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PERSISTENCE IN CORPORATE NETWORKS

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Persistence in corporate networks

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Abstract

We examine the bipartite graphs of German corporate boards in 1993, 1999 and 2005, and identify cores of directors who are highly central in the entire network while being densely connected among themselves. The novel feature of this paper is the focus on the dynamics of these networks. Germany's corporate governance has experienced significant changes during this time, and there is substantial turnover in the identity of core members, yet we observe the persistent presence of a network core, which is even robust to changes in the tail distribution of multiple board memberships. Anecdotal evidence suggests that core persistence originates from the board appointment decisions of largely capitalized corporations.

Introduction

We study the time evolution of German corporate director interlocks between 1993 and 2005, and detect a persistent core of directors who are highly central in the network while being densely connected among themselves. The statistical properties of the network core show little variation over time in spite of significant changes in corporate governance and considerable turnover in the identity of core directors, leading to questions about the mechanisms that are responsible for the origin of a persistent network core.

Traditional research in organization and management science has investigated the influence of shared directorships or ownerships on firm performance, profitability, and corporate strategy, including acquisition behavior, choice of financing, the magnitude and direction of political and charitable contributions, the adoption of poison pill practices, and many more, in fact generating such an abundance of results to warrant several recent surveys on different aspects of the subject [12, 13, 18, 31, 4, 19, 26]. Another strand of research, inspired by the interdisciplinary work of [3], has emphasized the statistical properties of corporate networks [5, 14, 30] and concludes that director interlocks exhibit the *small world effect*, whereby the interpersonal distance between any two directors is several orders of magnitude smaller than the number of directors in the network. In addition, [17] argue that director interlocks are also characterized by a high degree of clustering, typical of the *small world networks* introduced by [41].[1] Subsequently, however, [15] and [33] have shown from different yet complementary viewpoints that the high degree of clustering is present by construction, and should as such not be an unexpected feature in director interlocks. Intuitively, the reason is that directors of the same company are linked by definition to all their colleagues, while the large majority of directors serves only on a single board in the network.

Conventional wisdom has it that the *small world effect* typically stems from the presence of 'hubs', i.e. nodes with a large number of links to other nodes in the network, and the degree distribution of nodes has been shown to obey a power law in many complex networks [27, 34], yet corporate interlocks do not fall into this category, raising the question where the well-established small world property of corporate networks comes from then.

Finally, the idea that a core of director interlocks influences the degree of interest group formation has previously been put forward by [25], and several authors have suggested procedures to classify or identify a core of key players in complex networks, both in the social sciences [10, 11] and in interdisciplinary physics [21]. But the existence of a network core also has implications beyond pressure group formation, particularly for a class of diffusion processes sometimes referred to as *duplication in walks* [9]. An important illustration of such a process is the diffusion of states (e.g. expectations, tastes, opinions, trading positions, etc.) in a system made up of a large number of interacting heterogeneous agents. One can show, perhaps

[1]See [39] for a review of small world networks in the social sciences.

somewhat unexpectedly, that the existence of a core is often sufficient for the system-wide propagation of fashions and fads in such systems [2, 1]. The presence of a hierarchical core-periphery structure can oftentimes lead to system-wide conformity, including the possibility that the social interactions of core agents lead to coordinated "animal spirits" in a system that is several orders of magnitude larger than the size of the core. We believe that the potential implications of our empirical results are best discussed in light of this latter point.

While [24] show the existence of a network core in a more recent year, they lack observations on the time evolution of the board and director networks. One of the main findings of the present paper is that institutional ties among the largest German companies are maintained over time in spite of considerable turnover in the identity of directors. In light of this turnover, it becomes important to understand the origins of a persistent network core, and our analysis suggests that both the reconstruction of broken ties among large corporations, as well as their preference for recruiting experienced directors with multiple board memberships, are responsible for the time persistence of a network core. To our knowledge this paper is the only one besides the much earlier study by [23] that analyzes the dynamics of board networks.

Our findings are of particular interest in light of recent claims that the corporate network (especially in the German case) is in a "state of decline" due to increased shareholder orientation of companies, the strategic reorientation of large banks, and the decreasing influence of the state in the infrastructure sector [20, 6, 16].

In the remainder of this paper we will see that this decline is apparent in the basic descriptive statistics of the corporate network, yet the selective hiring mechanism of companies that we detect here manages to preserve a core network whose influence is, if anything, increasing over time.

After a description of the dataset, we proceed by identifying and comparing cores of directors among our subsamples. Hereafter we analyze how the re-wiring process among companies takes place throughout the years. We conclude by discussing more general implications of our findings that go beyond the realm of corporate networks.

The Dataset

Our compilation of board composition data aimed for Germany's one hundred largest publicly traded companies in 1993, 1999, and 2005. The thirty largest companies are listed in the German stock index DAX (*Deutscher Aktienindex*), while the next largest companies are listed in the Mid-Cap-DAX, or MDAX. The MDAX was founded in 1996, containing the seventy largest companies that were not included in the DAX, which we also used in the 1993 sample. In 2003, the number of companies in the MDAX was reduced from seventy to fifty, so we used the survivors among the twenty companies that left the MDAX in the 2005 sample, or replaced those that no longer existed with the next largest companies in 2005.

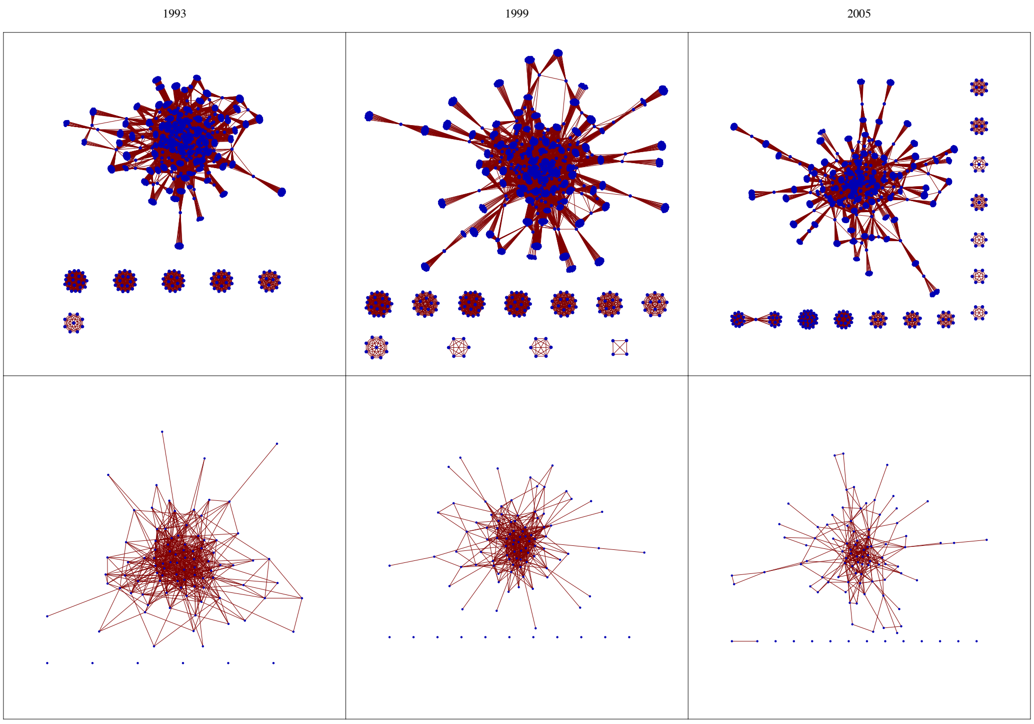

Figure 1: Company networks
The network of German director (top panel) and company (bottom panel) interlocks in 1993, 1999 and 2005. Only a few companies are isolated from the large connected components, and a casual graphical inspection already suggests that each network has a core-periphery structure.

For the purpose of our study, corporate boards consist of executive management (*Vorstand*) and supervisory board (*Aufsichtsrat*). According to the pertinent German legal code, they have to meet at least four times per year (§ 94(3) of *Aktiengesetz*, or *AktG*). Executives are appointed for a maximum of five years, and both appointment as well as potential reappointment need to be approved by the supervisory board (§ 84 *AktG*). In light of the five-year limit, we chose equally spaced intervals of six years to increase the likelihood of observing changes in the composition of corporate boards. We compiled

	1993	1999	2005
# of companies (distinct: 176)	97	100	100
# of directors (distinct: 3884)	1744	1711	1593
# of mandates	2143	2044	1833
average board size	22.9	20.4	18.3
average mandates per director	1.23	1.17	1.15
Company links (total)	803	657	375
Company links (unweighted)	597	490	291

Table 1: Corporate mandate statistics
The descriptive statistics of our sample illustrate a slight decrease in the number of directors and mandates over time, and a highly nonlinear decrease in the number of links between companies that are formed by multiple board membership. The non-linearity is caused by the fact that a director with b mandates creates $l(b) = b!/2(b-2)!$ links among companies. If, for instance, a director with $b = 6$ mandates retires and is replaced by different single-mandate directors each time, then $l(6) = 15$ links are removed from the network.

the data by consulting various archives that keep records of the annual reports of these companies, and by writing to companies for whom we could not locate annual reports. Three companies in the 1993 sample did not reply to our inquiry, all three of them with relatively minor market capitalization, leaving us with 97 companies in that year.

The descriptive statistics of our sample, reported in Table 1, show a decreasing average board size over time, which is mainly due to M&A activity among very large corporations,[2] and also to the fact that 2005 additions had only about half the sample's average board size in that year.

Let n be the number of directors in a year, and let c be the number of companies in that year. Then the *incidence matrix* M of dimension $n \times c$, with $m_{ij} = 1$ if director i is on the board of company j and zero otherwise, describes the corporate network in each year. The projection onto directors, $D = MM^T$, is the weighted adjacency matrix of director interlocks. Its diagonal entries equal the total number of board memberships of director i, while non-zero entries off the diagonal of D represent the weight of a link, showing on how many boards two directors serve together. Symmetrically the projection onto boards, $B = M^T M$, yields the weighted adjacency ma-

[2]Dresdner Bank, for instance, was acquired by Allianz in the financial sector, while VEBA and VIAG merged to EON in the utilities industry.

trix of company interlocks, its diagonal entries correspond to the board size of company j, and off-diagonal non-zero elements indicate the number of directors that two companies have in common. The resulting networks are displayed in Figure 1 and readily reveal the existence of a core in each period, but the figure also suggests that the number of core companies and directors decreases over time. The question is then, as one might perhaps intuitively expect, whether core directors also become less influential, less central or less densely connected among themselves.

Analysis of Cores of Directors

Random Benchmark and General Developments

To understand the origins of the small world effect in director interlocks, we find it instructive to follow the approach of [24].

We first need to identify a core of directors. For this purpose, it is instructive to consider the frequency of multiple board memberships, shown in Figure 2.

We start from the observation that the vast majority of directors serves on just one board, and conduct a simple thought experiment. Suppose that the directors in each sample are indistinguishable; then we can determine the probability of observing multiple board membership as a sequence of k independent Bernoulli trials, resulting in a binomial distribution for observing $B = b$ additional board memberships,

$$Pr[B = b] = \binom{k}{b} p^b \, (1 - p)^{k-b},$$

where p is the probability of success, i.e. of obtaining an additional board membership. To illustrate the procedure, consider for example the year 1993: there are 1744 directors in total, and the number of mandates is 2143, yielding $k = 2143 - 1744 = 399$, and $p = 1/1744$. Figure 2 illustrates the resulting binomial distributions and compares them to the empirical relative frequencies of multiple board membership.

For $b > 3$, the incidence of multiple board membership is several orders of magnitude higher than we would expect in a sequence of independent Bernoulli trials, which suggests that directors with three or more mandates are probabilistically distinct and thus in some sense special. One would expect to observe a network core if these directors were connected among themselves, thus we plot the network structure among directors with $B \geq b$ board memberships in Figure 3, which reveals that the resulting sub-graphs,

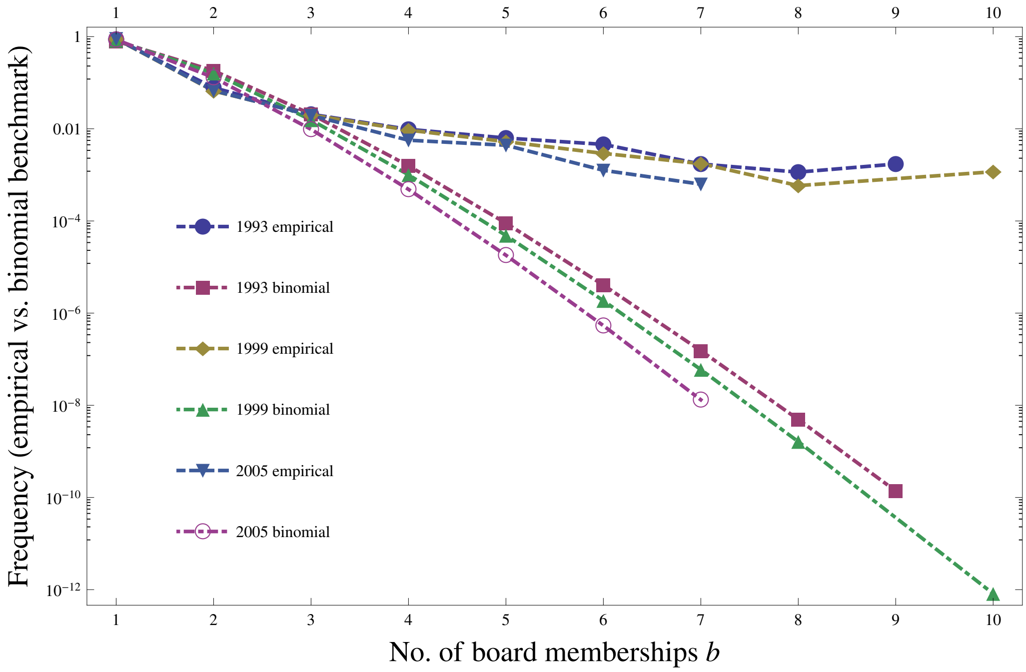

Figure 2: Board membership benchmark

The top curves show the empirical relative frequency of multiple board membership in each of the three years, while the bottom curves illustrate the binomial probability of observing multiple board membership in the respective sequences of independent Bernoulli trials described in the text. The semi-log scale reveals deviations on increasing orders of magnitude for $b > 3$.

or *b-cores*,[3] are indeed to a very large extent connected. We also observe that both the number of directors and the fraction of companies that are connected by the respective core directors decrease over time in our sample, shown in Figure 4.[4]

A major contributing factor to this development is certainly the recent reform of Germany's corporate governance code (DCGK).[5] The reform deals with a number of national and international criticisms that have been leveled against Germany's traditional corporate governance, mostly concerning the inadequate focus on shareholder interests, and the inadequate independence of supervisory boards, addressed for instance in DCGK paragraphs 5.1.2 (age limit for management board service), 5.4.2 (independence of supervisory board members), and 5.4.4 (deterrence of the hitherto custom that former chief executives serve as supervisory board chairmen). While the new code aims at standardizing best practices in corporate governance, it does not have the status of a formally binding law. Nevertheless, deviations from DCGK rules have to be explicitly justified and publicized on an annual basis (§ 161 *AktG*), and the observed decrease in the average number of board memberships is not an unexpected feature from this perspective. It is noteworthy that the code took effect in early 2002, while the pronounced decrease in average mandates indeed occurs between the 1999 and 2005 samples.

At this point, one can speculate whether the DCGK is the ultimate cause of these developments or not, yet over the years we do in fact observe a pronounced decline in executive managers' supervisory board memberships:[6] Table 4 in the appendix shows that in 1993 (1999, 2005), the 569 (441, 457) directors with executive positions additionally served on 228 (164, 83) supervisory boards of other corporations. The drop in the ratio of supervisory board memberships per executive ($228/569 = 0.4$ in 1993, 0.3 in 1999 and 0.17 in 2005) illustrates that corporate governance practices have indeed changed over the investigated time period. This brings us back to the question whether shrinking core sizes also implies that core directors become less influential over time.

[3]Notice that our b-cores differ from so-called k-cores, which are constructed using a node's minimum degree [36].

[4]Another method for this analysis would be to use a core-periphery model [11]. Since we only want to show that a set of nodes that we already identified is a core we use a different approach here.

[5]See http://www.corporate-governance-code.de/index-e.html.

[6]Current members of the management board must not simultaneously serve on the company's supervisory board (§ 105 *AktG*), but have routinely been allowed to serve as supervisory board members at other companies.

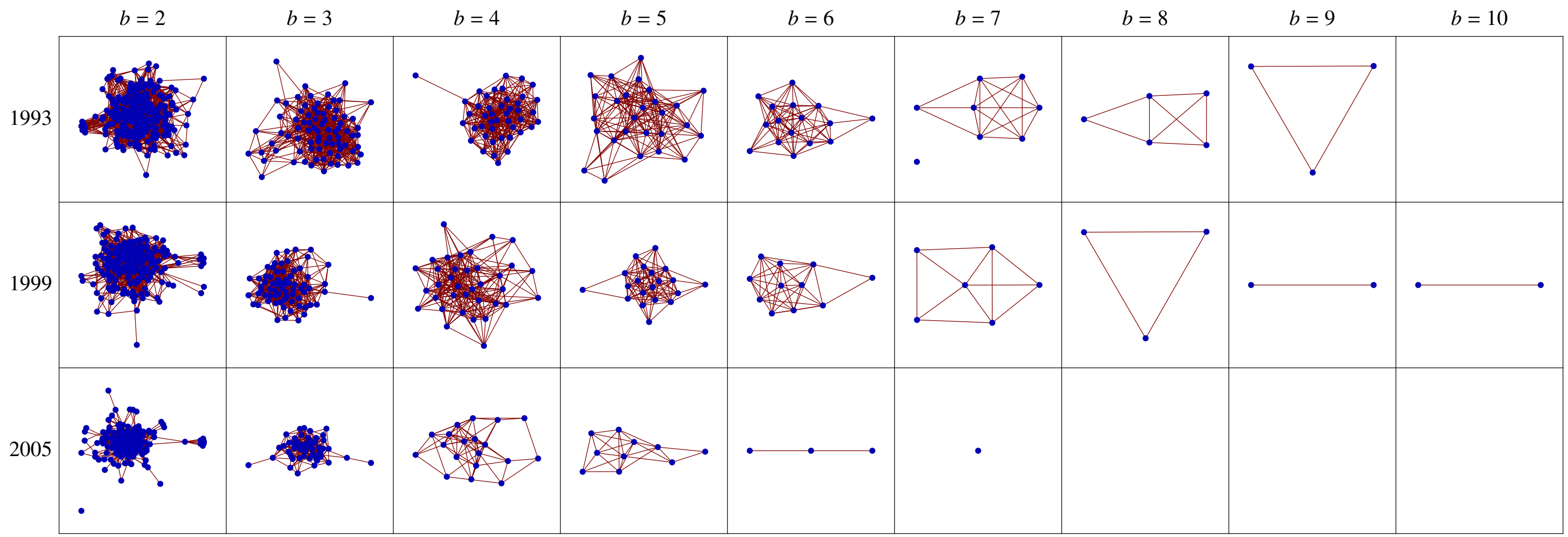

Figure 3: Network structure of successive b-cores

Network structures formed by considering directors with an increasing threshold of board memberships $B \geq b$. Notice that there is only one instance of an isolated director with $b \geq 3$ mandates in any of the years (Alfred Pfeiffer in the 1993 $b-7$ core). The size of cores and the overall fraction of companies in the respective cores both decrease over time, shown in more detail in Figure 4.

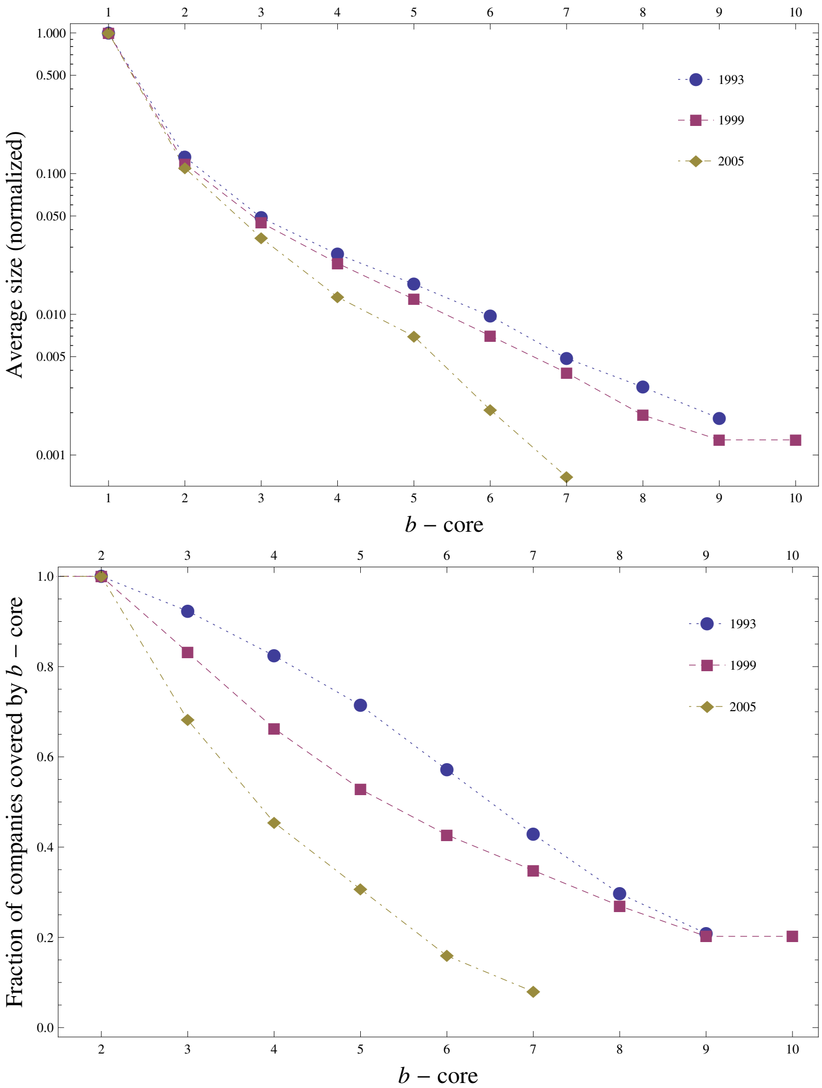

Figure 4: Directors and companies of b-cores
The number of directors in the respective b-cores (top panel) decreases over time, as well as the fraction of companies that are linked by the respective b-core directors (bottom panel).

Density and Corporate Reach

Intuitively a network core consists of directors that are highly central in the network and densely connected among themselves. The density of the (unweighted) graph **D** is given by the ratio of the existing number of links, denoted $|L|$, to the number of links in a complete graph of the same size, denoted $|N|$,

$$\text{density}_D = \frac{2|L|}{|N|(|N|-1)}, \tag{1}$$

which is by construction confined to the interval $[0, 1]$. The left panel of Figure 5 illustrates that (i) the density of b-core sub-graphs increases with b, and that (ii) the density of the respective cores remains fairly constant over time in spite of decreasing core size. In addition, we can assess the corporate reach per core director by the ratio of distinct core companies to the number of core directors, the rationale being that a core of densely connected directors probably yields the more institutional power the fewer individuals constitute the core, and the more companies they span. It is noteworthy that this measure of core power, shown in the right panel of Figure 5, actually increases over time, so the recent decline in the number of board memberships does not necessarily mean that core directors in Germany's corporate board network also become less influential. If anything, the statistics shown in the bottom panel of Figure 5 suggest the opposite.

Centrality

A complementary approach to measuring the importance or influence of directors is to consider their centrality in the overall network of director interlocks. Let C denote the respective adjacency matrices of the large connected components of D in the respective years, and let V denote the set of directors contained in C. A shortest path between two directors $u, v \in V$ is known as a graph *geodesic*, which is not necessarily unique, and the length of the geodesic $d_C(u, v)$ is known as the graph *distance* between the pair (u, v).

The first centrality measure we consider is *closeness centrality*, which measures the distance of a node to all other nodes in the network, and is typically defined as the reciprocal of the sum of geodesics to all other nodes in the network,

$$\text{closeness}_u = 1/\sum_{v \in V} d_C(u, v)\,. \tag{2}$$

Since we would like to compare the centrality of directors across years, we divide by the closeness score of the director with maximal closeness centrality

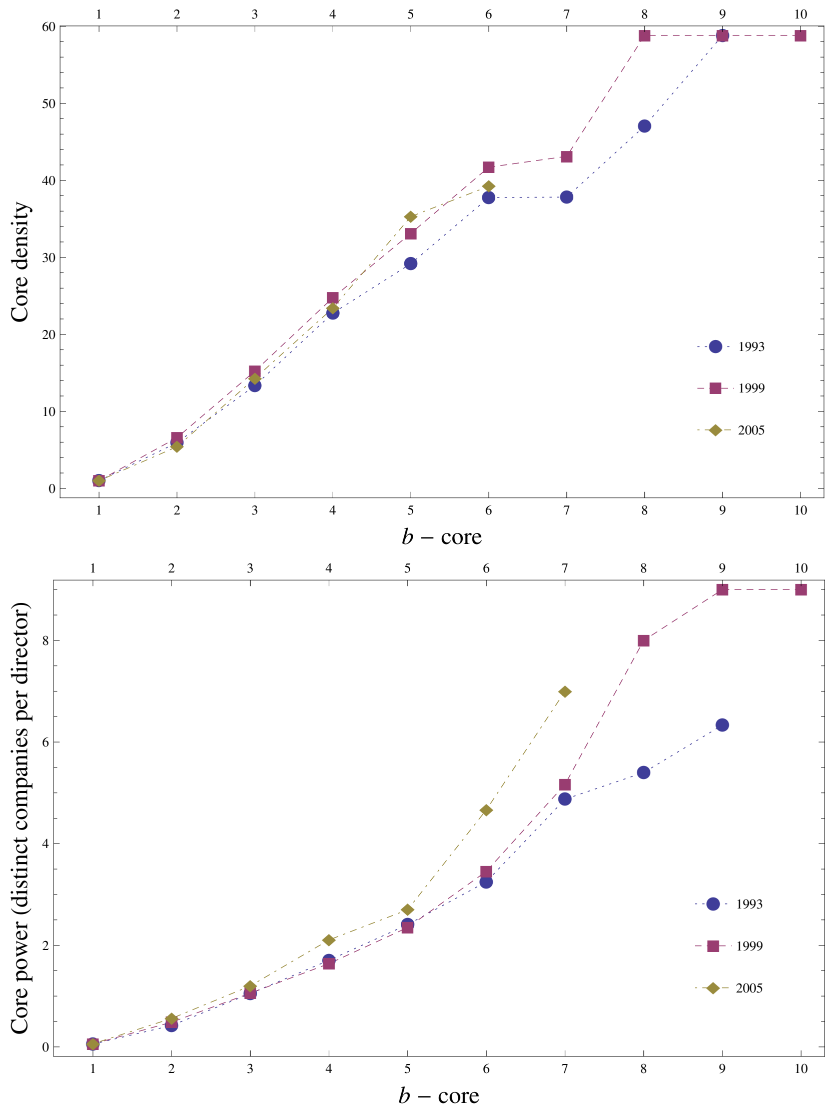

Figure 5: Core companies and core directors
The density of b-cores (top panel) remains fairly stable over time, as well as the ratio of distinct core companies to core directors (bottom panel). If anything, core density and core power increase despite the decrease in the average number of mandates over time.

in each year in order to normalize the scores. Directors who are more central in this sense should in principle be better able to reach out into the entire network or be faster in doing so.[7]

Another measure of the centrality of node u is *degree centrality*, constructed by summing the number of links that each node has, $\text{degree}_u = \sum_{v \in V} C_{uv}$. Intuitively, directors who have many links compared to their peers are in an advantageous position if they are able to influence many of their peers, or if they have better access to resources through their many links. But degree centrality only takes immediate ties of directors into account, and lacks information about the distance to directors that are not immediate neighbors. Moreover, directors with many board memberships have a relatively large degree by construction since the board size distribution has a characteristic scale that is well captured by its mean.

Therefore, we also compute the eigenvector centrality [8] for all nodes in V. *Eigenvector centrality* assigns scores of relative importance to directors in the network, based on the principle that connections to high-scoring directors contribute more to a director's score than equal connections to low-scoring peers. Hence the idea behind eigenvector centrality is that the quality of links is important, because directors who are connected to many influential peers can be expected to be important themselves. Suppose the eigenvector centrality score of node u, denoted e_u, is proportional to the centrality score of its neighbors,

$$e_u = \frac{1}{\lambda} \sum_{v \in V} C_{uv}\, e_v\,, \tag{3}$$

where λ is a constant. Then we can write the vector of centrality scores in matrix notation as $\lambda e = C \cdot e$, which shows that e is an eigenvector of C with corresponding eigenvalue λ. It is convenient to consider the eigenvector corresponding to the largest eigenvalue of C since its elements are all non-negative according to the Perron-Frobenius theorem. Again, we divided all scores by the maximum score in each year to normalize the data. Figure 6 shows that core directors are not only densely connected among themselves, but that they are also increasingly central in the entire network, which is another characteristic that one intuitively expects in the definition of a network core.

[7]For a discussion of the finding that power and centrality are not equivalent see [7].

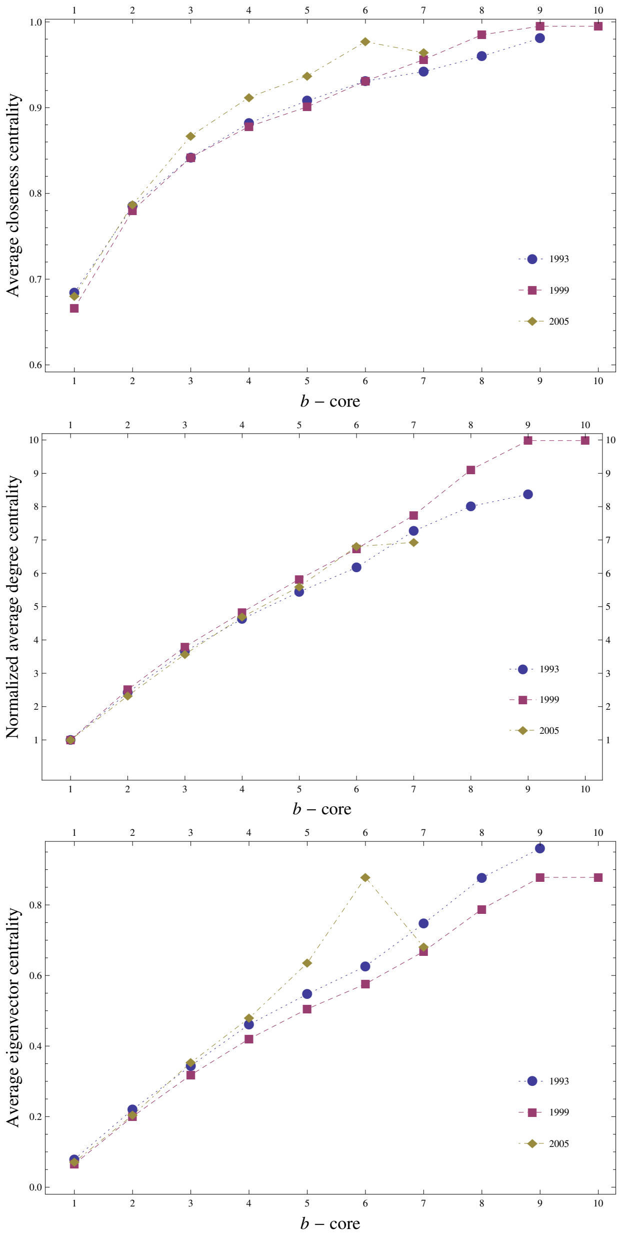

Figure 6: Centrality of directors
Normalized average centrality measures of b-core directors in the respective years: closeness (top), degree (center) and eigenvector (bottom) centrality remain fairly stable over time.

Core Persistence and Individual Turnover

We have argued that the structural characteristics of director interlocks are stable over time in spite of changes in corporate governance and a decrease in the average number of mandates. Motivated by the persistence in network structure, we want to investigate whether or how the core structure depends on the destinies of particular agents.

Evolution of Company Links

An important aspect here concerns the links in the company network. Their time evolution over consecutive periods reveals some noteworthy patterns: in 1993 (1999) the company network had 597 (490) unweighted links, 290 (195) of which were with companies that remained in the sample in the next period (and had not merged in the meantime), while 141 (95) of these links were still in place in the following period. In 78 (62) cases, at least one director was constantly part of both boards. In addition, 24 (10) directors had been recruited to reinforce existing links, then serving on average on 4.38 (4.04) boards. Out of these 24 (10) directors, 18 (7) were already serving on at least one board in the previous period, when they held 2.22 (2.68) appointments on average.

In the remaining 63 (33) cases, where an existing link was maintained through the recruitment of new directors, a total of 50 (26) directors was appointed to the 63 (33) positions, and these directors then served on average on 4.04 (3.77) boards. Out of these 50 (26) directors, 34 (20) had at least one mandate in the previous period, when they served on average on 2.68 (2.75) boards. So we observe that about half of the company links are being maintained between periods, which is consistent with earlier findings by [35] on the reconstitution of German corporate interlocks.[8] Keeping in mind that a substantial number of links in the (initial) company network might be unintended,[9] and given that the sample periods have been chosen sufficiently far apart to warrant board (re)appointment decisions, the observed

[8]The percentage of reconstructed ties among German companies is four to five times higher than previously observed in the US [38]. From a network core perspective, it would be interesting to clarify whether the percentage of reconstituted ties among the very largest (core) corporations in the US is substantially higher than in the original Stearns and Mizruchi sample.

[9]Imagine a director with three mandates and suppose that she is on the board of company A, which manages to place her on the board of company B for strategic reasons, e.g. to oversee A's interests. If she also serves on the board of a third company C, we consider the link between A and B intentional, while the links AC and BC are unintentional byproducts.

reconstitution of links would certainly seem to indicate planned or strategic connectivity among German corporations.

Secondly, these figures suggest that companies seem to prefer the appointment of directors who already serve on several other boards, which is particularly true for the maintenance of institutional links over time. It is rather doubtful that these directors were appointed for purely supervisory purposes since the effort involved in monitoring a handful of DAX companies is surely considerable, and in all likelihood becomes increasingly prohibitive if one of the appointments is an executive position. The frequency distribution of executives' supervisory board memberships in Table 4 nevertheless shows that some executives additionally served on up to ten other boards.

Director Survival

Since multiple board memberships seem to be essential for both the existence and the persistence of a core network, we also investigate the survival of directors over time. Out of a total of 1517 directors in the 1993 sample, 518 are still present in 1999, which is a survival rate of 34%. For the 1999-2005 transition, this figure is 32%. During the first (second) transition period, 12.6% (13.1%) of the surviving directors gained mandates, while 12.4% (11.3%) lost at least one mandate. But these percentages conceal that directors with multiple memberships have a markedly higher survival probability than the vast majority of directors with a single mandate: the survival probability conditional on the number of existing mandates is 31% (28%) for board members with a single mandate, it increases to 51% (47%) for directors with two mandates, 69% (76%) for those with three mandates, and 70% (78%) for those with four or more mandates.[10] It seems fair to say that the persistent structure of Germany's corporate network is driven by the recruitment decisions of large companies, which are characterized by a process of "selective replacement" that expresses itself in the figures on the maintenance of links among companies and the conditional survival of directors. While the vast majority of directors enters and exits the corporate network without ever being particularly central in it, a small number of highly connected key directors warrants a persistent network core over time. Moreover, fluctuations in the destines of key players are mitigated by the reconstitution of ties among large corporations, who favor directors with multiple memberships. To corroborate this claim, we consider the turnover in the centrality of companies and directors between periods.

[10]These figures are easily calculated from the transition matrices in Appendix .

			companies		directors	
			1993 – 1999	1999 – 2005	1993 – 1999	1999 – 2005
	survivors		91	89	1645	1562
	dropouts		34	38	1087	1081
average centrality	closeness	out	0.5034	0.4438	0.4950	0.4450
		in (t)	0.7789	0.7317	0.7014	0.6850
		in $(t+1)$	0.7521	0.7497	0.6845	0.7082
	eigenvalue	out	0.0023	$<$0.001	0.0513	$<$0.001
		in (t)	0.2766	0.2339	0.1040	0.0926
		in $(t+1)$	0.2458	0.3301	0.0866	0.0978
	normalized degree	out	0.0250	0.0270	0.0313	0.0207
		in (t)	0.3636	0.3121	0.1509	0.1358
		in $(t+1)$	0.3295	0.3431	0.1352	0.1857
change in centrality	closeness	avg. $\lvert\Delta\rvert$	0.0547	0.0730	0.0521	0.0684
		avg. Δ	-0.0268	0.0100	-0.0169	0.0233
		benchmark	0.1348	0.1598	0.1057	0.1208
	eigenvalue	avg. $\lvert\Delta\rvert$	0.0883	0.1161	0.0513	0.0571
		avg. Δ	-0.0308	0.0962	-0.0175	0.0052
		benchmark	0.2702	0.2912	0.1174	0.1199
	normalized degree	avg. $\lvert\Delta\rvert$	0.0953	0.1039	0.0543	0.0855
		avg. Δ	-0.0341	0.0310	-0.0157	0.0499
		benchmark	0.2651	0.2838	0.0954	0.1143

Table 2: Turnover activity in company and director networks

The notation for average centrality measures refers to averages for dropouts ("out"), and the centrality of survivors in the current ("in (t)") and next ("in $(t+1)$") period. The benchmark values for the changes in centrality have been calculated according to the procedure described in this section.

Turnover in Company and Director Centrality

We start by calculating the change in each of the three centrality measures for surviving nodes in the connected component. Table 2 illustrates that about two thirds of the companies but only one third of the directors survive consecutive periods. The life span of directors is biologically limited while the same is not (necessarily) true for corporations, thus the fact that about 70% of directors but less than 40% of companies drop out between periods is by itself not unexpected. In spite of the expected difference, we find that the *mean absolute change in centrality*, as a measure of variability among survivors, has the same level of magnitude for both companies and directors, and that absolute changes in centrality are rather small in both cases.[11]

In order to properly compare the turnover activity between companies and directors, we need to scale the absolute changes in centrality with a benchmark measure of persistence in centrality that accounts for the different scales of the company and director networks. In our benchmark case, we assume that each node's centrality could change to every observed centrality in next period's sample with equal probability, corresponding to a uniformly random rewiring of nodes. Thus the average absolute change in centrality would be zero in the case of a perfect conservation of the relative position of nodes, and would be equal to the benchmark value in case of a completely random rewiring of surviving nodes. For all centrality measures, we observe that the ratio of the benchmark to the observed value is larger for companies than for directors (about 3:1 vs 2:1), showing that surviving companies exhibit significantly less churning in their centrality than surviving directors. Comparing dropouts with survivors, both for companies and directors, we find that survivors are always substantially more central than dropouts (also reported in Table 2), and that the normalized degree is about one order of magnitude higher for survivors than for dropouts. Moreover, the very low average eigenvector centrality of dropouts further implies that the importance of the dropouts' few neighbors is also very low on average. In summary, both highly central companies as well as directors tend to stay central, while dropouts are located in more peripheral positions of the network. Company networks exhibit less turnover activity than director networks both in the share of surviving nodes but also in the centrality changes among survivors.

[11] All three measures of centrality exhibit a slight decrease in *average centrality* between 1993 to 1999, and an increase between 1999 to 2005, as reported in Table 2. This is in line with the visual inspection of the network structure in Figure 1, which shows an increasing number of peripheral nodes in 1999, and denser cores in 2005.

Identifying Company Groups

Our findings so far strongly suggest that as far as the persistence of a corporate network core is concerned, idiosyncratic director characteristics are far less important than the selective recruitment practices of companies. Hence we would like to identify cliques among the companies in our dataset, and see if persistent structures also exist in the network of company interlocks.

To identify and visualize connected groups, we essentially use principal component analysis and combine it with a matching algorithm. The procedure is described in detail in the supplementary information.

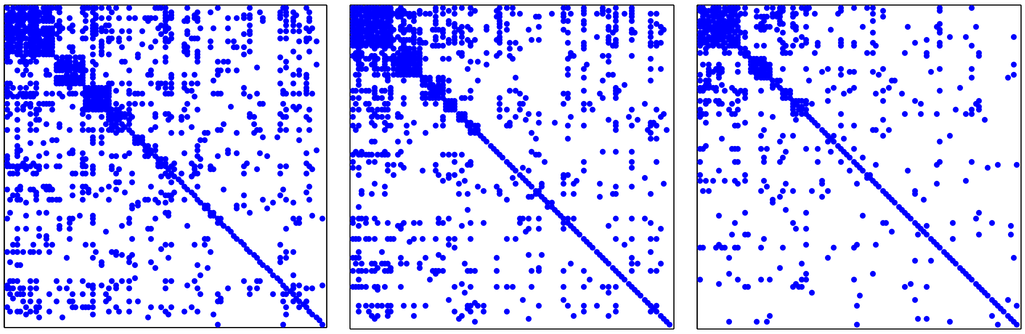

Figure 7: Resorted adjacency matrices for 1993, 1999, and 2005
The number and size of identified cliques is decreasing, leading to a sparser corporate network over time, yet with an intact core network of about a dozen mega-cap corporations.

Figure 7 shows the resorted adjacency matrices for 1993, 1999, and 2005. We plot company cliques with descending size from left to right, followed by firms that do not exhibit specific connection patterns. Hence, every black dot in row i and column j represents a link between company i and j, while a white space in position i, j would indicate that the two companies have no directors in common.

In 1993 it is still possible to divide the core of our network into overlapping subgroups, but this structure seems to be fading away over time. The number of firms that belong to a clique is about 30 in 1993, yet this number as well as the size of identified cliques is shrinking over time. There are, however, ten traditional German corporate heavyweights that persistently show up in the largest cliques: *Allianz, Bayer, Commerzbank, Deutsche Bank, Hochtief, Linde, Lufthansa, Siemens, Thyssen Krupp,* and *Volkswagen* (see Appendix for details). This visualization confirms our earlier findings that even if the overall level of connectedness in the corporate network is decreasing, highly central firms tend to consolidate their ties and thus remain or become even more central in the network.

Discussion

What have we essentially accomplished here? We show that the corporate network exhibits a core that does not become less influential over time in spite of significant reforms in corporate governance, and substantial churning and entry-and-exit dynamics among corporate directors. There are strong indications that selective recruitment decisions of the very largest companies are responsible for core persistence, and this process appears largely independent of idiosyncratic director characteristics and does not require a "mighty conspirator" who pulls the strings in the background.

In addition, the existence of a core provides an alternative explanation for the small-world effect in corporate interlocks in the absence of a scale-free degree distributions among directors or firms. Since in numerous social contexts the number of links and nodes exhibits a characteristic scale and does not span enough orders of magnitude to be a power law, one could speculate that core structures are important in other socio-economic contexts as well, and we will return to an important illustration of this point in financial markets below.

A remaining issue concerns the validity of our findings beyond Germany's one hundred largest publicly traded companies. So can we reasonably expect our results to be representative of the entire German corporate network? Judging from the more recent results by [24] for a single year, there is reason for optimism: considering the largest 284 German companies in 2008 (accounting for more than 95% of that year's market capitalization of Germany's stock exchange *Deutsche Börse*), they observe very similar magnitudes in the maximum number of board memberships, and in the density and average centrality of successive b-cores. Second, and more importantly from the viewpoint of the present study, they find that the pronounced core structure in Germany's corporate network is clearly formed by mega-cap companies, thereby justifying our focus on the one hundred largest companies here. Since they examine the corporate network in a single year, however, they lack information on the time evolution of the network.

As our dynamical analysis reveals, on the other hand, it is indeed the largest thirty to forty corporations that are responsible not only for the existence but also the *persistence* of a core over time. While the presence and persistence of a core originate from the appointment decisions of the largest corporations, our findings leave little room for the relevance of individual directors from a macroscopic point of view: the churning and entry-and-exit dynamics of individual directors in our sample instead emphasize the (seemingly exclusive) importance of the number of mega-cap mandates that a director has accumulated at a given point in time.

The self-reinforcing mechanism of selective replacement of board members has remained in place in spite of the apparent "decline of the corporate network." To paraphrase Mark Twain, reports of the corporate network's death are greatly exaggerated since selective recruitment maintains the small world characteristics of the corporate network in the absence of a power-law degree distribution among directors or companies, which has traditionally been invoked to explain the small-world property in other empirically observed networks.

To be clear, the main contribution of this paper is certainly of a descriptive nature. We have not performed any confirmatory statistical exercises in the traditional sense, so there is very little (if anything) we can say here about how or whether a corporate core influences the destinies of individual firms over time, but these are questions that could surely be addressed in future research. Nevertheless, we would like to believe that the existence and persistence of a core is interesting in its own right, and goes well beyond concerns regarding pressure group influence, especially when it comes to issues of a system's resilience or fragility. It has been shown that the system-wide coordination of opinions, expectations, or even animal spirits is enormously facilitated by the existence of a network core, and could very well make a system more fragile exactly because of that. Think, for instance, of the large-scale coordination of trading positions in financial markets that precede every financial crisis. From our perspective, it is rather noteworthy that financial markets also exhibit a core network structure, at least in the few instances for which data have been available so far [37], hinting at the broader importance of core networks in the social sciences.

Finally, if policymakers were to aim for a reduction of pressure group influence, core characteristics should be at the forefront in the design of policy and corporate governance, particularly in light of the fact that the legal restrictions that are already in place, like the number of simultaneous board appointments or the maximum term of service before potential reappointment, are obviously not sufficient to decrease the influence of the core network, much less prevent its existence.

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Supplementary Information

Clustering Procedure

There are essentially two streams of literature which deal with the detection of groups in networks [28]. The traditional approach is called *graph partitioning* and splits the network into a fixed number of subgraphs, e.g. by spectral decomposition of the so-called Graph Laplacian [40].[12] Graph partitioning algorithms can become very time consuming, but the more serious concern is conceptual because the number of useful partitions is generally not known. *Community detection* therefore has developed algorithmic procedures that endogenize the number of subgraphs in the partitioning process, for instance by iterative edge removal based on the calculation of betweenness scores [29]. We employ a mixture of these two approaches here, starting from a principal component analysis (PCA) and combining it with a scoring algorithm that creates groups based on the largest components without predefining the number of subgraphs. A combination of PCA and some matching algorithm represents a reasonable alternative to community detection algorithms in the social sciences, because the size of datasets is generally smaller than in the natural sciences.[13]

First we take the adjacency matrix of the firm network and standardize its columns by subtracting the column-wide mean and dividing by the standard deviation of column entries. This new adjacency matrix is denoted $\hat{\mathbf{B}}$. The column entries can now be interpreted as the relative weights of the companies' links, while the row entries resemble the relative attention a firm receives through links from other firms. We measure the correlations of link patterns by calculating the empirical correlation matrix $\mathbf{R} = c^{-1}\hat{\mathbf{B}}^T\hat{\mathbf{B}}$, allowing us to infer which firms have similar relative weights in their link patterns. Based on this correlation matrix we compute our new variables, the principal components $\mathbf{F}$, which are linear combinations of the original variables such that $\mathbf{F} = \mathbf{Y}\hat{\mathbf{B}}$. The column vectors in $\mathbf{Y}$ carry the weights for each new variable, and it can be shown that the solution to this problem amounts to solving for $\mathbf{F} = \mathbf{V}\hat{\mathbf{B}}$, where $\mathbf{Y} = \mathbf{V}$ contains the eigenvectors of the correlation matrix $\mathbf{R}$ ordered by descending eigenvalues [22]. The eigenvectors with the highest eigenvalues account for a large amount of the variance in the data, while low eigenvalues stand for eigenvectors and components that

[12] The Laplacian is a special form of the adjacency matrix of the network, where the trace of the Laplacian corresponds to the number of links between nodes.

[13] The benefit of this method is that PCA is more parsimonious and transparent than community detection algorithms, and perhaps also better known among social scientists than the latter. The foundations of our subsequent analysis can be found in [32].

contribute very little to the variance and are consequently neglected.

To illustrate the principles that we use to form groups, assume that based on some decomposition we have approximated the adjacency matrix $\mathbf{B}$ by a dimensionally reduced matrix $\hat{\mathbf{E}}$, where both matrices would contain ones for links and zeros otherwise, and $\hat{\mathbf{E}}$ being of lower rank than $\mathbf{B}$ due to the dimensional reduction. To decide whether $\hat{\mathbf{E}}$ is a good representation of the original network, we essentially have to compare and score by rewarding the correct matching of links (and non-links) in $\mathbf{B}$ and $\hat{\mathbf{E}}$, and symmetrically by punishing the matching of links to non-links in either direction. A weighting scheme will be helpful for the comparison because the adjacency matrices of our corporate networks are sparse, therefore actual links are more informative than non-links. These considerations result in a *scoring (or error) function* of the form

$$
\begin{aligned}
s(\hat{\mathbf{E}}|\mathbf{B},\Omega_{1\ldots4}) = \quad & \underbrace{\sum_{i=1}^{c}\sum_{j=1}^{c}\Omega_{1,ij}\hat{\mathbf{E}}_{ij}\mathbf{B}_{ij}}_{\text{links to links}} - \underbrace{\sum_{i=1}^{c}\sum_{j=1}^{c}\Omega_{2,ij}(1-\hat{\mathbf{E}}_{ij})\mathbf{B}_{ij}}_{\text{non-links to links}} \\
& - \underbrace{\sum_{i=1}^{c}\sum_{j=1}^{c}\Omega_{3,ij}\hat{\mathbf{E}}_{ij}(1-\mathbf{B}_{ij})}_{\text{links to non-links}} + \underbrace{\sum_{i=1}^{c}\sum_{j=1}^{c}\Omega_{4,ij}(1-\hat{\mathbf{E}}_{ij})(1-\mathbf{B}_{ij})}_{\text{non-links to non-links}}
\end{aligned} \tag{4}
$$

where $\Omega_{1\ldots4}$ represent weighting matrices. It is common to focus on the matching of links in the reference matrix $\mathbf{B}$ by equally weighting the (mis)matching of original links, $\Omega_{1,ij} = \Omega_{2,ij} > \Omega_{3,ij} = \Omega_{4,ij}$ [32].

The algorithmic procedure that we employ here to visualize the groups in the network follows exactly the principles illustrated above. The only difference is that instead of comparing two (binary) adjacency matrices, we compare the adjacency matrix with the principal components (which are not binary) in column-wise fashion. The groups to which we want to match firms are given by the largest components resulting from the PCA. Since $\hat{\mathbf{B}}$ describes links, the components in $\mathbf{F}$ describe those links in which companies differ. If we normalize the column vectors in $\mathbf{F}$ and allow for a sign change in every column, we get a new matrix of *link profiles* $\hat{\mathbf{F}}$ with dimensions $c \times 2c$, where each even column contains the entries of the previous (i.e. original, now odd) column with reversed signs. This sign change is necessary since the principal components describe only a new axis within the variable space, but do not inform us of the direction. As detailed below, we will only use the first few columns of $\hat{\mathbf{F}}$ and calculate a similarity score for each group (represented by a column in $\hat{\mathbf{F}}$) and each firm (represented by a column in $\mathbf{B}$). This results in a matrix $\mathbf{S}$ of scores for all firms and groups with dimension $c \times 2k$, where k is the number of included principal components (more on k below). Given the sign change, the number of groups will be $G \leq 2k$. The weights of links and non-links can be approximated by the number of links versus non-links in the original network. Since the graph density is only about 0.03, we set $\Omega_3 = \Omega_4 = 0$. Furthermore, we do not need to differentiate the weight of each single link as it is already contained in the respective values of the link profile $\hat{\mathbf{F}}$, hence $\Omega_1 = \Omega_2 = 1$. Notice, however, that in contrast to $\hat{\mathbf{E}}$ the link profile $\hat{\mathbf{F}}$ is not a binary matrix (it contains the relative weights of links), therefore we lastly introduce a matrix $\mathbf{\Theta}$ that translates $\hat{\mathbf{F}}$ into a binary form such that $\Theta_{jg} = 1$ if $\hat{\mathbf{F}}_{jg} > 0$ and zero otherwise:

$$\mathbf{S}_{ig} = \sum_{j=1}^{c} \mathbf{\Theta}_{jg}(\hat{\mathbf{F}}_{jg}\mathbf{B}_{ji})^2 - \sum_{j=1}^{c}(\mathbf{\Theta}_{jg} - 1)(\hat{\mathbf{F}}_{jg}\mathbf{B}_{ji})^2 \quad (5)$$

for all $i = 1, \ldots, c$ and $g = 1, \ldots, G$. Thus $\mathbf{\Theta}$ ensures that we sum over all positive entries in $\hat{\mathbf{F}}$ in the first sum, and over all negative entries in the second sum. For every correct match, that means if a firm i has a link to company j where the link profile would suggest one, we increase the score that firm i obtains for group g by the squared entry in the link profile. Symmetrically, if the firm has a link where we do not expect one, we deduct this score. Each firm is now assigned to the group for which it has the highest score (by identifying the maxima in each row of $\mathbf{S}$). There are, of course, quite a few firms that score rather poorly in all of the groups, simply because they do not belong to any. These firms either have very few links to begin with, or have a unique link pattern. To filter out such firms, we set a threshold in our scoring procedure, yet it turns out that the grouping is quite robust with respect to the exact value of this threshold.[14]

A critical point in any PCA analysis concerns the question how many components k to include in the first place. In our context, we find it instructive to check how many components will create groups containing at least three firms, representing a rather conservative criterion for defining a group.

[14]If the best score of a firm is not greater than the mean plus half a standard deviation of scores within a group, we do not match the firm to the group. The exact tuning of this threshold is of course arbitrary: choosing a much larger threshold level leads to smaller but more homogeneous groups (some firms might not be matched at all because of a single differing link), while a much lower threshold will inflate groups by matching peripheral firms that only show marginal similarity in the link pattern. Since neither group turns out to be large compared to the entire set of firms, our quantile-approach is quite robust, and the exact value of the threshold is not crucial for the overall results. In larger datasets, this parameter could certainly be endogenized.

Our algorithm iteratively increases the number of principal components and stops when the last included component no longer produces an additional group. It turns out that only a fraction of the firms can be mapped to groups, with the greatest eigenvalues accounting for roughly 10 percent of total variance, and the smallest relevant eigenvalues accounting for roughly 3 percent of total variance. We never include more than the five largest eigenvalues to create significant groups, that is to say that the inclusion of more than the largest five eigenvalues leads to groups of size smaller than three.

Adjacency Matrices Sorted by Cliques

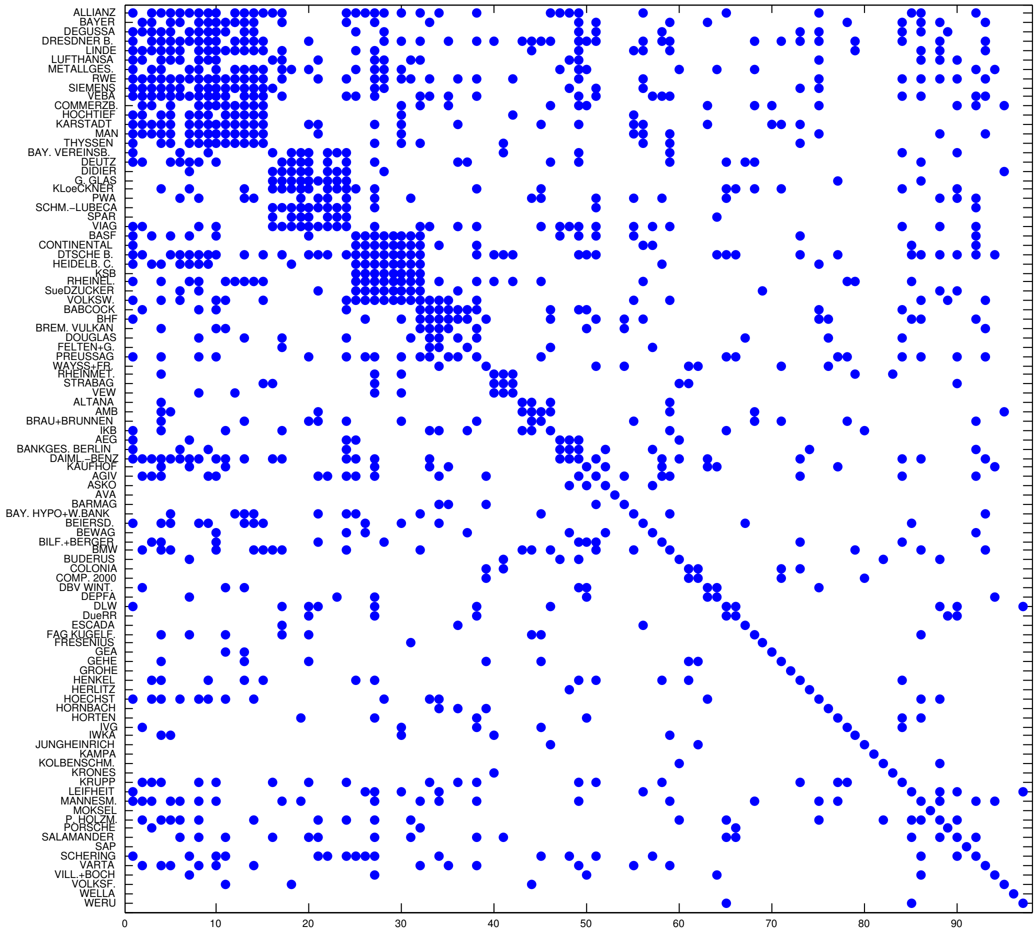

Figure 8: Resorted adjacency matrix 1993

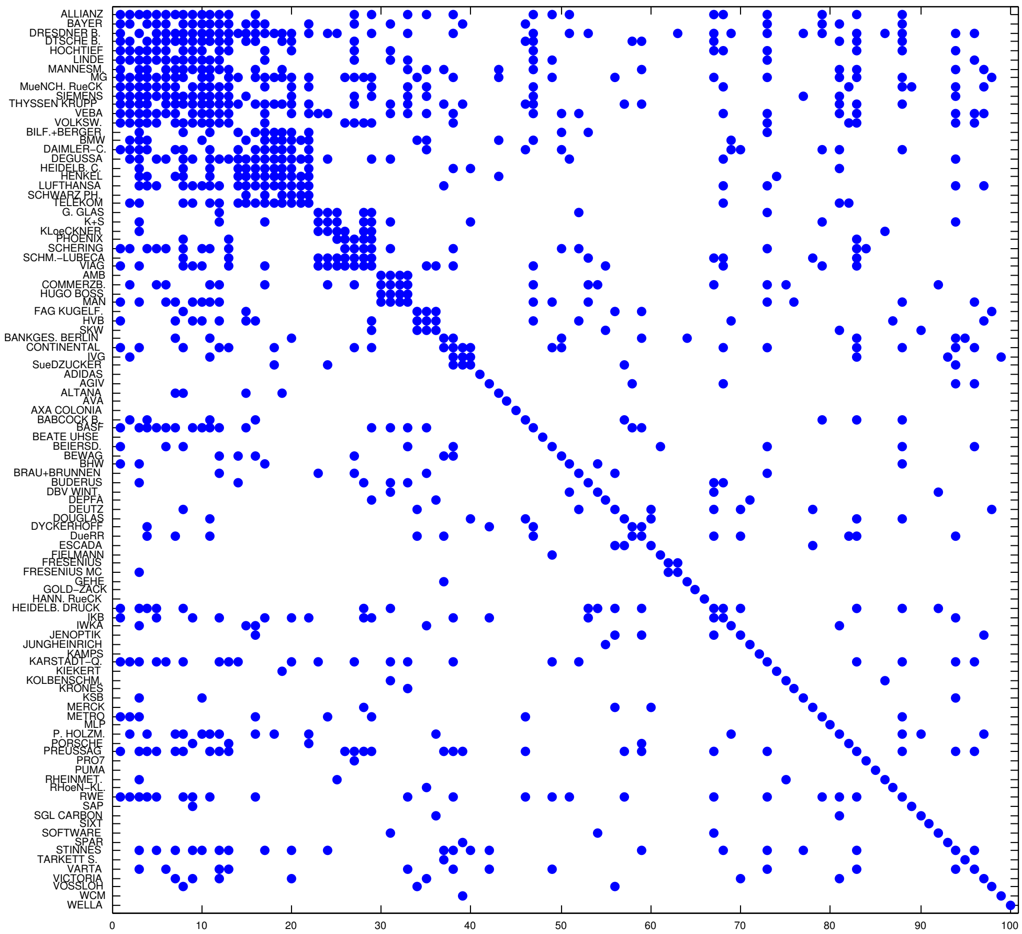

Figure 9: Resorted adjacency matrix 1999

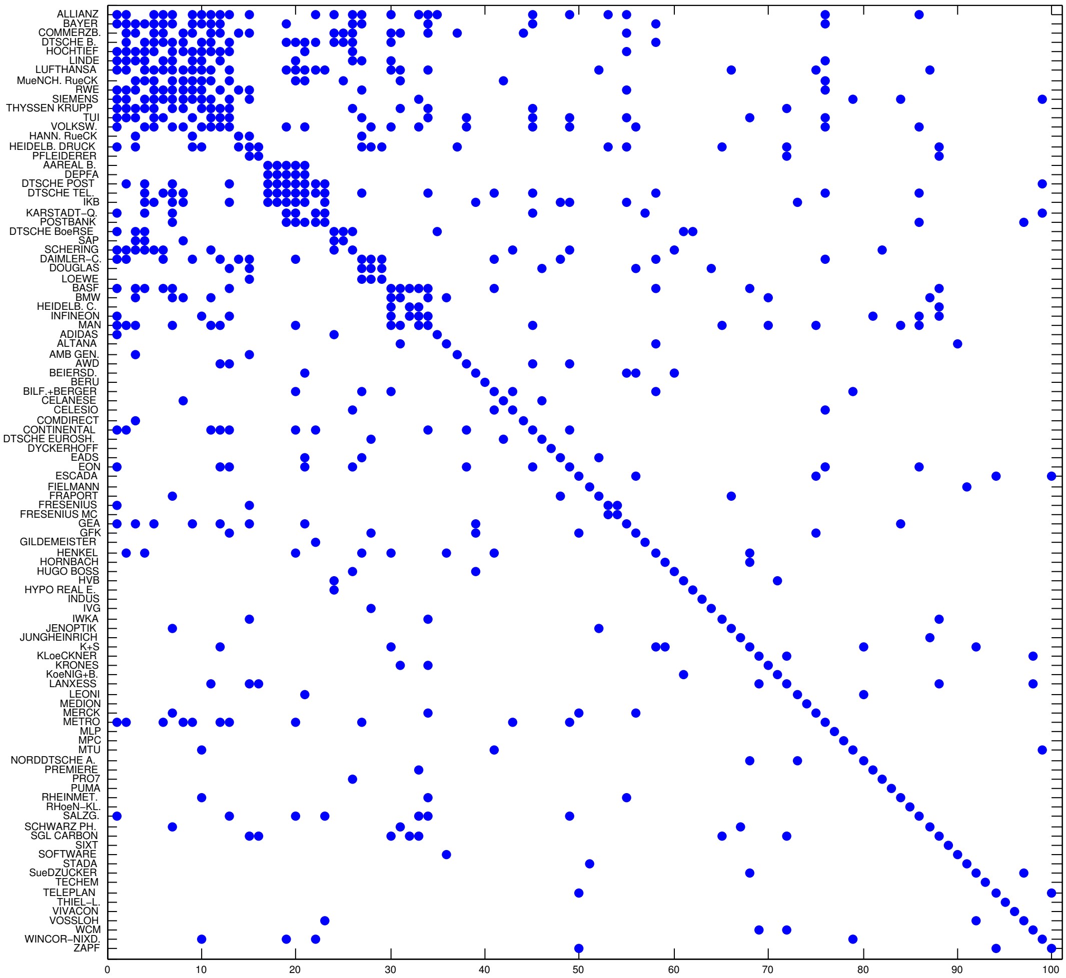

Figure 10: Resorted adjacency matrix 2005

Supplementary Information Tables

	1	2	3	4	5	6	7	8	9	10
1993	1528	136	36	17	11	8	3	2	3	0
1999	1528	113	34	16	9	5	3	1	0	2
2005	1436	107	31	9	7	2	1	0	0	0

Table 3: Overall frequency distribution of mandates

year	executives	# of additional mandates									
		0	1	2	3	4	5	6	7	8	9
1993	565	456	58	21	12	8	5	2	1	2	0
1999	539	469	29	19	9	5	4	1	1	0	2
2005	456	401	37	11	3	4	0	0	0	0	0

Table 4: Frequency of executives' supervisory board memberships

		# of mandates in 1999										
		0	1	2	3	4	5	6	7	8	9	10
# of mandates in 1993	0	906	911	39	7	4	0	0	0	0	0	0
	1	920	369	30	10	4	1	1	2	0	0	0
	2	59	33	14	4	1	0	0	0	0	0	0
	3	9	8	6	3	1	2	0	0	0	0	0
	4	4	7	2	1	1	1	0	0	0	0	0
	5	4	1	2	0	1	1	1	0	0	0	0
	6	1	0	0	0	0	2	2	1	0	0	1
	7	1	0	1	0	1	0	0	0	0	0	0
	8	0	0	0	0	0	1	0	0	0	0	0
	9	1	0	0	2	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0

Table 5: Transition matrix for board membership during 1993–1999.

		# of mandates in 2005										
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
# of mandates in 1999	**0**	977	883	34	10	1	0	0	0	0	0	0
	1	961	328	32	6	2	0	0	0	0	0	0
	2	49	24	16	3	1	1	0	0	0	0	0
	3	6	9	3	4	1	3	1	0	0	0	0
	4	4	5	3	1	0	0	0	0	0	0	0
	5	3	2	2	0	0	0	0	1	0	0	0
	6	0	0	1	1	1	1	0	0	0	0	0
	7	0	1	1	0	0	1	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	1	0	0	0	0	0	0	0

Table 6: Transition matrix for board membership during 1999–2005

		# of mandates in 2005										
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
# of mandates in 1993	**0**	662	1125	58	16	4	2	0	0	0	0	0
	1	1197	104	25	7	1	3	0	0	0	0	0
	2	94	11	4	0	1	0	1	0	0	0	0
	3	21	4	4	0	0	0	0	0	0	0	0
	4	12	2	0	1	0	0	0	1	0	0	0
	5	8	1	0	0	0	1	0	0	0	0	0
	6	2	2	1	2	0	0	0	0	0	0	0
	7	2	1	0	0	0	0	0	0	0	0	0
	8	0	1	0	0	0	0	0	0	0	0	0
	9	2	1	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0

Table 7: Transition matrix for board membership during 1993–2005