Working styles of students in a computer-based environment. Results of a DFG1 project

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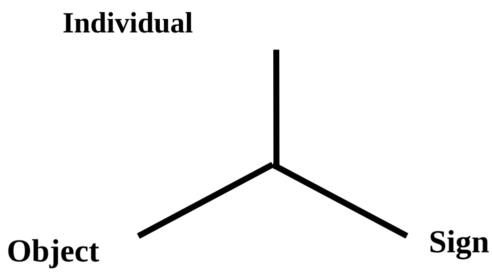
The aim of this applied project was to document and analyse working styles of students when solving problems in a computer-based environment in contrast to traditional paper and pencil activities. The results of the project should help to better understand individual computer-supported activities and consider them appropriately with respect to future curricular decisions.

Working Styles and Representations

Working styles are sequences of actions affecting and altering mathematical objects, for instance changing variables, multiplying an equation by a number or differentiating a function.

Real actions can only refer to real representations. Concerning the relationship between object and representation we refer to Peirce, whose basic hypothesis was that mental objects and their real representations are inseparably connected. “Every thought is a sign” (Peirce 1967, P. 175). To express the relationship between the mental object, the representation (sign) and the individual, Peirce used the semiotic triangle: Object – Individual – Sign.



We see the concept of mathematical working styles in two ways: first it has to be seen on the level of representation, which describes working with representations like symbols, graphs, tables, diagrams or pictures and second on the level of (computer) tools describing working with menus, buttons and scroll bars.

Further, we refer to Peirce's pragmatic maxim which says that the meaning of concepts and signs can only be clarified by operating with the signs. The computer is a tool with special mathematical notions for the objects and special menus, or mouse-driven actions and commands. It allows a person to work in new ways with objects – or the representations of these objects on the screen. Because of the novelty of this new tool (e.g. you can now create representations by pressing a button or clicking the mouse), a lot of traditional questions also have to be posed in a new way.

During this investigation, we were interested in operations which were orientated towards the solution of a problem. In order to be able to distinguish such activities from non-reflected guess and check strategies, actions must be seen in relation to the understanding that correspond to those actions. We refer to the models of understanding of Vollrath (1984) and

[1] DFG means “Deutsche Forschungsgesellschaft” (“German Research Society”). The project “Working styles of students in a computer-based environment” was approved on January 1st 1998 for a term of 2 years. The project started in March 1998. From February 1999 to August 1999, the work was intermittent because of a research sabbatical in the USA. However, during this time the test materials were evaluated with American pupils and students. As a result we obtained an interesting international comparison study between American and German students. The project was concluded in September 2000.

Skemp (1979). According to Vollrath, understanding of a concept develops on different levels, e.g. an intuitive level, a contents-related level and a formal level. Skemp distinguishes between "instrumental" and "relational" understanding.

Sequences and Spreadsheets

Sequences are fundamental objects in mathematics with a long tradition in its history. They are tools for the development of other concepts (e. g. the limit concept), tools for the mathematization of real-life situations (e. g. growth processes), but sequences are also interesting objects with a lot of surprising properties (e. g. Fibonacci sequence, sequence of prime numbers, sequence of polygonal numbers). In the last few years, with the increased importance of computers in mathematics and mathematics education, *discrete mathematics* and hence sequences have gained importance. This is also emphasised in the NCTM-Standards (1989), where discrete mathematics is a separate standard: "Sequences and series ... should receive more attention, with a greater emphasis on their descriptions in terms of recurrence relations." (9-12, p. 177). The computer makes it possible to generate sequences, to create symbolic, numerical and graphical representations and to change between these different representations. We think that a spreadsheet is the correct tool to work with in the field of discrete mathematics, especially when working with sequences.

Empirical Investigation

Questions in the investigation

The following questions about the changes in working styles were central to our investigation.

1. Question: How do students work with representations while solving problems and how do they argue with respect to the represented objects?

On the one hand, we distinguish representations according to symbolic, numeric and graphic forms and on the other hand we distinguish them according to the content of the represented objects. We used situations like mobile phoning, sports (basketball) and air-temperature as well as geometric contexts. The mathematical contents (in particular recursively-defined sequences and regression curves) were not familiar to the students.

2. Question: Local and global working styles?

a) According to which strategies do the changing of the considered quantities and variables and the selection of the representations occur?

b) On what kind of understanding - with respect to the meaning of the various quantities – are the working styles based?

We speak of *local working styles* if special terms of the sequence are changed (in particular the initial values of a recursively-defined sequence). If the variation of all terms of the sequence is observed, we speak of a *global working style*, e.g. finding sequence approximations and curves of best fit.

3. Question: What are the possibilities and the limits of "computer protocols"?

Within the scope of this study, the working styles of the participants were saved with the help of a "computer protocol". While the students worked on the computer, the programme ScreenCam2 ran in the background and saved all keyboard and mouse inputs made by the

[2] ScreenCam is a product of the company Lotus: http://www.lotus.com/home.nsf/welcome/screencam

user. Compared to videotapes and interviews, computer protocols offer the possibility of observing a large group of students simultaneously. Computer protocols are produced during the problem-solving situation, hence they are at the centre of the learning process. The computer protocol can be replayed as a 'film' which shows all screen activities of the pupil in real time. The protocols can then be analysed to determine

- whether and how a student has solved a problem,
- how much time he spent on one problem,
- how many, and what kind of representations, he has seen on the screen, and
- when and how often he switched over to another representation.

Moreover, it is possible to evaluate less successful student strategies. On the one hand this investigation should show how computer protocols can be realised technically during the experiment while on the other hand it should show the possibilities and difficulties of the analysis and the evaluation of such protocols.

The Learning Programme

With the help of the spreadsheet Excel, a teaching and learning programme was developed which introduces the subject. The level of difficulty is increased in a step-by-step manner and the programme allows different solution strategies to be selected (cf., Weigand 2001).

Version 1.1

Version 1.1 of the learning programme required students to work with sequences in real-life situations and explain different methods for determining regression or "best fitting" curves.

The programme consists of nine problems in four different groups.
Group I: Reading of representations and transfer between representations
Group II: Search processes – sequences defined by the equation $x_{n+1} = C \cdot x_n$
Group III: Search processes – Linear Regression
Group IV: Curve fitting

Version 2

In order to be able to evaluate the working styles of the students i.e. whether they are oriented towards a special goal or whether they are left more to chance, questions concerning contents-related understanding were integrated. Furthermore the contents-related (semantic) meaning of the representations was expanded by integrating real-life and geometrical problems. We used the following types of sequences:

- Linear growth $a_{n+1} = a_n + B$
- Exponential growth $a_{n+1} = A \cdot a_n$
- Limited growth $a_{n+1} = a_n + P \cdot (B - a_n)$.

As representations we used graphs, tables and formulas. Both geometric examples and real-life examples were considered.

Participants in the investigation

Both American and German students and pupils participated in this study. A first version of the learning programme (Version 1.1) was tested at a university in the USA with 26 student teachers of mathematics and with 20 pupils from two high school classes. The same programme was tested at a university in Germany with 25 student teachers of mathematics and with 16 grade 12 students. The results of this comparative study, and in particular the method of computer protocols, were presented at ICTMT4 in Plymouth in August 1999 (see Weigand 1999).

In addition, 28 students from two grammar school classes (grade 12) took part in an empirical investigation with an expanded version (Version 2.0). They all had a good or very good knowledge of Excel and had already worked with (arithmetical and geometric) sequences in the 11th grade when the concept of limit was introduced.

Results

Working on the tools level

While working on the level of the spreadsheet tools, we noticed the following differences in working styles compared to paper and pencil activities.

- Operations with cell names (e.g. A3) as names of variables did not cause difficulties as long as the content of one cell could be seen as a function of the content of another cell. Sometimes it was merely that the "=" sign was forgotten. Copying of formulas caused greater difficulties since the variables are automatically changed with relative cell references. If this automatic variation is to be prevented one has to work with absolute cell references. In this case, operating with self-defined variable names was very helpful. In Excel you can give the name “time” to a cell e.g. B5. If you refer to the variable “time” in a formula, it will not be changed when the formula is copied to other cells.

- The offer of free experimental working with parameters was not used very much if it was not related to a concrete problem. In particular the effects of convenient change aids (scroll bar) were only analysed very superficially. Only when difficulties arose during the problem-solving process did some students use the possibility of experimenting with representations more often, while other students demonstrated non-reflected activities like continued button pressing or zapping through the programme. The possibility of experimentation was used very often – to our surprise – for control purposes and for checking the results obtained.

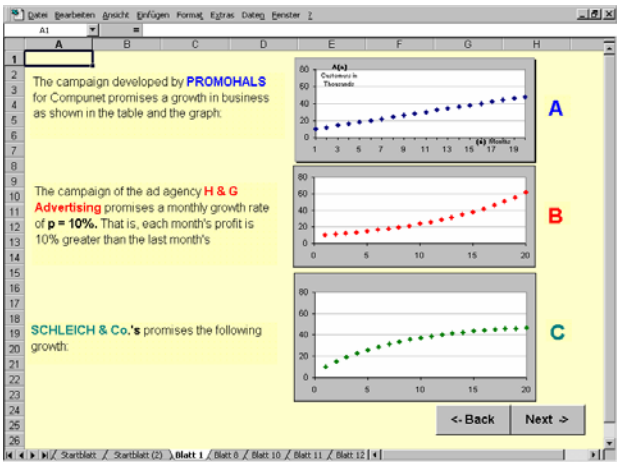
- The length of time participants spent on reading a particular page of the screen enabled conclusions to be drawn about the thoroughness of the reading. However if they were unable to solve a problem they read the remaining pages of the module only very superficially. Therefore – in the case of computer-supported environments – one requires "anchor points" or "entry points" which offer learners a place to start afresh. We rarely observed variation of strategies and the search for alternative solution methods. If a student did not make progress with their problem-solving strategy, they gave up working on the problem.

- The information density of the programme was very high. Some difficult questions (e.g. why does the straight line approximation not correspond to reality?) were overlooked and some questions obviously could not be remembered. It is therefore necessary to break questions down into smaller parts or repeat them. Redundancy seems to be a crucial point in the design of learning programmes.

Level of representation - test with Version 1.1

In this section we refer to the comparative study done with Version 1.1.

Reading representations



The starting-point was a real-life situation: "The company COMPUNET provides Internet connections to its current 10,000 consumers. They are interested in hiring an advertising agency to develop a campaign to increase the number of consumers. COMPUNET has three different advertising agencies to choose from: PROMOHALS, H & G ADVERTISING and SCHLEICH & Co. Each company guarantees an increase in profit for COMPUNET, but at different rates. Your job will be to look at which agency is best for COMPUNET."

Problem: "For which months does H & G Advertising's campaign project more customers than the other two ad agencies?"

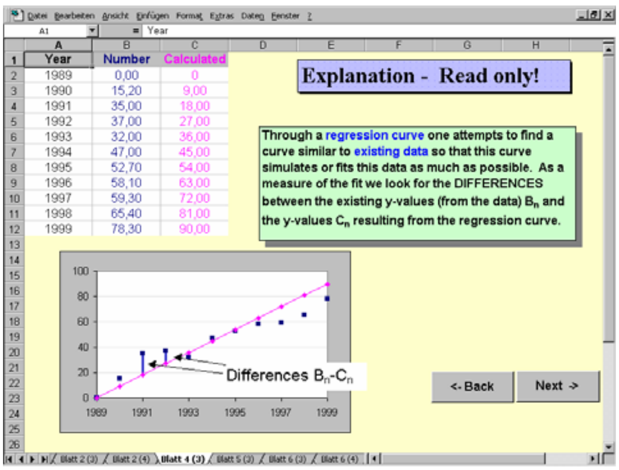
Problem: "In which month is the customer growth at H & G Advertising the greatest? In which month is it greatest at Schleich & Co?"

To solve these problems, the students could either work with the table or the graph. We were interested in the representations they used to solve the problems, whether they were able to read these representations and how they switched between them.

The USA students worked much more with graphs than tables. This was also true for the USA high school students, who worked significantly more with graphs than the German students. This may be a result of the NCTM standards which emphasize this kind of graphical working. It is interesting that the American students required only a few trials when solving problems experimentally. Again this might be an indication of how these students are used to working with graphical representations (see Weigand 1999). During this first investigation the kind of understanding was not yet checked by written arguments.

Local and global working styles

Problem: Search processes with $x_{n+1} = C \cdot x_n$ - Changing initial values



The growth in customers of the Internet provider is – for an exponential growth rate and a constant growth rate – dependent on the initial number of customers in the first month. The students had to find the smallest initial number of customers (to one decimal place) so that a special advertising campaign would project the greatest customer base for each subsequent month.

Problem: Search process – Linear Regression – Changing the growth factor

A second real-life situation was about a soft drinks company "6-down" which was founded in 1989. The number of bottles sold (in millions) over the last 10 years was given and we were looking for a **constant** representing the yearly growth to find a good fit to the actual stock sales.

To solve this problem the concept of regression was explained. We decided to take the smallest mean error regression line, instead of the smallest mean square error regression line, because it is easier to explain and the working styles do not differ within the two methods.

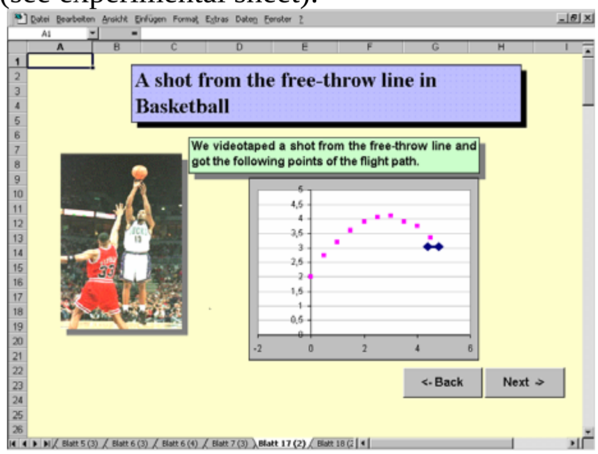
The results in the American classes varied more than in the German classes, however American students as a whole were more confident with search strategies. The advantage the German students had was in the handling of symbolic-graphic relations. This becomes especially evident in the case of the global working styles, while approximating a given sequence by means of a regression curve with varying parameters (cf., Weigand 1999). This confirms the results of the TIMS-Study which showed that German students are quite good at solving routine tasks.

The solution strategies, in the case of local search processes, showed a wide range of individual working styles (oscillating approximation, step-by-step linear-sequential approximation). In particular there was a lack of knowledge concerning the graphical meaning of the parameters of the equation, e.g., $f(x) = a(x-b)^2 + c$ or $f(x) = \sin(bx) + c$. The solutions of this investigation showed that many pupils (and students) have no reasonable strategy for varying several parameters of a functional equation, and thus their working styles are quickly reduced to unreflecting activism with random changes. The high number of American pupils who were not able to solve the problem at all is surprising. In this area the German students achieved significantly better results than their American counterparts.

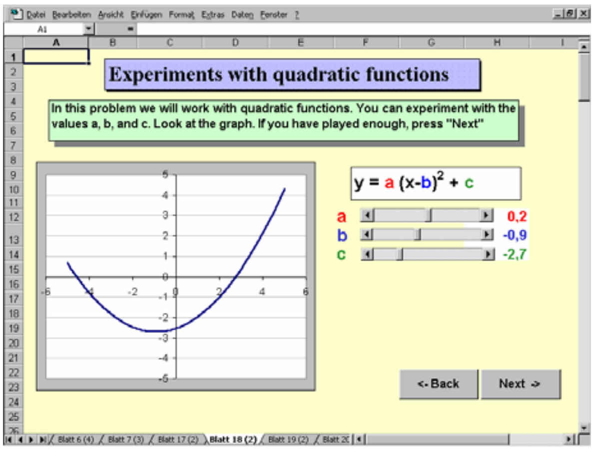
Curve fitting

Problem: Quadratic Regression

The explanation sheet showed some discrete points on the flight curve of a basketball after a shot from the free-throw line. The students were asked to fit the best parabola to the given values. They were first allowed to experiment with the variables of a quadratic function (see experimental sheet).

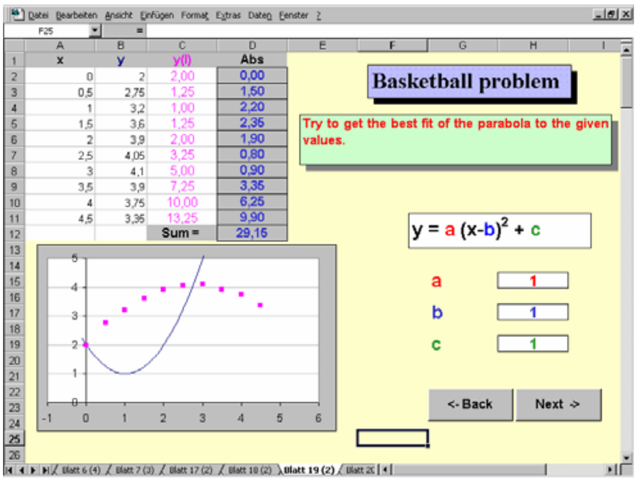


Explanation sheet



Experimental sheet

The students had to vary the three parameters in the quadratic equation $f(x) = a(x-b)^2 + c$. We were interested in how close they came to the best fit and we looked for the strategies they used when changing the values.

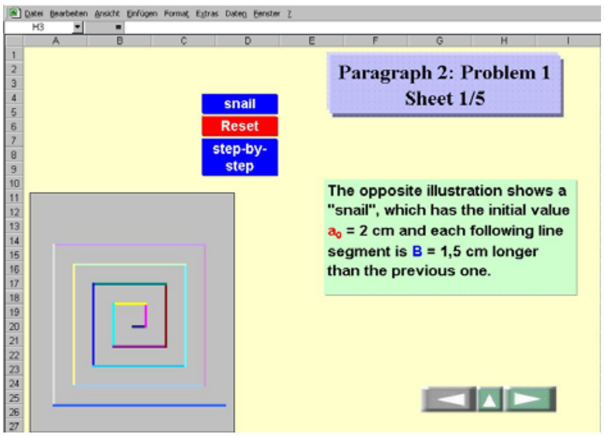


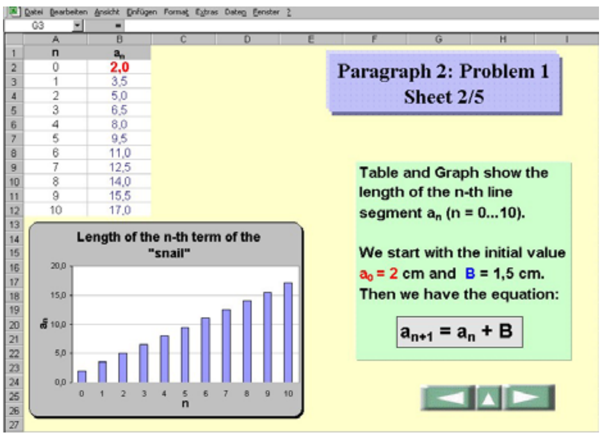
A successful problem-solving strategy is recognised by the use of special "anchor points" which are entered immediately. One sees that the parameter "a" is negative and smaller than 1, "b" is around 3 and "c" is around 4. Many student strategies cannot be classified as "good strategies" in this sense because students found the right values “only” by guess and check. The solutions of the students differ in the number of trials they used to get a good fit and in the way they handled the change of the three parameters. Some students were out of their depth. There were no significant differences between the working styles of the American and German students.

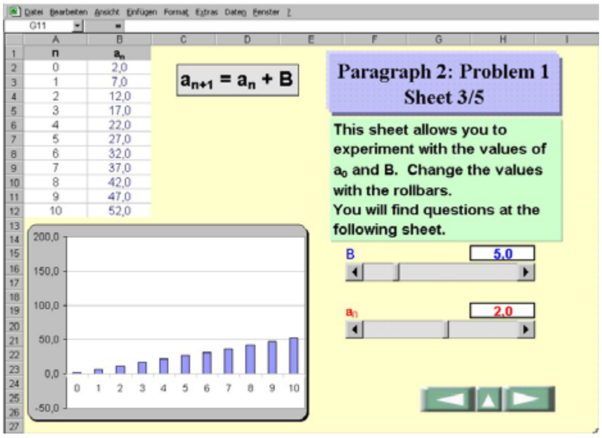
Level of representation - test with Version 2

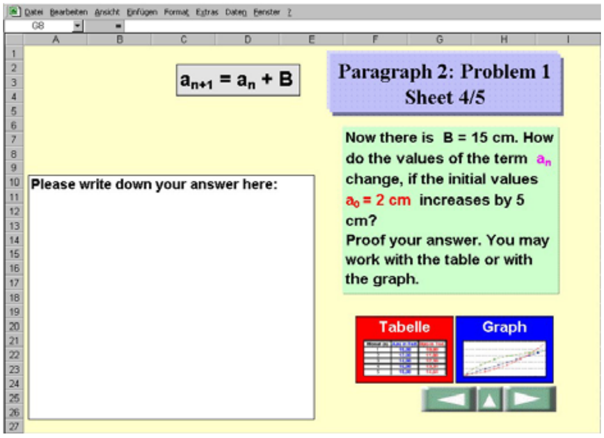
The sequence $(a_n)_{IN}$ with $a_{n+1} = a_n + B$

The question about the changes of the terms of the sequence (an), with $a_{n+1} = a_n + B$, while increasing the initial value a0 is about the understanding of local working styles. It was answered correctly by 82% of the participants. The students worked significantly more often with the table than with the graph. Some pupils worked exclusively with the table whereas others used the graph more often as time went on. The representation influences the arguments concerning the properties of the viewed sequence. The graphic representation goes along with a global view of the sequence. However the verbal responses very often only described the observed behaviour of the sequence and did not give arguments or proofs for its special properties.









The sequence $(a_n)_{IN}$ with the equation $a_{n+1} = A \cdot a_n$

The linear relationship between the terms a_n of the sequence and the initial value a_0 was recognized correctly by 78% of the students. In contrast, the question concerning the changes to a_5 while doubling the value of B was – as expected – not solved so well (solution percentage 35%). The students worked mainly with the table; indeed some did not consider the graph at all. Others frequently switched back and forth between graph, table and the description of the problem. On the one hand their arguments show step-by-step iterative thinking (this means that the students proceed successively from a_0 to a_5). On the other hand they argued about the exponential behaviour which is especially represented by the graphic and the symbolic level ($a_{n+1} = a_0 \cdot B^n$).

The comparison of the results of the two problems (4.3.1 and 4.3.2) showed that

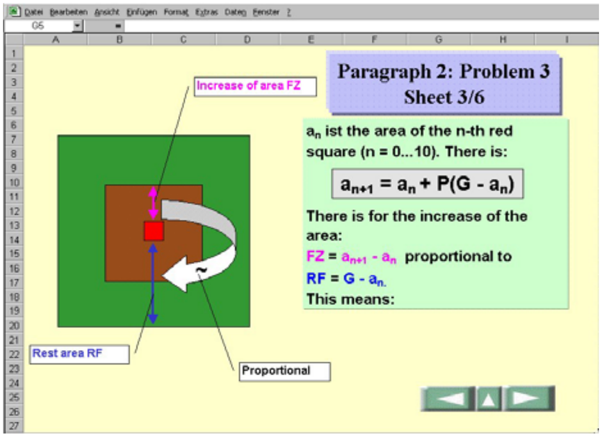
- two thirds of the students recognized the difference between the additive and the multiplicative relation of the recursively-defined equations,
- they understood the difference between the linear and the exponential growth, and
- nearly one half were able to use these properties while solving problems.

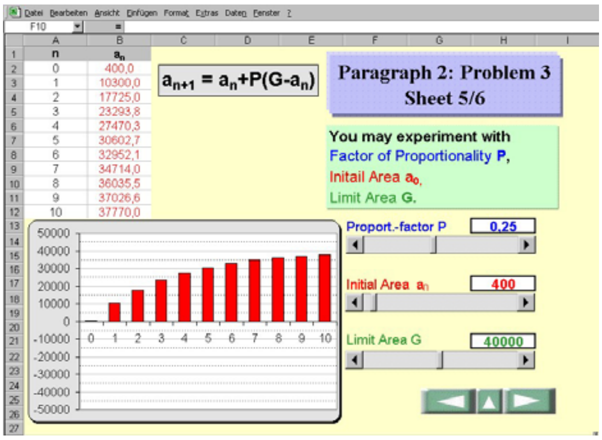
The sequence $(a_n)_{IN}$ with $a_{n+1} = a_n + P \cdot (B - a_n)$

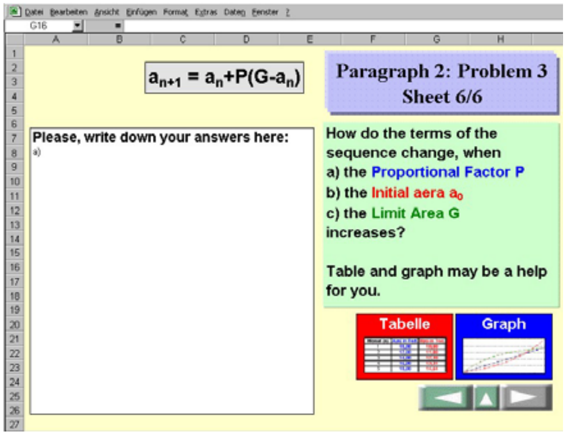
The first two problems were related to linear and exponential functions, a topic with which students were familiar. In contrast, the sequences describing limited growth were completely new to the students. The dependence of the terms an of the sequence on the quantitative variables a0, P, G cannot be described by simple relations. Therefore we posed the questions in a way that provoked intuitive descriptions. This is shown in the following two examples.

Geometric example: Limited Growth

The dynamic view of the students' descriptions are expressed by "getting closer", "quickly approaching" or "successively approaching". Compared to the previous problems the students worked more intensively with the graph. Some barely considered the table and some did not work with it at all.

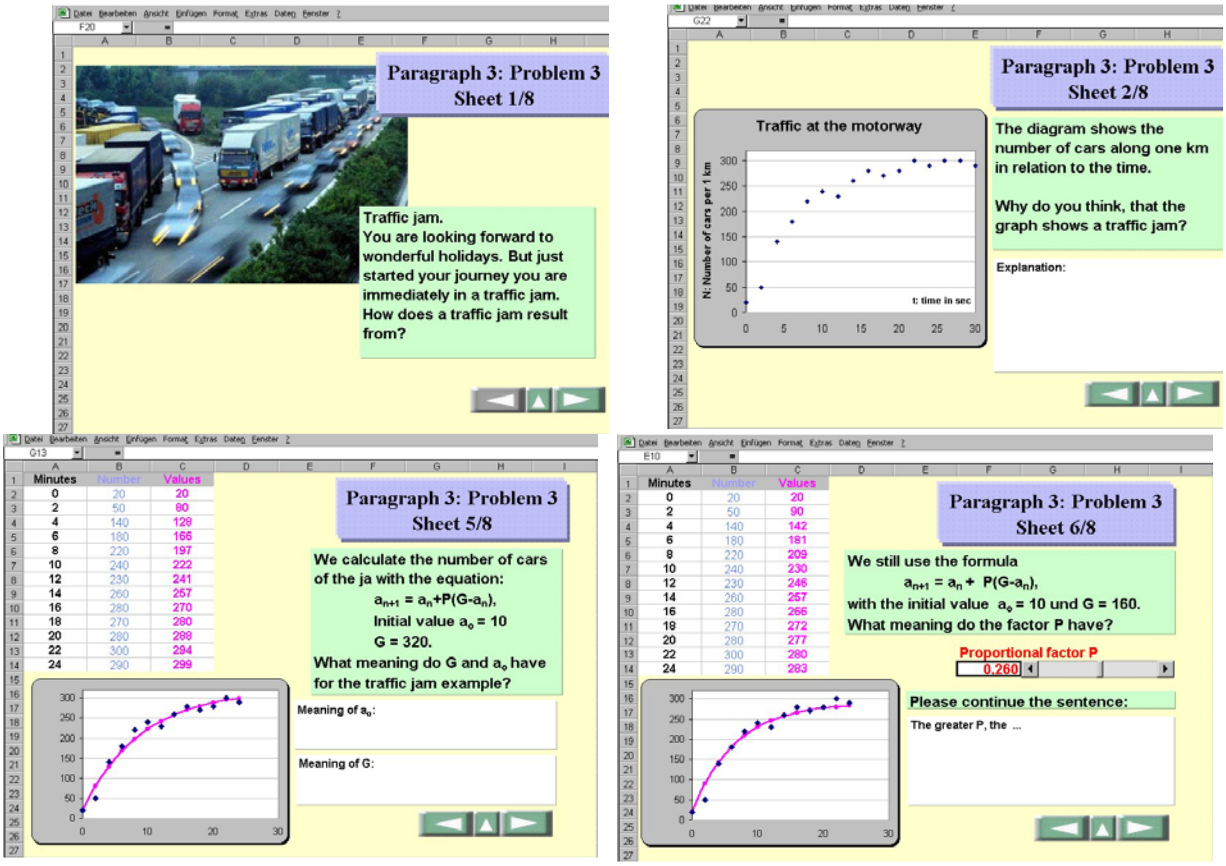






The Traffic Jam Problem: An application to $a_{n+1} = a_n + P \cdot (B - a_n)$

The interpretation of the representations of the traffic jam problem caused bigger difficulties than the previous one. The parameters were interpreted on different aspects. For the initial term " a_0" the answers contain expressions like "Number of cars per metre and per second" or "number of cars in usual traffic conditions". Connections to the geometric example are made (for a_0: "initial field – number of cars before the jam started"; for G: "Limited area that can be filled with cars"). Some students described only the effect of the change of the parameters ("the less a_0, the lower the curve"). The explanation of the parameter P and the continuation of the sentence "The larger P the ..." was made in a qualitative style by all participants. In particular any relationship to the difference $a_{n+1} - a_n$ was almost non-existent. However, the responses reflected the dynamics that were provoked by the learning programme: "More cars are getting into the jam" or "The number of cars is increasing".



Importance of the Computer Protocols

The recorded computer protocols allowed us to observe the working styles of individual students because all their activities were saved by the computer (keyboard entries, mouse movements, selection of the representations). Successful and unsuccessful problem-solving strategies can be recognized. To simplify the evaluation we had already recorded numerical data during the working phase at the computer: input values, selected representations and changes of the representation. Therefore these data were available for a quantitative evaluation immediately after the empirical investigation. The ScreenCam records served as a qualitative interpretation of the working styles of an individual. The computer protocol shows the working styles – like a videotape – in real time, therefore problem-solving strategies can be documented. In addition, problem-solving strategies employed by an individual student working on different problems, or different students working on the same problem, can be compared.

Compared to other empirical investigation methods like the use of videotapes, interviews or written tests, computer protocols have some special features:

- They are generated – in contrast to interviews – as the student solves the problem. They are therefore at the heart of the solution and learning process.
- Compared to videotapes and interviews, a larger group of students can be observed simultaneously at the same time.

The evaluation of the computer protocols also caused problems:

- We did not use a microphone in front of the computer. However this was not very important since the students worked at the computer on their own.

- Activities are not registered if there have been no keyboard entries or mouse movements. We did not know whether the students talked to each other about the problem, or whether they did not concentrate on the problem-solving process at all.
- To get transcripts of the electronic data, the computer protocol has to be written down in paper form. We had to develop criteria to construct a relationship between activities and signs and the verbal language. The following table shows part of such a transcript.

		Representation					
Time	Problem No.	Text of the problem	Table	Graph	No. of the Excel-Sheet	Comment /Description of the activities	Keyboard entries
00.00	1	X			Chap. 2 Probl. 1 Sheet 4		
00.53			X		Sheet 4a	Setting the initial value	
2.13				X	Sheet 4b	Cursor movements only	
3.05		X			Sheet 5	Input of the answer	"Concerning .....
4.15	2	X					
...							

Summary of the Results

The aim of the project was to document and analyse working styles of students in a computer-based environment while solving problems.

On the one hand, the results of the project show problems and difficulties while learners work with a computer. The high density of information presented requires concentration during the working process. The results of this investigation show that this only happens if the students solve a problem successfully. In this case the results are also controlled by the available experimental methods. On the other hand, unsuccessful strategies very often lead to cancelling the problem-solving process of this module. Variations of strategies and searching for alternative solution methods are rather rare. Concerning the construction of a learning programme, it is very important to have "anchor points" or "entry points" to enable learners “to re-enter the learning system”.

The results of the project also show the possibilities of operating with representations of sequences. A comparative study between high school and university students in the USA and in Germany showed that the American students obtained better results while using search strategies and they worked significantly more frequently with graphic than numeric representations. On the other hand the strength of the German students was in working on the symbolic level and in the transfer between symbolic and graphic representations. The arguments, concerning the properties of sequences, of both the American and the German students were influenced by the content and the kind of the representation. Beyond the level of heuristic operations students must be forced to think about the represented objects otherwise button-pressing activities are encouraged. For this reason convenient operating aids (varying through scroll bars) should be used very thoroughly.

Finally the empirical investigation has shown that it is technically possible to record and to analyse computer protocols. The transcripts of the computer protocols in the form of "time diagrams" give a good insight into the working styles and the solution strategies of the students.

The results of the project will have effects on mathematics instruction. They offer a better understanding of how individuals work on the computer and they show spreadsheet programmes as step-by-step, expandable and powerful didactic instruments for mathematics instruction.

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