

STRUCTURAL ANALYSIS WITH  
INDEPENDENT INNOVATIONS

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# *Structural analysis with independent innovations*

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## **Abstract**

Structural innovations in multivariate dynamic systems are typically hidden and often identified by means of a-priori economic reasoning. Under multivariate Gaussian model innovations there is no loss measure available to distinguish alternative orderings of variables or, put differently, between particular identifying restrictions and rotations thereof. Based on a non Gaussian framework of independent innovations, a loss statistic is proposed in this paper that allows to discriminate between alternative identifying assumptions on the basis of nonparametric density estimates. The merits of the proposed identification strategy are illustrated by means of a Monte Carlo study. Real data applications cover bivariate systems comprising US stock prices and total factor productivity, and four couples of international breakeven inflation rates to investigate monetary autonomy of the Bank of Canada and the Bank of England.

*Keywords:* Structural innovations, identifying assumptions, SVAR, Cholesky decomposition, news shocks, monetary independence.

*JEL Classification:* C32, G15

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# 1 Introduction

Vector autoregressive (VAR) models have generated a widely adopted and rather flexible toolkit to in-sample investigate the dynamic relations of socioeconomic time series, and for out-of-sample forecasting (Lütkepohl 2005). Regarding the former field of theoretical and applied econometrics, VAR models are descriptive in the sense that they offer a profound understanding of reduced form features of the data. When it comes to a contemporaneous perspective on the linkage of economic variables, structural VAR models (Amisano and Giannini 1997) have to borrow from economic theory, ad-hoc decompositions of reduced form covariance matrices, or simply from (economic) a-priori reasoning. Opposite to traditional simultaneous equation modeling, structural VAR models are mostly characterized by employing just identifying restrictions. Against this background, imposing zero restrictions on some instantaneous effects (Sims 1980), or on long-run effects of the shocks (Blanchard and Quah 1989) have been suggested for identification. More recently, the imposition of theoretically motivated sign restrictions upon impulse responses has become a popular approach to either fully identify all structural relations or, more agnostic, to leave some room for a few not directly restricted structural relations (Faust 1998, Uhlig 2005). Fry and Pagan (2011) review empirical studies employing sign restrictions. While sign restrictions are often considered to be rather mild, identification by means of sign restrictions also provoked a critical discussion (Fry and Pagan (2007, 2011) and Paustian (2007)).

Generally, distinct settings of identifying restrictions might compete for the understanding of simultaneous economic relationships. For instance, in the multivariate Gaussian framework distinct orderings of the system variables arrive at identical model diagnostics, but at distinct implied impulse response functions when using Cholesky factors to determine on impact dynamics. Put differently, upper and lower triangular decompositions of reduced form covariance matrices are observationally equivalent although they carry markedly distinct implications for the recursive ordering of structural innovations. Hence, it is a particular shortcoming of the Gaussian model framework that identifying restrictions cannot be tested against each other, or that there is no loss measure at hand that ranks such competing assumptions (or alternative variable orderings) according to data based criteria.

Introducing additional assumptions on the innovation generating distributions may offer both, implications for the underlying structural relations, and diagnostic tools that allow to evaluate if the data are in line with assumptions and respective conclusions. Along these lines Lanne and Lütkepohl (2010) assume mixed normal distributed model innovations to identify the structural shocks and impulse responses. Rigobon (2003) and, similarly, Lanne and Lütkepohl (2008) propose an identification scheme distinguishing states of lower and higher variance for which the causation structure is assumed identical. The approach in Lanne and Lütkepohl (2008) has been further generalized towards a Markov switching model (Lanne, Lütkepohl and Maciejowska 2009) formalizing the dynamic pattern (and recurrence) of distinct variance regimes. While accounting for particular data characteristics offers the extraction of statistically founded structural relations, however, such approaches are not applicable if the data lack the presumed characteristics (e.g. mixed normal distribution, covariance shifts).

This work proposes a data based approach to identification that applies under a non-Gaussian iid setting. Thus, in some sense and in comparison with the Gaussian VAR literature, the approach is general as it does not rely on additional assumptions on the data generating process. Rather, it mitigates the common assumption of conditional normality by excluding exactly this model from the considered space of distributions behind the empirical data. Henceforth, we refer to the proposed identification scheme as *Independence Targeted Structural Innovations* (ITSI). ITSI proceed under the presumption that available (vector valued) reduced form residuals are serially uncorrelated, and can be traced back to structural innovations that are independent and identically distributed (iid) over the cross equation dimension.<sup>1</sup> Hence, ITSI might be seen to provide an assessment of structural assumptions in a scenario where statistical tools based on distributional time heterogeneity lack applicability. Similar to the Markov switching approach in Lanne, Lütkepohl and Maciejowska (2009), ITSI provide statistical loss measures that can be used to evaluate competing assumptions on the generation of contemporaneous reduced form correlations. It associates, for instance, distinct losses to alternative recursive pat-

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<sup>1</sup>It is worthwhile to point out that such an assumption is implicit in the widespread use of impulse response functions if data lack joint normality. In such cases, assuming isolated unit shocks to occur in single variables is generally not in line with conditional expectations unless structural innovations are assumed independent.

terns characterizing the transmission from structural to reduced form model information. Since ITSI are derived for serially uncorrelated and contemporaneously independent innovations, simplest resampling schemes, i.e. iid resampling with replacement or Monte Carlo sampling, can be adopted to quantify the uncertainty attached to the diagnosis of a particular direction of instantaneous causality.

To preview some performance features of the proposed identification approach, ITSI offer discriminatory content in the non Gaussian framework with regard to rival decompositions of covariance matrices that are observationally equivalent in the multivariate Gaussian model. By means of simple resampling techniques the analyst is able to powerfully discriminate among alternative decompositions with well ascribed probabilities of type I decision errors. In empirical applications ITSI are found to support the view that TFP shocks bear the properties of news shocks. In Beaudry and Poirier (2006) such an argument has been put forth on the grounds of a-priori reasoning. Interestingly, in a bivariate system of TFP and stock market innovations the rival recursive causation scheme is powerfully rejected conditional on US data. As a second illustration, large sample analysis of innovations in international breakeven inflation rates show that the Bank of England is more capable to pursue an independent monetary policy in comparison with the Bank of Canada.

In the next Section the independence based approach to structural identification is outlined. Section 3 provides Monte Carlo evidence on the strength of the ITSI method to distinguish alternative causal relations in structural models. In Section 4 two empirical applications consider the contemporaneous interaction of international breakeven inflation rates in the first place. Secondly, a bivariate system of US stock prices and total factor productivity (Beaudry and Portier 2006) is subjected to the detection of a data supported news process. Section 5 concludes.

## 2 Independence based identification

Consider for notational convenience the case of a bivariate contemporaneously correlated but serially uncorrelated process, denoted  $u_t$ , such that

$$u_t = D\xi_t, \text{ with } \xi_t = (\xi_{1t}, \xi_{2t})', \quad (1)$$

$$\xi_{jt} \stackrel{iid}{\sim} F(0, 1), j = 1, 2, F \neq \Phi, \text{ and } \xi_{1t}, \xi_{2t} \text{ independent, } t = 1, 2, \dots, T. \quad (2)$$

In (2)  $F$  is short for any distributional model, excluding, however, the Gaussian distribution ( $\Phi$ ). The process  $u_t$  could be considered as the true or estimated error process entering or extracted from a VAR model. Likewise, the  $u_t$  process could be a stacked residual process gathered from a set of single equation models. In one of the two empirical applications that are discussed in Section 4, for instance,  $u_t$  consists of single equation GARCH implied model innovations. By implication, it follows from (1)

$$\xi_t = D^{-1}u_t, \text{ Cov}[\xi_t] = I_2, \text{ Cov}[u_t] = DD' = \Omega. \quad (3)$$

The typical problem in structural VAR analysis is to determine the matrix  $D$  that relates (latent) structural model innovations  $\xi_t$  with disturbances  $u_t$  which can be estimated consistently by means of OLS or ML estimators. For the so-called AB model, set out by Amisano and Giannini (1997) in the structural VAR framework, we have, for instance,  $D = A^{-1}B$ . While also the matrix  $\Omega$  can be estimated consistently from the data, the decomposition in the right hand side of (3) is not unique. Thus,  $D$  cannot be determined without further assumptions. Identifying assumptions might be either made with regard to the underlying stochastic properties of the data, or implications of economic theory could be used for the purpose of identification. While the former are often testable, the latter may leave no room for the data to speak against the imposed identifying relations. A particular important case arises if innovations  $\xi_{1t}$  and  $\xi_{2t}$  are jointly Gaussian. Then, any rotation of  $\xi_t$  and, thus, of  $u_t$  will be observationally equivalent such that the identification of  $D$  has to rely on a-priori restrictions.

The identification strategy proposed in this work exploits the idea that if  $u_t$  can be traced back to cross sectionally independent innovations with mean zero and unit variance, these innovations can be distinguished from covariance preserving transformations, since the independence feature is unique for the true innovations. Henceforth, the independence

condition in (2) is considered as the basis for the identification of structural innovations and, thus, of  $D$ .

The remainder of this section outlines, firstly, the statistical framework applied to characterize structural innovations, and to attach loss statistics to particular identifying assumptions. The concept of ITSI will be illustrated for two separate modelling purposes, the comparison of rival a-priori assumptions on the one hand, and the detection of a causation scheme that arrives at innovations with 'weakest dependence' on the other hand. ITSI will rely on log-quotes of nonparametric density estimates introduced in the second place. Since the entire approach relies on the assumption of iid residuals, thirdly, a simple resampling scheme is outlined that supports an analyst to quantify the uncertainty associated with decisions in favor of particular instantaneous causation schemes.

## 2.1 Contrasting a-priori assumptions

Without loss of generality assume that one is interested in the simultaneous relations between standardized reduced form disturbances, such that

$$\Omega = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \rho \neq 0, |\rho| < 1.$$

Assume further that the true relation linking structural and reduced form innovations is given by a lower triangular recursion ( $D_l$ ), and that prominent alternative scenarios formalize a symmetric contemporaneous relation ( $D_s$ ) or an upper triangular ( $D_u$ ) scheme, i.e.

$$D_l = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix}, D_s = \Omega^{1/2} = \frac{1}{2} \begin{pmatrix} \sqrt{1+\rho} + \sqrt{1-\rho} & \sqrt{1+\rho} - \sqrt{1-\rho} \\ \sqrt{1+\rho} - \sqrt{1-\rho} & \sqrt{1+\rho} + \sqrt{1-\rho} \end{pmatrix} \quad (4)$$

$$D_u = \begin{pmatrix} \sqrt{1-\rho^2} & \rho \\ 0 & 1 \end{pmatrix}, \text{ with } D_{\bullet} D'_{\bullet} = \Omega, \bullet \in \{l, u, s\}. \quad (5)$$

In (4)  $\Omega^{1/2} = \Gamma \Lambda^{1/2} \Gamma'$ , where  $\Lambda$  is a diagonal matrix comprising the eigenvalues of  $\Omega$  and the columns of  $\Gamma$  are the respective eigenvectors. Since

$$u_t = D_l \xi_t,$$

presuming an upper triangular or symmetric contemporaneous causation scheme arrives at the following implied structural innovations  $\tilde{\xi}_t^{(\bullet)}$

$$\begin{aligned}\tilde{\xi}_t^{(u)} &= D_u^{-1}u_t = D_u^{-1}D_l\xi_t \\ &= \begin{pmatrix} \sqrt{1-\rho^2} & -\rho \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \xi_t, \\ \tilde{\xi}_t^{(s)} &= D_s^{-1}u_t = D_s^{-1}D_l\xi_t \\ &= \frac{1}{2}\sqrt{1-\rho^2} \begin{pmatrix} (1/\sqrt{1+\rho} + 1/\sqrt{1-\rho}) & (1/\sqrt{1+\rho} - 1/\sqrt{1-\rho}) \\ -(1/\sqrt{1+\rho} - 1/\sqrt{1-\rho}) & (1/\sqrt{1+\rho} + 1/\sqrt{1-\rho}) \end{pmatrix} \xi_t.\end{aligned}$$

For both falsely imposed causation schemes the elements of the implied innovation vectors  $\tilde{\xi}_t^{(\bullet)}$ ,  $\bullet \in \{u, s\}$ , involve both iid components of  $\xi_t$  and, thus, lack independence in general.<sup>2</sup> Of course, dependence of elements in  $\tilde{\xi}_t^{(\bullet)}$  arises only if  $\rho \neq 0$ .

## 2.2 Targeting independent innovations

As outlined in (4) and (5) violations of independence show up for falsely recovered structural innovations. In practice, an analyst might not be willing to assume a particular causation scheme a-priori. Rather she might be interested in detecting particular tuples of structural innovations showing weakest dependence patterns.

For such a case consider two innovation tuples where  $D$  refers to some 'initial/benchmark' matrix linking independent structural and reduced form innovations,

$$i) \xi_t = D^{-1}u_t \text{ and } ii) \tilde{\xi}_t = R\xi_t = RD^{-1}u_t. \quad (6)$$

In (6)  $R$  is a rotation matrix with typical elements  $r_{ij}$  such that  $RR' = I_2$ . Thus, with  $r_{ij} \neq 0$ , the elements in  $\tilde{\xi}_t$  generally read as,

$$\tilde{\xi}_{1t} = r_{11}\xi_{1t} + r_{12}\xi_{2t}, \quad \tilde{\xi}_{2t} = r_{21}\xi_{1t} + r_{22}\xi_{2t}. \quad (7)$$

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<sup>2</sup>The assumption of unit marginal variances of  $u_t$  largely facilitates to explicitly show the relation between  $\tilde{\xi}_t^{(\bullet)}$  and the underlying true structural innovations  $\xi_t$ . Moreover, considering  $D_l$  to comprise the true contemporaneous relations does not restrict the generality of the arguments. Similar arguments apply, if an analyst is interested in distinguishing a true upper triangular recursion from falsely supposed lower triangular or symmetric relations.



With similar arguments as raised above, therefore, pseudo innovations in  $\tilde{\xi}_t$  have mean zero and unit covariance but lack independence if  $\xi_{1t}$  and  $\xi_{2t}$  are independent.

Using the contrast in (6) for identification purposes one may start with a specific initialization,  $\xi_t = D_l^{-1}u_t$ ,  $\xi_t = D_u^{-1}u_t$  or  $\xi_t = D_s^{-1}u_t$ , and rotate these candidates systematically. Now, consider the space of systematically rotated innovations,

$$\xi_t^{(\theta)} = R_\theta D^{-1}u_t = R_\theta \xi_t = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \xi_t. \quad (8)$$

If for any rotation indicated with  $\theta_0$  the vector  $\xi_t^{(0)} = R_{\theta_0} \xi_t$  would comprise independent innovations, other rotations  $R_\theta, 0 \leq \theta < \pi/2, \theta \neq \theta_0$  fail to provide independent innovations. If the elements of a particular rotated vector  $\xi_t^{(0)}$  are diagnosed independent, the implied relation between  $\xi_t^{(0)}$  and  $u_t$  is considered as the impact relation between structural and reduced form residuals ( $u_t = D_0 \xi_t^{(0)}$ ). The result in (7) shows that the discriminatory content of ITSI comes from the fact that, generally, rotations of independent innovations result in linearly combined innovations. Noting that both elements  $\tilde{\xi}_{1t}$  and  $\tilde{\xi}_{2t}$  process the same structural information ( $\xi_{1t}, \xi_{2t}$ ) the elements of these rotations are dependent. However, the following rotation matrices imply that the elements in  $R_\theta \xi_t^{(0)}$  are observationally equivalent to those in  $\xi_t^{(0)}$

$$R_{\pi/2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, R_\pi = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \text{ and } R_{3\pi/2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The rotation by means of  $R_\pi$  amounts to multiplying the columns of  $D_0^{-1}$ , and, thus of  $D_0$ , with minus unity. Since in impulse response analysis it is common to trace the impact of positive structural shocks, one arrives at uniquely identified structural relations by assuming the diagonal elements of  $D_0$  to be positive. Except for the sign transformation, the effect of the rotations  $R_{\pi/2}$  and  $R_{3\pi/2}$  is to exchange the columns of the implied impact matrix  $D_0$ . Note that, nonuniqueness of the matrix  $D$  with regard to interchanging its columns is already implied by the decomposition of  $\Omega = DD'$ , and also applies to other data based identification approaches (Lanne and Lütkepohl 2008). Thus, to further disentangle  $D_0$ ,  $D_0 R_{\pi/2}$  and  $D_0 R_{3\pi/2}$  an analyst has to rely on theoretical economic considerations. In many of such scenarios, for instance, it appears natural to assume that the loading of structural innovations on reduced form disturbance is stronger (in absolute

magnitude) for own (diagonal) effects in comparison with cross variable (off diagonal) effects. Henceforth, to extract data driven impact matrices  $D_0$  the space of nontrivial rotations in (7) corresponds to the support  $0 \leq \theta < \pi/2$ .

As introduced, the ITSI scheme is purely data driven. Clearly such an approach deserves a measure of (in)dependence of elements in a sample of some innovation vector candidates  $\xi_t$ . In the following the identification criterion, or put differently, dependence assessment is made explicit. As a side note, minimizing the dependence criterion will be suggested for the determination of the transition scheme  $D_0$ . Moreover, the dependence measure might lack uniqueness in the sense that dependence quotes for a set of candidate innovation vectors can hardly be distinguished by inferential criteria with some deserved significance. Therefore, bootstrap based diagnostic and inferential tools for identification are provided subsequently.

### 2.3 Assessing innovation independence

Irrespective if an analyst wishes to distinguish rival decomposition matrices,  $D_l, D_u$  or  $D_s$  say, or is in search for some minimal dependent structural innovations  $\xi_t^{(0)}$ , ITSI deserve a statistical means to assess independence or dependence of the elements of candidate structural innovation vectors. Let  $\xi_t, t = 1, 2, \dots, T$ , be a sample of such candidate innovation vectors  $\xi_t^{(\bullet)}, \bullet \in \{l, u, s, 0\}$ . Independence of the elements in  $\xi_t$  implies, for instance,

$$f(\xi_{1t}) = f(\xi_{1t}|\xi_{2t}), \quad (9)$$

where the left (right) hand side term in (9) is the unconditional (conditional) density of  $\xi_{1t}$ . Nonparametric estimates of the unconditional and the conditional density of  $\xi_{1t}^{(\bullet)}$  read, respectively, as (see e.g. Rosenblatt 1969)

$$\begin{aligned} \hat{f}(\xi_{1t}|\xi_{2t}) &= \frac{\hat{f}(\xi_{1t}, \xi_{2t})}{\hat{f}(\xi_{2t})} \\ &= \frac{\sum_{\tau=1}^T K_h(\xi_{1t} - \xi_{1\tau})K_h(\xi_{2t} - \xi_{2\tau})}{\sum_{\tau=1}^T K_h(\xi_{2t} - \xi_{2\tau})} \text{ and} \end{aligned} \quad (10)$$

$$\hat{f}(\xi_{1t}) = \frac{1}{T} \sum_{\tau=1}^T K_h(\xi_{1t} - \xi_{1\tau}). \quad (11)$$

In (10) and (11)  $K_h(v) = \frac{1}{h}K(v/h)$  is a kernel function and  $h > 0$  is the bandwidth parameter.<sup>3</sup> Due to estimation errors in finite samples the relation in (9) should hold approximately  $\hat{f}(\xi_{1t}) \approx \hat{f}(\xi_{1t}|\xi_{2t})$ . As a practical consequence, finite samples ITSI diagnostics may suffer from the local biases that are typical for kernel based estimates. Asymptotically, i.e. with  $h \rightarrow 0$  as  $T \rightarrow \infty$ , ITSI diagnostics take full advantage of the consistency of kernel based estimates. The following loss statistic is proposed to indicate actual dependence of components of  $\xi_t$ :

$$\mathcal{L}(\xi_1, \xi_2) = \mathcal{L}(\xi) = \sum_{t=1}^T \left| \ln \left( \frac{\hat{f}(\xi_{1t})}{\hat{f}(\xi_{1t}|\xi_{2t})} \right) \right| = \sum_{t=1}^T |\ln(\hat{f}(\xi_{1t}) - \ln(\hat{f}(\xi_{1t}|\xi_{2t}))|. \quad (12)$$

Small values of  $\mathcal{L}(\xi)$  are in favor of independence, while the larger is  $\mathcal{L}(\xi)$  the less is the likelihood for having innovation vector candidates that comprise independent elements. As a practical rule and depending on the purpose of the analysis, one may consider the innovations implied by  $\min\{\mathcal{L}_l, \mathcal{L}_u, \mathcal{L}_s\}$  or  $\xi_t^{(0)}$  to be the true structural model innovations where

$$\theta^0 = \operatorname{argmin}_{\theta} \mathcal{L}(\xi_t^{(\theta)}). \quad (13)$$

Henceforth, loss measures also indicate to which causation scheme they refer. To be explicit,  $\mathcal{L}_l, \mathcal{L}_u, \mathcal{L}_s$  and  $\mathcal{L}_0$  are determined on the basis of innovations obtained from triangular decompositions of covariance matrices, the eigenvalue decomposition and from the minimum dependence measure in (13), respectively.

## 2.4 Inference

On the one hand, any selection of  $\xi_t^{(\bullet)}$  might be characterized by a lack of independence even if it minimizes the dependence statistic in (12) among rival a-priori choices, or conditional on systematic evaluations of a space of matrix rotations. On the other hand, if the elements in  $\xi_t^{(\bullet)}$  are characterized by independent components, it is unclear if not other choices (e.g. neighboring rotations) differ only 'insignificantly' from the selected innovation tuple. Against this background it is of immediate interest to have some inferential tool at hand that helps to uncover if elements in  $\xi_t^{(\bullet)}$  can be reasonably considered

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<sup>3</sup>In this study, the Gaussian kernel defined as  $K(v) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}v^2)$  is employed throughout. The bandwidth parameter is set to  $h = 1.06(T-1)^{-0.2}$ , since the variance of innovation estimates is unity by construction.

independent. Moreover, a statistical test is desired to highlight if (slight) rotations of  $\xi_t^{(\bullet)}$  might be considered to consist of independent innovations.

For inferential purposes a simple resampling procedure can be employed. Suppose under the null hypothesis that the elements in a sample of a particular innovation vector candidate  $\xi_t$ ,  $t = 1, 2, \dots, T$ , are independent. In this case one may imagine that elements  $\xi_{2t}$ , say, have been attached to elements  $\xi_{1t}$  in a purely randomized manner. As a result the loss measure  $\mathcal{L}(\xi) = \mathcal{L}(\xi_1, \xi_2)$  will differ only unsystematically from a loss  $\mathcal{L}(\xi_1, \xi_2^*)$ , where the random variables in  $\xi_2^*$  are either drawn with replacement from  $\xi_{2t}$  or assigned by means of Monte Carlo techniques. The following resampling scheme is proposed for testing the null hypothesis of innovation independence:

1. Determine  $\mathcal{L}(\xi_1, \xi_2)$  from the data.
2. Draw with replacement a sample from the second elements in  $\xi_t$  denoted  $\xi_{2t}^*$  and obtain  $\mathcal{L}(\xi_1, \xi_2^*)$ .
3. Repeat step 2 sufficiently often, say  $R = 1000$  times.
4. Reject the null hypothesis of contemporaneous independence with significance level  $\alpha$ , if  $\mathcal{L}(\xi_1, \xi_2)$  exceeds the  $(1 - \alpha)$ -quantile of the distribution of  $\mathcal{L}(\xi_1, \xi_2^*)$ , denoted  $\mathcal{L}^*(\xi_1, \xi_2)$ .

Henceforth, bootstrap loss statistics are indicated as  $\mathcal{L}_\bullet^*$  and the  $(1 - \alpha)$ -quantile of the bootstrap distribution is denoted  $\mathcal{L}_{\bullet, 1-\alpha}^*$ ,  $\bullet \in \{l, u, s, 0\}$ .<sup>4</sup>

### 3 Monte Carlo study

To illustrate the potential of the proposed identification scheme to detect factual linkages between contemporaneously independent structural innovations  $\xi_t$  and reduced form disturbance vectors  $u_t$  this Section offers Monte Carlo evidence. First, the data generating sampling model is briefly introduced and performance criteria are stated. Then, simulation results are discussed.

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<sup>4</sup>The proof of the asymptotic validity of the bootstrap scheme is straightforward noting that the distributions of  $\xi_2$  and  $\xi_2^*$  are identical under the null hypothesis of independence.

### 3.1 Data generation and performance evaluation

The following model is used to draw bivariate independent structural innovation processes:

$$\xi_t \stackrel{iid}{\sim} (0, I_2), \xi_{jt} \sim t(\nu_j) \frac{\sqrt{\nu_j - 2}}{\sqrt{\nu_j}}, j = 1, 2, \xi_{1t}, \xi_{2t} \text{ independent}, t = 1, 2, \dots, T,$$

where  $I_2$  is the two dimensional identity matrix,  $t(\nu_j)$  is short for the Student- $t$  distribution with  $\nu_j$  degrees of freedom. In the Monte Carlo study degrees of freedom  $\nu = 4, 8, 16$  and  $\nu = 32$  are distinguished. While lower degrees of freedom might be thought to represent strong violations of joint normality of the structural innovations, scenarios where  $\nu = 32$  might be hard to distinguish from observational equivalence of  $\xi_t$  and its rotations that holds under joint normality. With regard to the sample size  $T = 100, 250$  and  $T = 500$  are distinguished. Since the detection of dependence patterns relies on nonparametric estimates of conditional and unconditional densities, one would expect that with increasing sample size and with the bandwidth parameter shrinking to zero, the accuracy of the identification strategy improves in terms of selective strength.

After the generation of structural innovations these are transformed to reduced form error vectors such that their covariance matrix is

$$\text{Cov}[u_t] = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \rho = 0.25, 0.50, 0.75.$$

Distinct covariance levels govern the linear dependence of reduced form disturbances. For the transition of structural innovations to reduced form disturbances a recursive causal structure is employed such that the true transition matrix and its inverse are lower triangular ( $\Omega = D_l D_l'$ ). In the simulation study it is of interest under which conditions an analyst may distinguish the lower triangular recursive transition scheme from the assumption of a 'symmetric' transition which is implied by setting  $\Omega = D_s D_s'$ , or from an upper triangular recursion,  $\Omega = D_u D_u'$ . Intuitively one may expect it more difficult to distinguish between a recursive and symmetric transitions the smaller is the level of correlation ( $\rho$ ). For instance, in the case of weakest correlation considered,  $\rho = 0.25$ ,  $D_l$ ,  $D_s$  and  $D_u$  are rather 'similar' to each other,

$$D_l = \begin{pmatrix} 1 & 0 \\ 0.25 & 0.968 \end{pmatrix}, D_s = \begin{pmatrix} 0.992 & 0.126 \\ 0.126 & 0.992 \end{pmatrix}, D_u = \begin{pmatrix} 0.968 & 0.25 \\ 0 & 1 \end{pmatrix}.$$

After the generation of reduced form disturbances  $u_t$  the three rival decompositions of unrestricted estimates  $\widehat{\Omega} = 1/T \sum_t u_t u_t'$  are employed to arrive at implied structural innovations

$$\xi_t^{(l)} = \widehat{D}_l^{-1} u_t, \xi_t^{(u)} = \widehat{D}_u^{-1} u_t \text{ and } \xi_t^{(s)} = \widehat{D}_s^{-1} u_t, t = 1, 2, \dots, T.$$

Then, all estimated innovation samples are used to determine respective loss statistics  $\mathcal{L}_l$ ,  $\mathcal{L}_u$  and  $\mathcal{L}_s$ .<sup>5</sup> Moreover, the loss statistics are subjected to resampling to test the null hypothesis of independence. The number of bootstrap replications is 1000. The following means of indicators  $I()$  are used to assess the performance of the proposed identification scheme:

- (i) Mean estimates  $I(\mathcal{L}_l < \mathcal{L}_s)$ ,  $I(\mathcal{L}_l < \mathcal{L}_u)$ ,
- (ii) Mean estimates  $I(\mathcal{L}_l > \mathcal{L}_{l,1-\alpha}^*)$ ,  $\alpha = 0.10, 0.05$ ,
- (iii) Mean estimates  $I(\mathcal{L}_s > \mathcal{L}_{s,1-\alpha}^*)$ ,  $I(\mathcal{L}_u > \mathcal{L}_{u,1-\alpha}^*)$ ,  $\alpha = 0.10, 0.05$ .

While the criterion in (i) mimics the situation where an analyst just looks for a minimal dependence statistic, the criteria in (ii) correspond to the type one error of a formal test of the null hypothesis of independence. In case the proposed resampling schemes apply, one would expect empirical means for these criteria of 10% and 5%. Similarly, the criteria in (iii) correspond to the power of formal independence tests, since the imposed transition scheme differs from the true one, such that the extracted innovations  $\xi^{(s)}$  and  $\xi^{(u)}$  lack independence.

## 3.2 Simulation results

Simulation results are documented in Table 1. As one may expect diagnosing the true lower diagonal recursive structure ( $D_l$ ) is more frequently successful if one contrasts this

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<sup>5</sup>Opposite to econometric practice the performance of ITSI in ranking rival patterns of contemporaneous causation the simulations in this study are performed with 'true' rather than estimated reduced form innovations. In this context it is worthwhile to notice that estimated disturbance terms are consistent for their true counterparts under the weak condition of finite innovation variance. Unreported results show that in finite samples extracting VAR residuals from the generated disturbance terms  $u_t$  has only very minor impacts on the discriminatory strength of the proposed identification scheme.

relation against an upper triangular recursion ( $D_u$ ) as it is the case for the symmetric alternative  $D_s$ . Henceforth, the discussion of identification outcomes refers to contrasting  $D_l$  against  $D_s$ . The detection of the true lower triangular recursion (when testing against  $D_s$ ) is facilitated, the stronger is the violation of joint normality, i.e. the smaller is the degrees of freedom parameter  $\nu$ , the larger is the sample size  $T$ , and the stronger is the contemporaneous correlation. For instance, conditional on  $T = 100$  and  $\nu = 4$  the loss statistic  $\mathcal{L}_l$  is smaller than  $\mathcal{L}_s$  in 84.9% and 59.6% of all replications if the level of contemporaneous correlation is  $\rho = 0.75$  and  $\rho = 0.25$ , respectively. Conditional on  $T = 100$  and the medium correlation level  $\rho = 0.5$ ,  $\mathcal{L}_l$  is smaller than  $\mathcal{L}_s$  in 74.8%, 56.4% and 51.0% of all Monte Carlo experiments if the degrees of freedom parameter is  $\nu = 4, 8$  and  $\nu = 32$ , respectively. Thus, in the case  $\nu = 32$  ITSI can hardly distinguish the standardized Student- $t$  model from the (nonidentified) Gaussian reference case. Generally the power properties of the proposed resampling scheme are satisfactory. Conditional on  $T = 100$ ,  $\rho = 0.75$  and with 5% significance the null hypothesis of independence of innovations is rejected in 41.0% and 11.0% of all Monte Carlo replications when testing against  $D_s$  and if the underlying Student- $t$  degrees of freedom parameter is  $\nu = 4$  and  $\nu = 8$ , respectively. With regard to the empirical size features of the proposed resampling scheme, it turns out that inference on independence is somewhat conservative in small samples. For  $T = 100$  and a nominal significance level of  $\alpha = 0.05$  the empirical rejection frequencies vary between 2.6% and 6.6% depending on the particular data generating model. To explain the tendency of undersizing of the bootstrap independence test it is worthwhile to point out that small sample estimation errors for the elements in  $\hat{\Omega}$  may bias the stochastic properties of the elements in  $\xi_t$  towards some 'remaining' dependence even if the true causation scheme is used for the extraction of structural innovations.

With increasing sample size the discriminatory content of ITSI identification improves for almost all scenarios. However, contrasting  $D_l$  against  $D_s$ , the largest sample size considered,  $T = 500$ , is not sufficient to effectively distinguish independent standardized Student- $t$  distributions with  $\nu = 32$  degrees of freedom from the uninformative bivariate Gaussian case. For instance, with  $T = 500, \nu = 32$  and  $\rho = 0.75$  in only 52.9% of all Monte Carlo replications loss measures for  $D_l$  are smaller in comparison to those attached to  $D_s$ . For the same setting ( $T = 500, \nu = 32, \rho = 0.75$ ), however, the contrasting of

$D_l$  against  $D_u$  is more often successful, i.e. in 55.6% of all replications  $\mathcal{L}_l$  is smaller than  $\mathcal{L}_u$ . Depending on the degrees of freedom parameter, the resampling of ITSI loss statistics  $\mathcal{L}$  appears to suffer from local biases of Kernel estimates even conditional on the largest sample size ( $T = 500$ ). Interestingly and most intuitive, the effects of such local biases are more severe the less concentrated is the distribution of innovations around zero ( $\nu = 16, 32$ ).<sup>6</sup>

## 4 Empirical applications

To further illustrate the scope of ITSI implied loss statistics two prominent issues of empirical macroeconometrics are considered in this section. Firstly, ITSI loss statistics are used to uncover potential patterns of monetary autonomy in four systems of international breakeven inflation rates. Secondly, the ITSI approach is followed to assess competing notions of news processes in a bivariate system of US stock prices and total factor productivity. In Beaudry and Portier (2006) TFP innovations are regarded as news process largely on the basis of an ad-hoc assumption.

The empirical applications will cover both aspects raised in Section 2. In the first place, distinct presumptions on the contemporaneous causation scheme will be contrasted against each other. In the second place, a particular causation scheme is extracted from a large space of candidate specifications to arrive at structural innovations with minimum strength of dependence. For practical implementation of the ITSI identification two alternative choices for the matrix  $D$  in (6) are considered, namely a lower and an upper triangular recursion. Then, these two baselines are rotated by means of a grid of rotation matrices  $R(\theta)$ ,  $\theta = 0.0125\omega$ ,  $\omega = 1, 2, \dots, 39$ .

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<sup>6</sup>Unreported simulation based evidence for selected scenarios (e.g.  $\rho = 0.25, \nu = 16$ ) with very large samples, i.e.  $T = 2000$ , confirms both consistency of the identification scheme and asymptotic validity of the simple resampling approach detailed in Section 2. Detailed results for simulation experiments with large samples are not provided in this work noticing high computational burdens of resampling nonparametric density estimates.



## 4.1 International linkage of breakeven inflation

### 4.1.1 Monetary autonomy

A core concern of monetary policy is long term price stability. Since the 1990s inflation targeting has become a widely followed strategy to implement a moderate and predictable evolution of prices. The essential element in such a framework is to anchor long run inflation expectations. Hence, the extent to which central banks are able to implement stable, unique and definite beliefs is crucial. In small economies the targeting of inflation expectations by monetary authorities could be threatened by the neighborhood to or intense trading relationships with leading economies such as the US or the European Monetary Union (EMU). Particular central banks that might be subjected to cross market monetary transmission in this respect are the Bank of Canada or the Bank of England. In this section monetary linkage is empirically assessed in terms of the degree to which ex-ante inflation rates are determined on international markets comprising a subset of the G7, namely Canada, France, the UK and the US. Inflation compensation as implied by the (liquidity adjusted, Shen (2006)) difference between yields of long term 'Treasury Inflation Protected Securities' and conventional bonds is considered to measure future inflation prospects. Daily price quotes cover the time period 4/2/2001- 9/30/2008. Breakeven inflation rates analyzed below are available from the net (<http://www.bepress.com/snede/vol13/iss4/art5/>, see also Herwartz and Roestel 2009). Liquidity adjusted breakeven inflation rates are displayed in Figure 1. By graphical inspection liquidity adjusted breakeven rates appear consistent in the sense that all corresponding monetary authorities have been communicating inflation targets around 2% to 3% over the last decade.

### 4.1.2 Reduced form estimates

Overall, four bivariate systems of innovations governing daily break even inflation rate changes are considered. Below, these systems are labeled with roman numbers and comprise the following combinations of bond markets

I	II	III	IV
CA,US	FR,US	UK,US	UK,FR

The systems I, III and IV are ordered such that a presumably dominating market is listed after a dominated market. For the second system one might also a-priori consider the US bond market to informationally dominate the European counterpart. In light of respective empirical evidence (Ehrmann and Fratzscher 2004), it is, however, conceivable that the notion of US dominance (over the Euro Area) has seen some weakening over more recent time periods.

Modeling first and second order features of the ex-ante inflation expectations, changes of liquidity adjusted breakeven inflation rates show at most rather weak serial correlation but instead marked patterns of volatility clustering. Quantifying second order characteristics by means of GARCH(1,1) specifications (Engle 1982, Bollerslev 1986) turns out to approximate higher order features of the Canadian and UK breakeven rate changes accurately. With regard to the remaining two series diagnostic results indicate that GARCH(1,1) implied standardized residuals might show some (mild) remaining conditional heteroskedasticity. An expansion of the empirical model towards a GARCH(2,1) or GARCH(1,2), however, failed to provide reasonable (i.e. positive) GARCH parameters and/or to improve the diagnostic features of standardized residuals. Therefore, underlying market innovations are extracted from breakeven inflation rate changes by means of univariate GARCH(1,1) models. GARCH(1,1) implied standardized residual processes are stacked to obtain bivariate systems of reduced form bond market innovations. Estimation and diagnostic results and, moreover, the cross correlation matrix of univariate GARCH residuals are shown in Table 2. From the theoretical outset of the ITSI approach it is clear that its identification potential vanishes in case that reduced form residual processes are jointly Gaussian. It is well established that residuals of volatility models, though being iid distributed, often fail to exhibit a Gaussian distribution (Bollerslev 1987). For the considered systems of breakeven inflation innovations explicit tests on joint normality are not provided in detail. In fact, highly significant Jarque-Bera test statistics (Jarque and Bera 1980) determined for the four breakeven systems vary between 211.11 (CA/US) and 2589.23 (UK/US).

### 4.1.3 Structural analysis

According to the empirical results in Table 2 the correlation estimates obtained in the four dimensional system vary between 0.16 (UK/US) and 0.44 (CA/US). Noting that almost 2000 time series observations enter the analysis for each system one might expect that if the true contemporaneous market relation is of a recursive type the ITSI identification scheme delivers clear cut identification results. In the case that both markets of a system contribute to reduced form disturbances, the true scheme of instantaneous causality is not of a recursive structure. A-priori, given that monetary policy in the Euro area and the US might issue news of similar (international) weight, the second system could be considered to deliver some mixed evidence on eventual recursive patterns of processing structural innovations. Consequently, for this system a symmetric square root decomposition of the reduced form covariance might be convenient according to a-priori considerations. Empirical ITSI loss measures for systems of (GARCH standardized) breakeven inflation rates are documented in Table 3. For all bivariate systems the empirical analysis is performed with regard to full samples and, moreover, distinguishing two (almost) equal sized subsamples.

For the systems comprising Canadian and French breakeven rates jointly with US inflation expectations an upper triangular scheme is characterized by smaller loss statistics as the lower triangular counterpart. Thus, according to these statistics shocks in the US rate are more likely to impact on the remaining rate in comparison with a recursion where innovations of the US rate cannot be interpreted as a news process. For both systems and conditional on full sample information, the (upper triangular) extracted innovations lack independence with 5% significance, however. For both considered subsamples the upper triangular recursive scheme is confirmed to result in minimum dependence statistics among the alternative candidate innovation processes  $\xi_t^{(l)}, \xi_t^{(u)}$  and  $\xi_t^{(s)}$ . However, only for the second subsample of the CA/US system the extracted innovations are diagnosed independent with 5% significance. Regarding the French breakeven rate to represent EMU inflation expectation the diagnosed upper triangular scheme is at odds with more recent empirical evidence of a weakened impact of the US market on European interest rates (Ehrmann and Fratzscher 2004), and more in line with traditional views on interest rate transmission from the US to the German and smaller European markets (Katsimbris and

Miller 1993, Hassapis, Pittis and Prodromidis 1999).

Conditional on full sample information the third system comprising standardized innovations of US and UK inflation compensation is closest to the notion of symmetric instantaneous causality, i.e. with regard to inflation expectations particular recursive transmissions cannot be retrieved from these markets. On the one hand this result might be surprising in light of the literature available on US dominance. On the other hand, however, regarding the informational content of breakeven inflation rates, it is noteworthy that the trading of inflation protected securities has a by far longer tradition in the UK in comparison with all remaining markets. Against this background one might attribute specific informational content also to UK inflation expectations. From a statistical perspective, moreover, the relatively low level of correlation (0.159, UK/US) between reduced form information for these two markets might be a reason for the mixed evidence with regard to potential patterns of instantaneous causation. For innovations  $\xi_t^{(s)}$  and  $\xi_t^{(l)}$  the hypothesis of independence is rejected with nominal 5% significance but not at the more conservative 1% level. For innovations  $\xi_t^{(u)}$  the null hypothesis of independence is rejected with 1% significance. Conditional on subsample information, ITSI diagnostics are in favour of distinct recursive patterns supporting US dominance for the first subsample while a lower triangular scheme delivers smallest independence statistics conditional on the more recent subsample.

An argument relating to the fact the UK markets for inflation indexed securities are well established might also be put forth with regard to the last system of breakeven rate innovations. Considering the UK/FR system the evidence does not reflect a dominance of French/EMU breakeven rates. In fact, one diagnoses some evidence of a lower triangular relation characterizing this particular system (full sample and first subsample) or of a symmetric structural relation (second subsample). While independence of the diagnosed innovations is confirmed for both subsamples with 5% significance, for the full sample independence cannot be diagnosed with 5% significance but at the more conservative level of 1%.

Apart from comparing rival a-priori schemes to link structural and reduced form information ITSI could be used to detect structural innovations with weakest dependence. Following these lines we find for almost all subsample systems impact relations such that

the null hypothesis of independence cannot be rejected at common significance levels. Conditional on full sample information structural innovations in  $\xi_t^{(0)}$  lack independence with 10% significance for all subsystems, the elements of  $\xi_t^{(0)}$  are diagnosed dependent with 5% significance for the FR/US subsystem. In general, however, the actual dependence of elements in  $\xi_t^{(0)}$  appears markedly reduced in comparison with selected a-priori decompositions  $D_\bullet$ ,  $\bullet \in \{l, u, s\}$  for each market. As outlined in Section 2 the selection of  $D_0$  deserves further identifying assumptions, since the independence measure is invariant, e.g. with particular sign switches and the reordering of the columns of  $D_0$ . Assuming (i) that diagonal elements of  $D_0$  are positive and (ii) that structural shocks of either variable have the strongest impact on its own reduced form counterpart obtains the particular  $\widehat{D}_0$  estimates provided in Table 3. Allowing for such freely estimated feedback relations, it turns out that elements in  $\widehat{D}_0$  are numerically close to upper triangular recursions for the CA/US and FR/US systems. Evidence for a 'symmetric' relation is the strongest for the UK/US subsystem, while a  $\widehat{D}_0$  is closest to a lower triangular scheme for the UK/FR system. Thus, systematically selecting structural innovations with weakest dependence is to some extent supportive for US dominance over CA and FR, while the UK breakeven rate appears to dominate the FR rate on impact. In addition, the system UK/US is still best described by an 'almost' symmetric feedback relation.

Summarizing the results on monetary dependence for the two smaller economies considered, it turns out that in comparison with the Bank of Canada, the Bank of England is likely better able to target domestic long run inflation.

## 4.2 Is US total factor productivity a news process ?

### 4.2.1 Reduced form modeling

Beaudry and Portier (2006) have raised the issue if in a bivariate system comprising a technology measure (TFP) and stock prices (SP) as an indicator of future expectations about the business cycle, surprises i.e. news are released in expectations (i.e. stock prices) or technology. Interestingly, the former case would suggest that technological change is to some extent 'foreseen' or processed in stock prices. The quarterly data on US total factor productivity and stock prices spans the period 1947 to 2000 and can be drawn from the net (<http://www.aeaweb.org/articles.php?doi=10.1257/aer.96.4.1293>). To clarify the origin

of news, Beaudry and Portier (2006) also employ higher dimensional systems comprising consumption and/or hours worked which is beyond the scope of the ITSI approach as introduced in this work. For the bivariate system two alternative identification schemes are applied in Beaudry and Portier (2006) one of which relies on long term identifying restrictions that allow a non vanishing response of the news shock on total factor productivity. Alternatively, Beaudry and Portier (2006) use a Cholesky decomposition excluding on impact dynamics operating from stock prices on total factor productivity. It turns out that these two identification schemes obtain highly correlated news shocks. The Cholesky type lower diagonal identification scheme used by Beaudry and Portier (2006) can be subjected to loss comparison with an alternative upper triangular decomposition scheme. It is this particular aspect of the relationship between technology and stock prices that can be subjected to a loss assessment in the ITSI framework.

To investigate the empirical linkage of stock prices and factor productivity four alternative VAR specifications are used to extract reduced form residuals. According to standard model selection criteria (AIC, BIC, HQ) applied to level data with deterministic trend, lag order 2 is broadly supported. Accordingly, the four systems analyzed are a VAR(2) with trend for level data, a VAR(2) for level data without trend and a VAR(2) and VAR(1) model for first differences of stock prices and total factor productivity both excluding a deterministic trend. Reduced form residuals extracted from these model specifications are throughout rather similar and the empirical residual correlation in these bivariate systems is around  $\hat{\sigma} \approx 0.16$ . VAR estimates are provided in Table 4. Noting an only weak correlation of reduced form disturbances in the TFP/SP system and the relatively small time series dimension the discriminatory content of the ITSI loss statistics is likely limited on the one hand. On the other hand, documented statistics testing the presumption of joint reduced form normality are significant at any reasonable level such that ITSI loss statistics naturally apply for a comparison of distinct structural assumptions.

#### 4.2.2 Structural analysis

ITSI diagnostics for the alternative systems comprising reduced form disturbances of TFP and stock prices in the US are documented in Table 5. Detecting 'dependence' minimizing

matrices  $D_0$  by means of systematically rotating  $D_l$  or  $D_u$  obtains loss statistics and contemporaneous causation pattern that are close to the loss statistics attached to the lower triangular recursive structure. While for most  $\widehat{D}_0$  matrices some asymmetric feedback relations are detected, it is worthwhile to point out that the lower left element of these matrices is throughout larger in absolute magnitude than the upper right element. Distinguishing alternative loss statistics  $\mathcal{L}_\bullet$ ,  $\bullet \in \{l, s, u\}$  almost all estimated VAR systems deliver smallest loss statistics for the lower triangular decomposition ( $\mathcal{L}_l < \mathcal{L}_u$ ) implying that innovations in stock prices have no contemporaneous effect on reduced form innovations characterizing total factor productivity. Thus, total factor productivity is confirmed to bear the interpretation of a news process. Moreover, with regard to inferential results it turns out that elements of implied structural innovations  $\xi_t^{(u)}$  lack independence with 5% significance throughout. Thus, while it is difficult to distinguish the dependence level of structural innovations  $\xi_t^{(l)}$  and  $\xi_t^{(s)}$ , the empirical evidence is markedly at odds with the presumption that iid news enters the bivariate system through stock prices. Overall, however, the evidence in favor of symmetric instantaneous causation is weaker as it is for the most likely (i.e. the lower triangular) recursive scheme. Interestingly, Beaudry and Portier (2006) rely on this assumed recursion on the basis of a-priori reasoning. Combined with arguments in Beaudry and Poirier (2006) this also supports the view that the news shock impacts on TFP in the long run.

## 5 Conclusions

In this paper a loss functional is introduced that carries informational content to discriminate between competing structural relations in a non iid Gaussian framework. Independence targeted structural innovations (ITSI) can provide a ranking of alternative just identifying structural data representations. Thus, ITSI assist the analyst in determining a data supported structural view at the economy, or put differently, highlight to which extent particular identifying restrictions are not supported by empirical processes. The ITSI concept fully relies on the notion of data being independent and identically distributed over the time dimension such that simple resampling and Monte Carlo techniques are applicable to resolve inferential issues with regard to competing a-priori settings of

structural data relations. A Monte Carlo study underpins that ITSI may reasonably be used to contrast rival schemes of recursive causality even if sample sizes are small to moderate. Applied to empirical systems of break even inflation rates, ITSI diagnostics indicate that the Bank of Canada is more dependent on US monetary policy in comparison with the Bank of England. For the US system comprising total factor productivity and stock prices suggesting that news arrives through TFP innovations (Beaudry and Poitier 2006) is found to be in line with sample information.

For the purpose of simplicity and computational tractability the outline of ITSI in this paper has addressed the bivariate case exclusively. The generalization towards higher order systems is straightforward and feasible in principle. The determination of ITSI in higher dimensional systems is an interesting direction of future research. Moreover, in the framework of multivariate GARCH models, it has become a common practice to use the eigenvalue decomposition for the extraction of vector model innovations. Such innovations are typically found to exhibit remaining leptokurtosis as it has been the case in this study for the systems of univariate GARCH innovations extracted from breakeven inflation rates. In consequence, ITSI may also support an analyst to decide upon the most suitable, data driven decomposition of time varying covariance matrices in the multivariate GARCH framework. Opening a further direction of future research one might notice from the empirical analysis of systems of breakeven inflation rates - and in particular of the UK/FR subsystem - that the assumption of time invariant structural relations might lack support empirically. In this respect it is of interest in how far the notion of independent structural innovation could be helpful to uncover time variation in contemporaneous causal relations.

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			$D_l$ vs. $D_s$					$D_l$ vs. $D_u$		
$T$	$\rho$	$\nu$	$\mathcal{L}_l <$	$\mathcal{L}_l >$	$\mathcal{L}_l >$	$\mathcal{L}_s >$	$\mathcal{L}_s >$	$\mathcal{L}_l <$	$\mathcal{L}_u >$	$\mathcal{L}_u >$
			$\mathcal{L}_s$	$\mathcal{L}_{l,.95}^*$	$\mathcal{L}_{l,.90}^*$	$\mathcal{L}_{s,.95}^*$	$\mathcal{L}_{s,.90}^*$	$\mathcal{L}_u$	$\mathcal{L}_{u,.95}^*$	$\mathcal{L}_{u,.90}^*$
100	.25	4	.596	.066	.121	.078	.152	.714	.196	.304
		8	.548	.034	.079	.041	.099	.577	.068	.128
		16	.511	.029	.078	.029	.073	.533	.033	.083
		32	.493	.027	.071	.025	.069	.487	.031	.066
	.50	4	.748	.042	.104	.203	.313	.892	.551	.693
		8	.564	.033	.066	.041	.103	.671	.118	.226
		16	.524	.031	.077	.049	.099	.547	.056	.110
		32	.510	.029	.065	.031	.081	.524	.037	.087
	.75	4	.849	.060	.118	.410	.541	.909	.695	.790
		8	.635	.032	.079	.110	.180	.703	.200	.302
		16	.525	.036	.082	.045	.102	.554	.059	.125
		32	.539	.026	.066	.040	.083	.540	.041	.082
250	.25	4	.713	.051	.109	.110	.190	.895	.339	.476
		8	.579	.036	.083	.050	.103	.664	.083	.161
		16	.514	.033	.092	.029	.070	.547	.038	.089
		32	.518	.030	.060	.031	.073	.527	.035	.076
	.50	4	.887	.063	.120	.365	.510	.978	.917	.950
		8	.669	.050	.097	.110	.203	.760	.294	.417
		16	.566	.034	.072	.042	.086	.597	.063	.166
		32	.531	.034	.077	.044	.086	.518	.049	.103
	.75	4	.956	.058	.129	.777	.857	.989	.967	.985
		8	.712	.051	.110	.201	.318	.803	.368	.484
		16	.581	.034	.070	.054	.115	.622	.096	.188
		32	.518	.041	.084	.043	.093	.530	.070	.116
500	.25	4	.824	.063	.134	.165	.283	.956	.604	.735
		8	.627	.043	.091	.063	.117	.742	.130	.215
		16	.509	.039	.089	.034	.086	.560	.042	.102
		32	.496	.039	.074	.034	.076	.505	.034	.080
	.50	4	.964	.063	.119	.622	.749	.998	.991	.997
		8	.739	.047	.096	.137	.245	.882	.445	.592
		16	.582	.037	.077	.053	.108	.656	.103	.190
		32	.508	.033	.072	.039	.075	.551	.057	.108
	.75	4	.996	.062	.114	.973	.990	1.00	1.00	1.00
		8	.836	.047	.100	.314	.429	.903	.597	.729
		16	.642	.043	.089	.091	.167	.682	.144	.249
		32	.529	.041	.086	.045	.100	.556	.060	.117

Table 1: Simulation results: The table shows empirical frequencies for events indicated in the top row, i.e., frequencies of minimum loss measures obtained for  $\mathcal{L}_l$  (columns 4 and 9), size estimates for nominal significance levels 5% and 10% (columns 5 and 6) and power estimates with respect to these nominal levels (columns 7,8,10,11).

co.	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	LM1	LM5	corrs ( $\hat{\sigma}$ )		
CA	2.95E-06 (1.51)	.030 (3.92)	.968 (120.)	0.040 (0.84)	3.907 (0.56)	CA	FR	UK
FR	4.84E-06 (2.25)	0.069 (4.24)	0.922 (55.5)	5.387 (0.02)	12.46 (0.03)	.230		
UK	1.47E-06 (1.03)	0.024 (3.29)	0.973 (117.)	0.299 (0.58)	2.833 (0.73)	.207	.293	
US	2.19E-06 (1.10)	0.047 (2.61)	0.954 (60.1)	5.033 (0.02)	25.70 (0.00)	.440	.214	.159

Table 2: GARCH(1,1) parameter estimates and model diagnostics for changes of breakeven inflation rates in Canada (CA), France (FR) the UK and the US, denoted  $\varepsilon_t$ . For a particular variance process the conditional variance  $\sigma_t^2 = E[\varepsilon_t^2 | \mathcal{F}_{t-1}]$  characterizing the time series  $\varepsilon_t$  is,  $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ . Values in parentheses are either  $t$ -ratios (for parameter estimates) or  $p$ -values (for the LM-statistic testing against conditional heteroskedasticity in GARCH implied standardized residuals). The right hand side panel shows unconditional correlations for the standardized GARCH(1,1) residuals.

$D_{\bullet}$	$\widehat{D}$			$\mathcal{L}$			$\widehat{D}$			$\mathcal{L}$		
	CAUS						1st subs			2nd subs		
$D_u$	0.998	0.441	176.9**	0.999	0.517	110.3**	0.998	0.361	99.34*			
	0	0.899		0	0.843		0	0.949				
$D_l$	0.896	0	252.9***	0.851	0	158.7***	0.933	0	133.7***			
	0.440	1.002		0.523	0.989		0.355	1.015				
$D_s$	0.972	0.226	212.7***	0.961	0.270	141.7***	0.981	0.182	113.2***			
	0.226	0.976		0.270	0.951		0.182	0.999				
$D_0$	0.764	0.643	172.02*	0.679	0.733	100.50	0.877	0.476	93.75			
	-0.243	0.972		-0.267	0.952		-0.136	1.006				
	FRUS						1st subs			2nd subs		
$D_u$	1.001	0.213	192.8***	1.028	0.200	106.1**	0.972	0.228	106.0**			
	0	0.979		0	0.968		0	0.989				
$D_l$	0.978	0	211.7***	1.007	0	115.1**	0.947	0	111.7**			
	0.213	1.002		0.208	0.989		0.218	1.015				
$D_s$	0.995	0.107	197.3***	1.023	0.103	111.1**	0.965	0.112	106.6**			
	0.107	0.996		0.103	0.983		0.112	1.009				
$D_0$	0.930	0.302	187.26**	1.009	0.280	104.96*	0.900	0.293	97.20			
	-0.107	1.018		-0.076	0.965		-0.106	1.033				
	UKUS						1st subs			2nd subs		
$D_u$	1.001	0.159	188.1***	0.991	0.220	96.20	1.011	0.097	110.4**			
	0	0.989		0	0.964		0	1.010				
$D_l$	0.988	0	180.5**	0.966	0	107.0**	1.006	0	100.4			
	0.159	1.002		0.220	0.989		0.097	1.015				
$D_s$	0.997	0.080	176.7**	0.984	0.111	100.9*	1.010	0.049	102.5*			
	0.080	0.998		0.111	0.983		0.049	1.014				
$D_0$	0.985	0.078	176.5*	0.931	0.337	93.94	1.010	-0.031	100.25			
	0.080	1.011		-0.122	0.981		0.128	1.007				
	UKFR						1st subs			2nd subs		
$D_u$	1.001	0.293	216.5***	0.991	0.408	110.2**	1.011	0.175	98.87			
	0	0.957		0	0.944		0	0.956				
$D_l$	0.957	0	181.8**	0.909	0	103.6*	0.994	0	86.73			
	0.293	1.001		0.393	1.028		0.182	0.972				
$D_s$	0.990	0.148	185.9***	0.969	0.205	104.6**	1.007	0.090	82.92			
	0.148	0.990		0.205	1.008		0.090	0.967				
$D_0$	0.998	0.070	176.93*	0.990	-0.027	103.44	0.991	0.078	82.83			
	0.225	0.975		0.434	0.932		0.106	0.983				

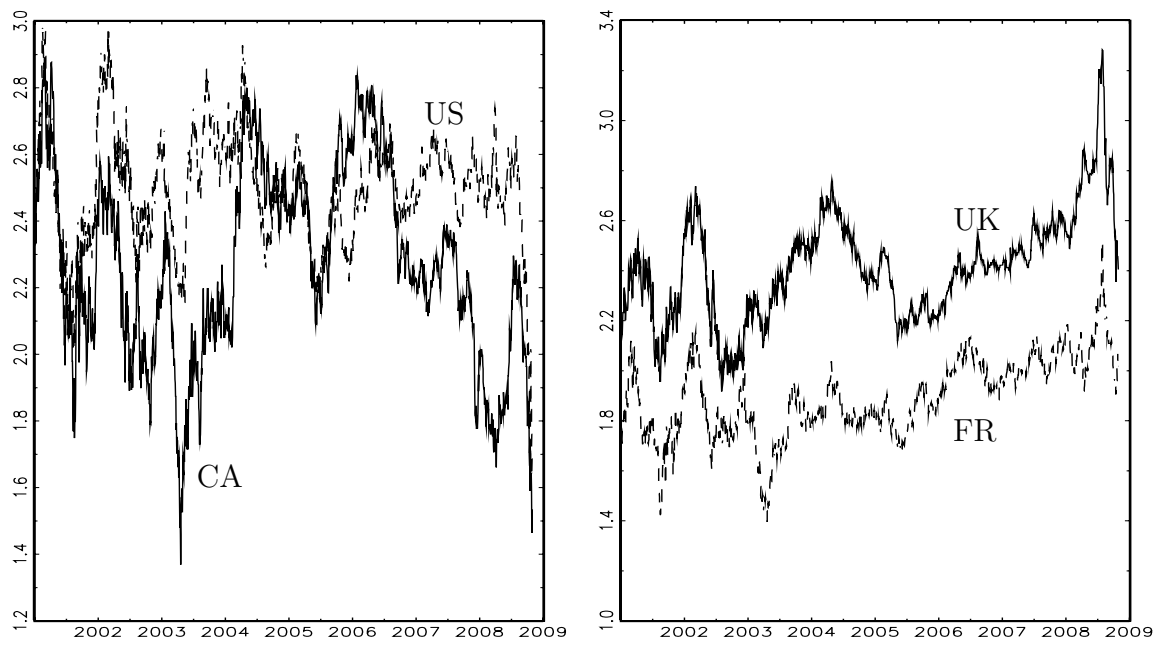
Table 3: ITSI estimates and diagnostics for bivariate systems of breakeven inflation. Matrices  $D_u$ , ( $D_l$ ),  $D_s$ , and  $D_0$  refer to upper (lower) triangular schemes, symmetric impact relations and impact relations resulting in minimum dependence of structural relations. Full samples comprises 1956 daily observations. 1st and 2nd subs refer to almost equal sized subsamples covering 1000 and 956 observations, respectively. Significant bootstrap based diagnostics against the independence assumption are indicated with \*\*\* (1% significance), \*\* (5% significance) and \* (10% significance). For further notes see Table 2.

	TFP	SP	TFP	SP	$\Delta$ TFP	$\Delta$ SP	$\Delta$ TFP	$\Delta$ SP
TFP(-1)	0.944 (13.7)	0.165 (0.42)	0.945 (13.8)	0.227 (0.57)	-0.036 (-0.51)	0.295 (0.73)	-0.024 (-0.35)	0.217 (0.55)
TFP(-2)	0.040 (0.58)	-0.414 (-1.06)	0.045 (0.66)	-0.225 (-0.57)	0.085 (1.24)	-0.603 (-1.54)	-	-
SP(-1)	0.057 (4.66)	1.283 (18.6)	0.058 (4.99)	1.346 (19.8)	0.054 (4.35)	0.367 (5.17)	0.058 (4.87)	0.345 (5.09)
SP(-2)	-0.055 (-4.47)	-0.288 (-4.12)	-0.057 (-4.83)	-0.354 (-5.15)	0.007 (0.53)	-0.042 (-0.56)	-	-
$c$	0.021 (1.20)	-0.030 (-0.31)	0.019 (1.14)	-0.072 (-0.73)	0.003 (3.71)	0.008 (1.82)	0.003 (4.33)	0.005 (1.36)
$t$	2.37E-05 (0.53)	0.001 (3.33)	-	-	-	-	-	-
LM 10	3.180 (0.53)		2.620 (0.62)		3.512 (0.48)		3.152 (0.53)	
JB	95.12		80.03		89.85		90.03	

Table 4: VAR parameter estimates for bivariate systems of US total factor productivity and stock prices (Beaudry and Portier 2006). Model selection criteria (AIC, BIC, HQ, not documented) are in favor of a VAR order two if the model is specified in levels and contains a linear trend). VAR parameter estimates are documented with  $t$ -ratios in parentheses. LM is the Lagrange Multiplier test ( $p$ -value in parentheses) for multivariate serial correlation up to lag order 10. JB is the Jarque-Bera statistic on joint normality of both reduced form residual processes. Under the null hypothesis of normality JB is  $\chi^2$  distributed with 4 degrees of freedom,  $p$ -values are not provided. Dependent variables are listed in the top row, where  $\Delta$  is short for the first difference operator. The left hand side column lists the conditioning variables from which TFP and SP are either in levels or in first differences. Moreover,  $c$  and  $t$  signify a constant and a trend, respectively, entering the VAR. Estimation and diagnostic results are obtained from Eviews 6.0.

$D_{\bullet}$	$\widehat{D} \cdot 1000$		$\mathcal{L}_{\bullet}$	$\widehat{D} \cdot 1000$		$\mathcal{L}_{\bullet}$
	levels, with trend			levels, no trend		
$D_u$	9.241	8.293	83.29***	9.248	8.770	85.31***
	0	51.50		0	52.95	
$D_l$	9.123	0	21.12	9.123	0	22.81
	1.469	52.16		1.511	53.67	
$D_s$	9.156	1.250	22.50	9.157	1.291	21.29
	1.250	52.14		1.291	53.66	
$D_0$	9.095	0.716	21.10	9.157	1.291	21.29
	-2.628	52.11		1.291	53.66	
	1st diff, VAR(2)			1st diff, VAR(1)		
$D_u$	9.344	9.532	92.10***	9.395	8.774	86.76***
	0	52.51		0	53.08	
$D_l$	9.194	0	23.57	9.270	0	23.86
	1.669	53.37		1.532	53.80	
$D_s$	9.235	1.423	23.91	9.304	1.307	24.43
	1.423	53.35		1.307	53.78	
$D_0$	9.081	-1.438	22.81	9.155	-1.450	22.62
	9.997	52.45		9.929	52.90	

Table 5: ITSI estimates and diagnostics for the US system comprising reduced form errors of TFP and stock prices. For further notes see Table 4 and Table 3



**Figure 1:** Liquidity adjusted breakeven inflation rates for Canada and the US (left hand side) and France and the UK (right hand side).