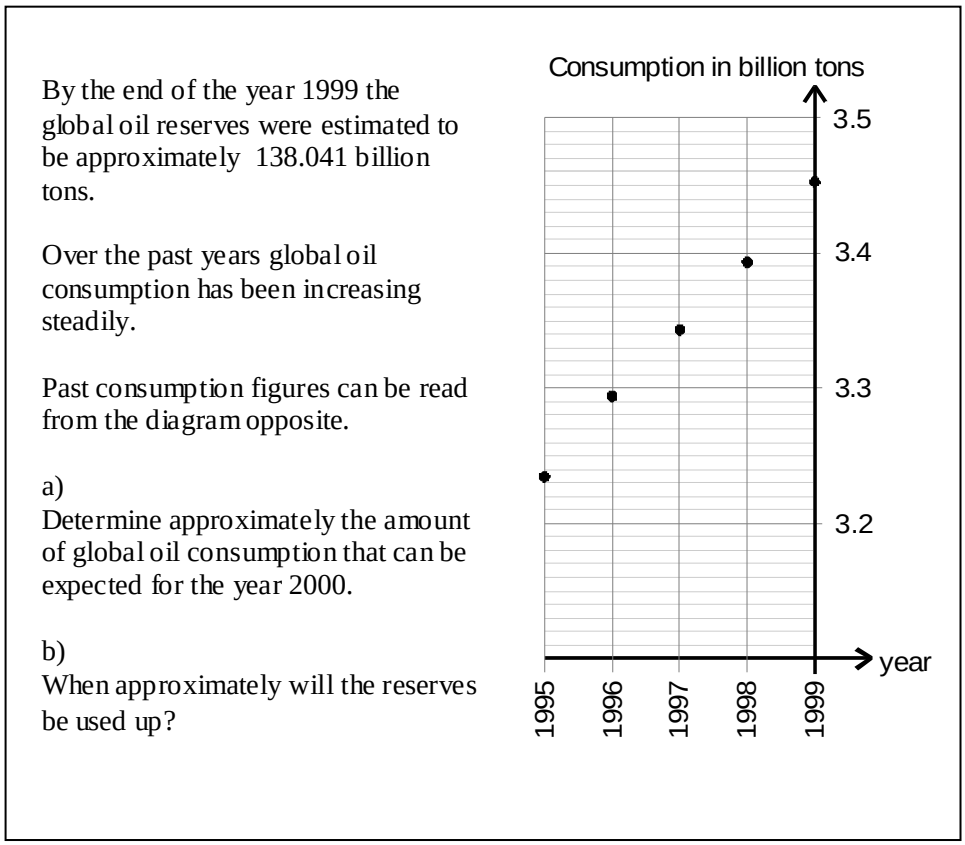

Figure 2. First Task of the Study.

Descriptive Results

The task was almost solved correctly by one pair of students through using a table. Two pairs used a promising approach but failed to apply the mathematical techniques properly. One pair did not really understand the task. We do not want to present the answers but rather give an outlined reconstruction of the statements made about the figurative context. We have restricted the number of students in this paper for the sake of clarity, but future publications will show more detailed results.

In the transcripts we present below, many filling words were omitted for reasons of better understanding (shown by dots in brackets). Words printed in italics were spoken with emphasis. A question mark in brackets indicates that the spoken word could not be heard properly.

Christine almost completely ignored the figurative context of the task. Only at the beginning did she mention a task that appears to be similar for her. This task was about cell growth. She tried to adapt well known formulae to the task to produce a result. She accepted her result of approximately 361 years without any comment. (Remember a correct answer would be approximately 32 years.) During the stimulated recall phase she made no statements about the figurative context (in contradiction to the instructions given) but explained her way of working on the task by focussing on technique. In the interview Christine was asked questions about the figurative context:

Interviewer: Have you ever dealt with this *topic* in another school subject? Or in another way through newspapers or-
Christine: Hm- for which topic?
Interviewer: What this is about here.
Christine: Mineral oil? Oh, you mean mineral oil?

Interviewer: If you think *mineral oil* is the topic, then that's what I mean.
Christine: Ehem. No. [laughing]

Later in the interview she said:

Christine: Mineral oil always sounds so uninteresting.

So for Christine the figurative context hardly seemed to exist when working on the task. Even later in the interview she seemed to think, for the first time, that the task even *has* a figurative context. Then (and not during the solution process) she gave contextual comments.

Ingo associated more than Christine. His first thought was the driving test his sister had passed on the same day. He reflected on his personal situation in the future when oil would be used up. He had an idea about the number of years left until this would happen and said that his knowledge was based on media publications. He talked about alternative technologies which could extend the life of the oil reserves and also mentioned a long discussion with his chemistry teacher about the subject. He mentioned the oil crises and the Gulf War and he said in the interview that he was concerned about the increasing oil consumption:

Ingo: At any *rate* I also would worry about it and I think, as it is mentioned in the media, it is interesting for *every* student of my age to know how it will be in the future. (...) And (...) well, I personally *worry* about that.

Later in the interview he said:

Ingo: Well, I personally have already thought about it quite often. It's really on my mind. The point where it will be finished.

He had some problems in applying mathematical techniques properly, so he failed narrowly to complete the task successfully.

Karla associated, in an emotional way, the problem of the destruction of our natural environment. Her chemistry lessons had provided her with knowledge about how oil comes into being, the limitations of natural resources and other ecological problems. She liked chemistry lessons and her chemistry teacher, as she described in the interview which follows:

Karla: Yes that was a topic I *am* interested in for its ecological aspects. And also chemistry is a school subject I like. I get on well with my chemistry teacher. (…) and therefore, it has been an important topic to me, also in *that* lesson. And I liked it, and we watched a film about when energy resources will be exhausted and (…) that actually did interest me and I would say *concerned* me.

Karla reflected on the assumption of continuously increasing oil consumption but would rather consider decreasing oil consumption. Given the emotional importance the problem of the destruction of natural environment had for her, one is almost tempted to interpret this as a wish. She said in the stimulated recall:

Karla: (...) yes, there one notices that the earth is already quite wrecked, and (low voice) in this respect it actually interested me and (louder) then I thought about it again in that situation. (...) if really every year zero point zero seven (…) billion tons *more* are consumed or if it is perhaps only for a while or (…) if one really may *say* it that way, that it always increases or may be it was (only?) (…) for three or five years like that (…) and that then it will decrease again or (…) what might happen somehow to solve the ecological problem (…).

Karla solved the task by using tables appropriately with only one little mistake.

Let us summarise: it becomes obvious that different students interpreted the figurative context offered in the task in very different ways. The differences appeared in the associated aspects of reality and in emotional attitudes.

It must be mentioned that the students' statements do of course not cover their internal representations of the figurative context offered in the task completely. One has to expect that many thoughts and associations remain unmentioned due to either missing consciousness of the thoughts or to a lack of willingness to tell them.

In spite of the limitations just mentioned the differences in the perception and interpretation of the figurative context offered in the tasks are obvious. These differences are based on individual knowledge, experience and attitude. Consequently reflections on the effects of figurative contexts must consider its individual nature. Following this line of thought we use the term *contextual idea* for the individual internal representation of the figurative context offered in the task.

First Results Concerning the Influence of Contextual Ideas

In the following we want to present the first results of possible contextual influences gained by the reconstruction of the contextual ideas.

The contextual idea can cause distraction from the task.

Although contextualised tasks are seen as generally helpful, distracting effects could also be observed. For example Ingo mentioned in the interview:

> *Ingo*: (...) somehow has reminded me of statements and such things I read about. (And?) then (…) at first I had to shake off this thought, so that I could concentrate (…) again (on the?) task because (…) for a moment I was tempted to think too *much* about it.
> *Interviewer*: It would have disturbed you?
> *Ingo*: (answering quickly) Yes.
> *Interviewer*: Because it-
> *Ingo*: Because (...) it does not help us to solve the problem and because then it is difficult to concentrate again on the problem. Because then one has another thought at the same time, and of course one associates personal experiences or what one has read. If one thinks too *much* about it, it is more difficult, then one must come back (…) to the problem.

This phenomenon of distraction can have different aspects so an emotional and a more matter-of-fact-related distraction could be found. Let us present an example for each of the two aspects mentioned.

Emotional involvement in the figurative context can influence or even disturb the work on the task.

This is remarkable because usually it is expected that familiarity with or interest in the figurative context of the task has a helpful effect. The opposite seems to be possible as well. The above mentioned student Karla, who showed a very emotional involvement, said in the interview that during the work on the task she always had to think about the films on environmental destruction shown in the chemistry lessons. Asked if she felt disturbed by that she answered:

> *Karla*: Yes, maybe, it happens often to me that somehow I have to think of different things totally while solving problems, also during tests. (…) And then I say to myself *concentrate* and then it *works*, but in that moment one thinks about these *films* and so one does not think about the *task* and how to solve it. That is a *different* problem.

This is interesting because there exist didactical approaches to support girls by using special figurative contexts which are assumed to be meaningful to girls. Further analyses have to be done in this field.

A rich contextual idea can be a disturbing factor when working on the task.

In a certain sense a rich contextual idea produces a world too large for the more or less narrow task. The contextual idea contains more information than the text of the task itself and so the student has to choose what is important and what is not. During the problem-solving process and in the stimulated recall phase the student Arthur mentioned a large variety of

real-world factors that could influence global oil consumption. Asked about that in the interview:

Interviewer: What influence did these thoughts have (…) on you while working on problems? Can you tell?
Arthur: (...) (low voice) while solving the problem they only made it more difficult because it was (?) difficult to *estimate*. I (somehow?) assumed a difference of ten years. That is hardly solvable *mathematically*.
Interviewer: Well, I understand that you were thinking of so many things, how complex it is-
Arthur: Yes.
Interviewer. And that has disturbed you?
Arthur: (answering quickly) Yes.
Interviewer: Hm. Okay.
Arthur: Well, I mean it has made me *thoughtful* because if (...) one *imagines* how it might be, the problem cannot be solved. If one is given (a formula?) how long might it last? I think after being given the formula a task could be solved *easier*. But this was (low voice) too (...) big a field to (?) simplify.

Here a distraction from the task caused by too much content information and not by emotional involvement, as described above, takes place. Again this seems to be a paradox: a task which is meaningful to the student becomes more difficult due to the host of associations.

Frequently the contextual idea is used to control whether a result is plausible. In conjunction with a misconception of the modelling process this can cause an error.

In the case we have in mind the student Josef - the partner of Arthur who was mentioned earlier - predicted global oil consumption for the year 2000 to be *lower* than the consumption for 1999. They used the mean of the consumption figures for the years 1995 to 1999 as a prediction for 2000. After feeling that this might be incorrect the student finally accepted this result giving the following reason: the result can be correct because the recently introduced German ecological tax leads to decreasing oil consumption. Later in the interview Josef put forward the idea of a figurative context through *ore deposits*. Asked how his argumentation would have changed he said in *this* case he would not accept the result because he did not know anything about ore deposits. Let us illustrate this by two short passages taken from the interview:

Interviewer: You found out that with this *calculation* the oil consumption would decrease, if I understood correctly? And then you have interpreted it. There might be less cars-
Josef: Exactly, yes.
Interviewer: There might be eco-tax.
Josef: Yes, the eco-tax aspect was caused by earlier lessons this year.
Interviewer: In maths?
Josef. No, in social studies. Eco-taxes as a general topic.
Interviewer: Hm.
Josef: And (...) that it *should* be achieved by this that the consumption decreases. Because of that we thought we could *leave* the calculation as it was.
Interviewer: It would *fit*.
Josef: It would fit, exactly.

Asked if he could imagine a different figurative context for the task Josef said:

Josef: It doesn't have to be *mineral oil* reserves. It may be ore deposits in the earth or something similar. Absolutely.
Interviewer: What would that have changed? Or *would* that have made a difference for you if the problem had been formulated with ore deposits for instance?
Josef: (answering quickly) Yes, it would have made a difference because we have calculated the average and we did not take into account the constant increase. We referred to the eco-tax, and with ore I would not know how it should be decreased.
Interviewer: If you got the same result with the ore problem and if there was a decrease, well then you would-

Josef: I would have calculated it *once more* and would have to find *another* way (?).
Interviewer: I see, and *here* you said it might be possible? Because of that (?)
Josef: [interrupting] Exactly. Here we *had* (...) a reason why it might decrease.

How can this be understood? The following explanation is possible: Josef had – probably not consciously – built and evaluated a mathematical model. After the evaluation (and not, as would be correct, while building the model) he added an additional figurative context assumption i.e. the introduction of the ecological tax. Hence he justified a result which appeared not to be correct at first sight. He did not, as proper modelling would require, start another circle of the modelling process, including the new assumption of the introduction of ecological taxes. Here a kind of modelling error took place to which the student was drawn by his contextual idea. It becomes clear how important it is to talk on a meta-level also about mathematical modelling in the classroom to avoid mistakes like this.

The contextual idea can cause motivation.

Although this seems to be common sense it should be mentioned here after all the negative effects noted earlier in this paper. Motivation is sometimes taken as the only effect. It remains an open question at the moment as to how strong the motivational effect is, if it carries through the work on the task or if it is just a mind opener in the sense that it catches the student's attention. The above mentioned student Ingo stated in the interview:

Ingo: (...) had the (...) problem *without* theme, just purely mathematical, well, *naked*, then for instance I would have been sitting down and thinking no, I don't feel like doing it. Well, in some way I would not have gone *into* the problem, very probably. (...)This topic has helped me *very* much to find a way for getting into the mathematics. Otherwise I would never have come to the point where I worked on it with enthusiasm.

Final Remarks

The hopes concerning the inclusion of contextual problems, such as the fostering of motivation and understanding, claimed in the theoretical debate on applications and modelling have to be seen in a more differentiated light. We point out that the figurative context offered in a task can give rise to individual contextual ideas dependent on the student's previous experiences and attitudes. It has to be considered that the contextual ideas can have a variety of influences, which might be fostering or hindering. One has to be more careful with psychological arguments concerning the inclusion of modelling and applications in mathematics education. More emphasis should be put on normative reasons for the inclusion of real world problems in mathematics education. Furthermore consequences for teacher education and in-service training have to be realised: we need to make student teachers and practising teachers more aware of the students' various individual interpretations of the figurative context offered in a task and the possible influences a figurative context can have on them.

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